Maths 761 Notes for Laboratory 9

In Lecture 16 and Worksheet 6, the following system was considered.

$$\begin{aligned} \dot{x} &= \lambda x + 2xy + xy^2, \\ \dot{y} &= 1 - x^2 - y^2. \end{aligned}$$

1. Use XPPAUT to draw a bifurcation diagram for this system as λ varies. You should find four local bifurcations, two pitchfork bifurcations and a pair of Hopf bifurcations (both Hopf bifurcations occur at the same parameter value but at different points in the phase space). Try various options for the vertical axis of your bifurcation diagram to ensure that all the equilibrium solutions are seen in your bifurcation diagram.

2. The Hopf bifurcations occur at $\lambda = 0$. We would like to follow the branches of periodic solutions arising from the Hopf bifurcations. To do this, Grab a point at which there is a Hopf bifurcation, then choose Run/Periodic to follow the periodic orbit.

   Look in the terminal window that accompanies your AUTO window (this is a black window that usually appears on your screen behind the XPP and AUTO interfaces). This window contains information about the solutions that AUTO is following, including the positions of the equilibria (if AUTO is following a branch of equilibria) and the period of a periodic orbit (if AUTO is following a branch of periodic orbits). What happens to the period of the periodic orbit as you move away from the Hopf bifurcation? Note that to get a nice bifurcation diagram you may need to set NPR=1 in the numerics menu in AUTO. (What does this constant do?)

   You can choose to plot the period of the orbit on the vertical axis of your bifurcation diagram. To do this, choose Period from the options on the axes menu, change YMAX to something large like 100, click OK, then click Redraw. (Note that when you do this, all equilibrium solutions will appear with period zero (i.e., on the λ axis).

   Show your bifurcation diagrams to your lecturer or tutor before going any further.

3. Now use Grab to get a point on one of the Periodic orbit branches. This also sets the corresponding parameter value in the main XPP window. Set up an X vs. Y coordinate system in the main XPP window, with ranges $[-2, 2]$ for both coordinates. In AUTO, use File/Import orbit to put the orbit into the XPP phase space window. Click Restore in the main XPP menu to plot the orbit once it has been imported. Use Initial Conds/Mice to draw more of the phase space diagram.

4. Work out where and how the branch of periodic orbits terminates as λ varies. (One end of the branch will be at the Hopf bifurcation – where is the other end?) Draw some phase portraits showing the qualitative changes that occur near the bifurcation at the end of the branch.