DEPARTMENT OF MATHEMATICS

MATHS 190 Lecture 18 Summary

In this lecture we illustrate two important observations:

- Simple repeating rules can lead to unexpected complex behaviour.
- A small change in where you start can lead to huge differences in where you end up.

Lecture 18 was based around the following question: **Why can we predict the tides but not the weather?**

It is very difficult to predict the weather over long periods of time. We started by discussing why this should be so. We decided that very complex systems should be very difficult to predict. But what about very simple systems? Can we predict those any better?

We discussed calculator precision, and agreed that calculators can only represent numbers to a certain accuracy. Different calculators will have different approximations to the same number. This gives us a clue that calculator arithmetic is not always very reliable.

We then took a simple arithmetic rule and repeated it 25 times on a calculator. Then we restarted the calculation from the middle and got a completely different result. Weird, huh? What's going on? Both the calculators are 'correct' but they don't agree.

This is called **extreme sensitivity to initial conditions**.

One problem with predicting the weather is that we do not understand the "state" of today's global weather accurately enough to be able to determine the future state (even if we understood the equations perfectly and could solve them quickly). For more detail you can read about the famous Lorenz equations.

The **butterfly effect** is the hypothesis that the flapping of a butterfly's wings in Hong Kong can cause a tornado in Germany. (Well, at least theoretically. Does this actually happen? I doubt it.)

Before you come to the next lecture: You should spend an hour or two thinking and reading about the ideas presented in the lecture. You should also:

- Read section 5.1

Other activities you could do if you have time are:

- Try out the calculator game on your friends or family. Do they understand just how wrong computers can be?
- Look up the Lorenz equations online and learn more about them.