190 MID-SEMESTER TEST (SEM. 2 2010)

① a) A prime number is a number only divisible by 1 and itself.

b) i) 1

ii) E.g. $p_1 = 2$, $p_2 = 3$ $\Rightarrow m = 7$ prime

$p_1 = 3$, $p_2 = 5$ $\Rightarrow m = 16$ not prime.

c) All we know is that none of the numbers $p_1, p_2, \dots p_{100}$ are divisors of n. It could be prime or not prime.

② a) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

b) i) 1, 1, 1, 3, 5, 9, 17, 31, 57, 105

ii) $P_n > F_n$ for $n = 6, 7, 8, 9, 10$

iii) Yes. Since $P_n = P_{n-1} + P_{n-2} + P_{n-3}$

$F_n = F_{n-1} + F_{n-2}$

we know that for $n = 8, 9, 10$, P_{n-1} and P_{n-2} are greater than F_{n-1} and F_{n-2}, so the sum $P_{n-1} + P_{n-2}$ will be greater than $F_{n-1} + F_{n-2}$. Adding P_{n-3} makes P_n even bigger (but is not in fact necessary at this point).

The statement then follows by induction.

(3) a) $39/8 = 4\ 7/8$. There must be at least 5 of one colour (if there were only 4 of each there could only be $4 \times 8 = 32$ sweets).

b) 0. They could all be blue.

c) $39/2 = 19\ 1/2$. One of Claire & Lou must get 20 sweets (if they only had 19 each would only be $19 \times 2 = 38$ sweets).

d) 0. Claire could get all the blue ones.

(4) a) An infinite set is one for which there are more members than any finite number n.

b) Pair items from the sets in a one-to-one correspondence until you run out of objects from one set. Whichever has objects left is the largest.

c) There is a one-to-one correspondence, given by, e.g.

$$\begin{array}{cccccc} \{1 & 2 & 3 & 4 & 5 & \ldots, \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ \{1, & -1, & 4, & -4, & \cancel{25}\ 9, & -9 \ldots \end{array}$$

etc.

d) Proof by contradiction:

- Assume that set of real numbers in [0, 1] has same cardinality as $\{1, 2, 3, \ldots\}$
- Then there is a one-to-one correspondence, so I can write a list: e.g.

$$
\begin{aligned}
1 &\Leftrightarrow 0.\underline{1}111111\ldots\ldots \\
2 &\Leftrightarrow 0.0\underline{1}01010\ldots\ldots \\
3 &\Leftrightarrow 0.12\underline{3}4567\ldots\ldots \\
4 &\Leftrightarrow 0.314\underline{1}519\ldots\ldots \\
5 &\Leftrightarrow 0.5000\underline{0}00\ldots\ldots \\
&\vdots
\end{aligned}
$$

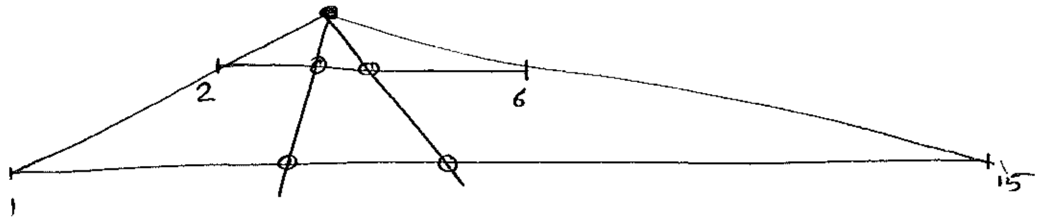
- ~~I can~~ Then consider the number formed by looking at the diagonal elements (underlined), and replacing these with a "2" if the number is a "4" and a "4" if the number is <u>not</u> a "4". In this example the number is

0.4444....

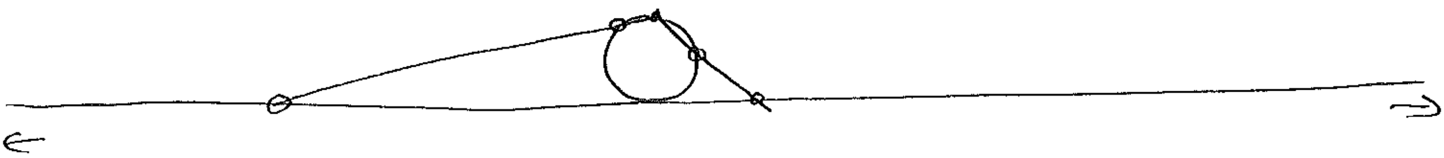
- This number is not in the list since it differs from each number in at least one position, but it is in the interval [0, 1].

- Contradiction! $\Rightarrow$ [0, 1] has a larger cardinality than $\{1, 2, 3, \ldots\}$.

(5) (a) [2, 6] and [-1, 15] have the same cardinality, since a one-to-one correspondence between pts in each interval can be found:



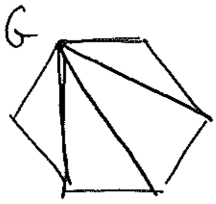
(b) Take a line segment and roll it into a circle with one point missing:



draw the real line underneath.

A one-to-one correspondence can be found be drawing lines from the top of the circle to the real line, as shown.

(6) (a)



The gallery can be triangulated so each triangle has one corner at G and another corner at all other corners of the gallery.

(b) 1, because G is a vertex of every triangle in the triangulation.

(c) 1. You can triangulate the gallery so that the corner from which you can see all other corners is a vertex in every triangle. This is the corner at which the guard should be placed.