DEPARTMENT OF MATHEMATICS

MATHS 190 **Lecture 14 Addon**

A HARD GALLERY

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In the lecture I was asked if an art gallery which looks similar to the one below was a counterexample to the *art gallery theorem*. Given a number of vertices v, according to the art gallery theorem as few as $\lfloor v/3 \rfloor$ guards should be enough to protect the gallery. For the example shown below this would be $\lfloor v/3 \rfloor = \lfloor 11/3 \rfloor = 3$. However, if we do not get it right it might seem to us that we need many more guards than these three.

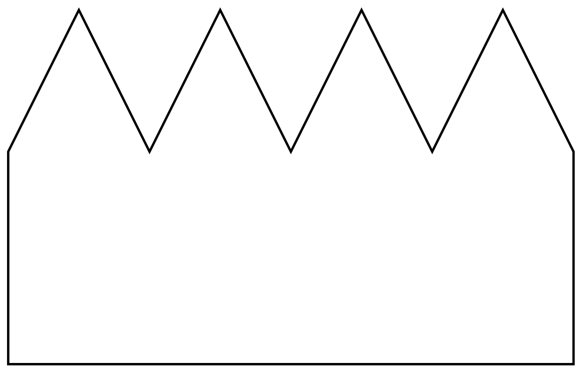

FIGURE 1. The comb-shaped challenge art gallery. How many guards do we need? Four, three or even as few as... TWO?

To check a question like this is really easy: While finding the best *minimal number of guards* might be hard for some floorplans, here we only have to go through the steps which we followed during the proof. So the first thing we have to do is to find a *triangulation* of our art gallery like the one below, see Figure 2. Remember that it is not important which kind of triangulation we choose because in the proof it was also enough to choose *any* triangulation. The only thing which we have to keep in mind is that the edges of our triangulation (i.e. the dashed lines in Figure 2) do not cross each other—otherwise we would get new vertices *inside* our art gallery.

After the triangulation we have to colour the vertices in three colours so that each triangle has vertices in all colours. For this example we

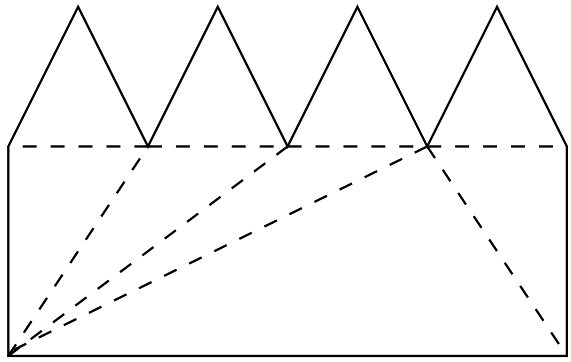

FIGURE 2. A triangulation of the comb-shaped art gallery. There are really no good or bad ones. In the proof we saw that any triangulation will do.

chose blue, yellow and red. Like with the triangulation it is not important how we colour the vertices as long as we have three different colours in each triangle.

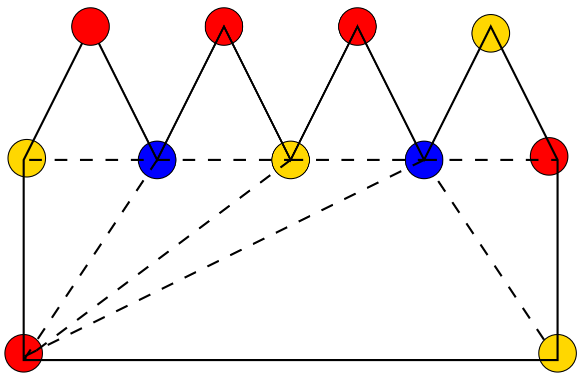

FIGURE 3. The colouring of the triangulated art gallery leads to a surprise: If we place guard at the blue vertices we even need just *two*! So... this gallery was not even the hardest gallery we could come up with!

Looking at the results of our colouring leads to a surprising discovery: If we place guards at the blue vertices it shows that we only need *two* rather than three! One can easily see that these two guards can "see" the whole gallery. So this gallery is not even the hardest gallery with eleven vertices which we can build.