DEPARTMENT OF MATHEMATICS
MATHS 761 Laboratory 4 notes

In this laboratory session we are going to use XPP for iterating maps. For discrete dynamical systems, the .ode file is similar to that of an ordinary differential equation. Instead of integrating the equations, XPP iterates them.

1. In this question you are going to investigate the stability of fixed points in linear two-dimensional maps. Write a .ode file for the map

$$x_{n+1} = ax_n + by_n \quad (1)$$
$$y_{n+1} = cx_n + dy_n \quad (2)$$

One way to do this is as follows:

```
#linear map
x(t+1)=a*x+b*y
y(t+1)=c*x+d*y
par a=0.5,b=0,c=0,d=2
x(0)=0.5
y(0)=0.5
@ meth=discrete
@ total=50
done
```

The line `@ meth=discrete` tell xpp that the equation is a discrete dynamical system.

Let A be the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. For the following matrices A, first calculate the stability of the zero fixed point using the methods discussed in class, then check your answers using XPP.

(a) $A = \begin{pmatrix} 0.5 & 0 \\ 0 & 2 \end{pmatrix}$

(b) $A = \begin{pmatrix} -0.9 & 0 \\ 0 & -1.1 \end{pmatrix}$

(c) $A = \begin{pmatrix} 0.375 & 0.125 \\ 0.125 & 0.375 \end{pmatrix}$

(d) $A = \begin{pmatrix} -0.4 & -1.6 \\ -0.8 & 0.4 \end{pmatrix}$

(e) $A = \begin{pmatrix} -0.25 & 0.75 \\ 0.75 & -0.25 \end{pmatrix}$

(f) $A = \begin{pmatrix} 0.7 & 0.7 \\ -0.7 & 0.7 \end{pmatrix}$

(g) $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Notes:

- You can find the fixed point and its stability in XPP in the same way as for continuous systems.
- You can change the linestyle to dots ((G)raphic stuff, (E)dit curve, hit Enter, then enter 0 in the Line type box). You may or may not find this helpful, depending on which sort of map you are investigating.

2. Write a .ode file for the Hénon map:

$$x_{n+1} = y_n + 1 - ax_n^2$$
$$y_{n+1} = bx_n$$

(a) Start with $a = 1.4$, $b = 0.3$ and plot a picture of 1000 iterates of x_n and y_n.

(b) Investigate changing the parameters to see what different sorts of behaviour you can find. You might like to try some of more or the following:

i. Find all fixed points, and show that they exist only if $a > a_0$, where a_0 is to be determined.

ii. Calculate the Jacobian, and find the eigenvalues.

iii. Show that one of the fixed points is always unstable.

iv. Investigate numerically what happens by keeping $b = 0.3$ and varying a between a_0 and $a \approx 1.1$.