

**DEPARTMENT OF MATHEMATICS**  
**MATHS 761 Worksheet 4 - Discrete dynamical systems**

1. For the following maps, find all fixed points and determine their stabilities. Describe the behaviour of orbits near each fixed point and draw a diagram showing some typical orbits near each fixed point.

(a)

$$x_{n+1} = 12x_n^2 - 1$$

(b)

$$\begin{aligned}x_{n+1} &= x_n(1 + x_n + y_n) \\ y_{n+1} &= -y_n x_n - 6\end{aligned}$$

(c)

$$\begin{aligned}x_{n+1} &= x_n(1/3 + y_n) \\ y_{n+1} &= y_n(4 - x_n^2)\end{aligned}$$

2. (a) Show that if  $\lambda$  is an eigenvalue of  $\mathbf{A}$  with corresponding eigenvector  $\mathbf{v}$  then  $\mathbf{y}_n = \lambda^n \mathbf{v}$  is a solution to the difference equation  $\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n$ , where  $\mathbf{A}$  is a real matrix.
- (b) Show that if  $\lambda$  is an eigenvalue of  $\mathbf{A}$  with multiplicity 2 and one eigenvector  $\mathbf{v}$  and one generalised eigenvector  $\mathbf{p}$  then  $\mathbf{y}_n = \lambda^{n-1}(n\mathbf{v} + \lambda\mathbf{p})$  is a solution to the difference equation  $\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n$ .