Automorphism groups of free groups: Properties, Presentations and Image-Restrictions

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In the study of free groups the automorphism group plays an important part and one way to get to know more about a group is to find a presentation for it. In his paper "Die Isomorphismengruppe der freien Gruppe" Jakob Nielsen was the first one to state a presentation for the automorphism groups of the free groups. From this time on various presentation had been stated, improving Nielsen presentations in many perspectives. In 1932 B.H.Neumann proved that for $n \geq 4$ $Aut(F_n)$ is generated by two elements and in 2008 M.F. Newman filled the gap by proving it for $n = 2$ and $n = 3$, where Nielsen needed 4 generators. Another approach made by K.Vogtmann in her 2008 paper "A presentation for $Aut(F_n)$" provides a presentation for $Aut(F_n)$ with $n + 1$ generators which are all of order 2. But up till now all those presentations weren't accessible in a computationally useful manner. My first aim was to get this done, so that for a given n magma would return a presentation for $Aut(F_n)$ and the related homomorphisms which generate $Aut(F_n)$. With those presentations at hand one can use Mamga to explore the subgroups of small index and the surjective images of the automorphism groups. During those exploration one realises that there seems to be some kind of restriction which images turn up and in their 2003 paper "Homomorphisms form automorphism groups of free groups" Bridson and Vogtmann prove that for $n \geq 3$ the homomorphic image of $Aut(F_n)$ either has cardinality at most 2 or contains an isomorphic image of the symetric group S_{n+1}. Since this paper doesn't provides any restrictions for images of $Aut(F_2)$ the question aroses which images do turn up. Computations on that matter show gaps in the orders of the images as well, suggesting there should also be some restriction. Addtionally those presentation can be used to explore other open question like if for $n \geq 4$: $Aut(F_n)$ has a finite-index subgroup G with infite abelianization.