Conformal and projective geometries of Einstein manifolds

Heather Macbeth

Supervisor:

A. Rod Gover
The University of Auckland

Einstein manifolds are of fundamental importance in differential geometry. To an Einstein manifold, as to any manifold equipped with metric, we can associate natural conformal and projective geometries. Roughly speaking, an Einstein manifold's conformal geometric structure is obtained by forgetting all data about lengths and volumes, leaving only knowledge of angles. Its projective geometry, on the other hand, is obtained by forgetting all data except what is needed to characterize straight lines. The tractor calculus of Eastwood, Gover et al. provides a framework for studying conformal and projective geometries.

In this work we use the tractor calculus to investigate the conformal and projective geometries of Einstein manifolds, and their relationships. In particular, we show here that an Einstein manifold's dual projective tractor bundle $\mathbf{T}^*_{\text{proj}}$ can be naturally embedded as a rank-$(n+1)$ sub-bundle of the manifold's conformal tractor bundle $\mathbf{T}_{\text{conf}}$. Restricting the conformal tractor metric on $\mathbf{T}_{\text{conf}}$ to $\mathbf{T}^*_{\text{proj}}$ therefore induces a canonical metric on $\mathbf{T}^*_{\text{proj}}$.

This construction motivates, and generalizes, the recent observation of Armstrong that there is a natural equivalence between the existence of a metric on a projective geometry's tractor bundle, and the existence of Einstein, non-Ricci-flat metrics consistent with that projective geometry.