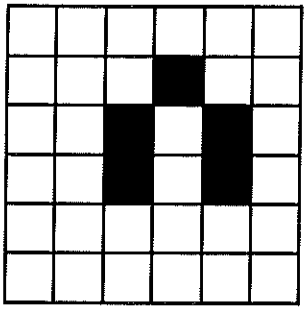
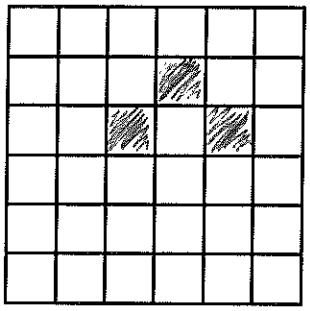
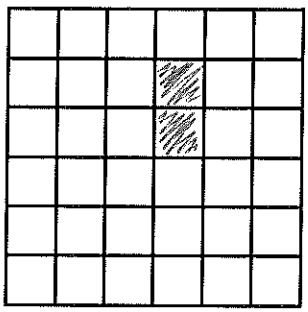
Department of Mathematics

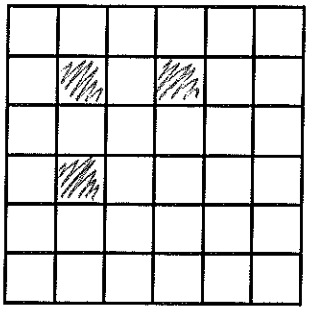
Maths 190 Assignment 4 Solutions

October 29, 2010 Due:

1. (10 marks)
   - A, J
   - B, F
   - C, E
   - D, H
   - G
   - I
2. (6 marks)
   (a) The object has two sides. When you cut it in half, you get two cylinders linked together, each with a double twist (720 degress) in it.
   (b) The object has one side and one edge. When you cut it in half you get a cylinder with lots of twists in it and a knot .
3. (6 marks)
   (a) The next two generations are:

   (b) For example:

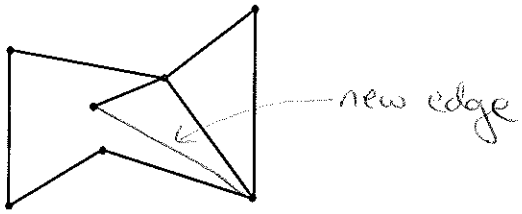


4. (10 marks)

(a) The graph has 7 vertices and 8 edges.

(b) Two of the vertices have odd order (number of edges), so there cannot be an Euler circuit.

(c) Only one edge needs to be added



(d) $V - E + F = 2$

The graph has 3 faces. So for this case we have 7-8+2=2, so Euler's formula is satisfied.

(e) If you add a new edge between two existing vertices, the number of faces increases by one, and the number of vertices stays the same.

Hence, E and F both increase by 1, ie. $E' = E+1$, $F' = F+1$ and $V' = V$ so in Euler's formula there is a cancellation and $V'-E'+F' = V-(E+1)+(F+1) = V-E+F = 2$.

Hence Euler's formula is still satisfied for the new graph.