

- ▶ Topic for today: **Numerical Patterns in Nature**

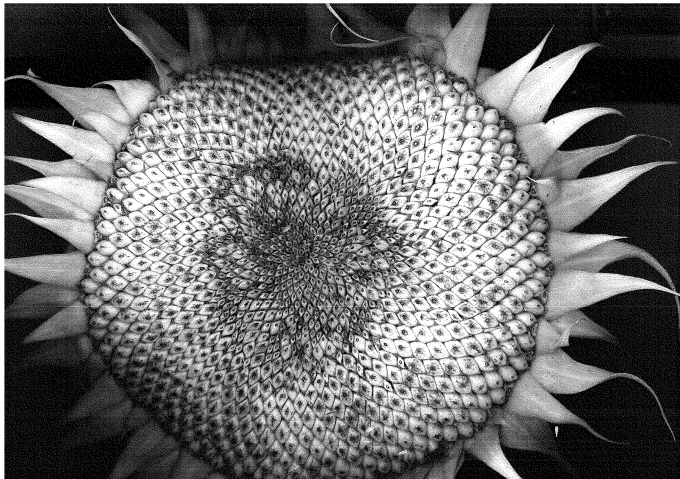
- ▶ **Vitally important question:**

What do rabbits, pineapples, daisys, sunflowers and pinecones have in common?

Pineapples and pinecones

- ▶ Look carefully at your pineapple.
- ▶ How many spirals does your pineapple have?

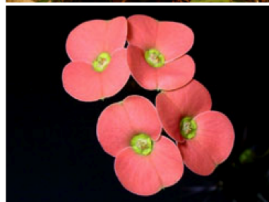
How many spirals in a sunflower?



How many spirals in a pinecone?



How many petals on a flower?



Is there a pattern?

- ▶ Put the numbers we have found in order.
- ▶
- ▶ Can you spot a pattern?

Fibonacci Numbers

- ▶ 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .
- ▶ Call these F_1, F_2, F_3 , etc
- ▶ Then

$$F_n + F_{n+1} = F_{n+2}$$

- ▶ This is an example of a *recurrence relation*.

She loves me, she loves me not...?

- ▶ Pulling petals off a flower, we'd like to know our chances of getting an odd number - "She loves me"!
- ▶ Most flowers have a Fibonacci number of petals.
- ▶ How many of the Fibonacci numbers are odd?

Which are odd?

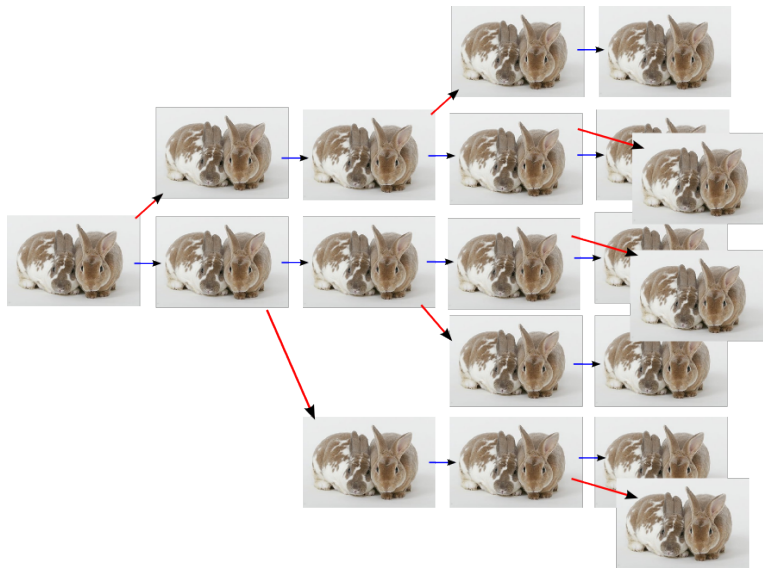
- ▶ 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...
- ▶ Looks like every third one is even.
- ▶ How can we show/prove this is always true?

What about rabbits?



- ▶ Suppose we start with one breeding pair of rabbits.
- ▶ Each pair of rabbits produces one pair of offspring each month.
- ▶ It takes the rabbits one month to mature.
- ▶ How many rabbits do we have?

How many rabbits?



The Golden Ratio

- ▶ What is the ratio F_{n+1}/F_n as n gets large?

n	F_n	F_{n+1}	$\frac{F_{n+1}}{F_n}$
1	1	1	$\frac{1}{1} = 1$
2	1	2	$\frac{2}{1} = 2$
3	2	3	
4	3		
5	5		
6	8		
7	13		
8	21		

The Golden Ratio

- ▶ Seems to converge...
- ▶ The Golden Ratio ψ .

Which is your favourite rectangle?



Fibonacci Building Blocks

- ▶ Claim: Every natural number is either a Fibonacci number, or can be written as the sum of Fibonacci numbers.
- ▶ Try it!

- ▶ In fact, every natural number can be written as the sum of *non-consecutive* Fibonacci numbers.
- ▶ Use a “greedy” algorithm:
 - ▶ subtract the biggest possible Fibonacci number to get a smaller number.
 - ▶ repeat as required.

Important ideas from today:

- ▶ Certain patterns, and sequences of number reappear throughout nature. One common one is the Fibonacci sequence.
- ▶ Rabbits breeding, the numbers of spirals on pinecones and daisies, all follow the Fibonacci sequence.

For next time

- ▶ Read 2.3 in the textbook, on prime numbers.