Maths 190 Lecture 19

- **Topic for today**: Rubber sheet geometry
- **Question of the day**: Can you take off your vest without taking off your jacket?

Rubber band shapes

A rubber band can be distorted into a triangle, square, etc.

A cut rubber band can be distorted into the letters S, U, V, etc

Rubber band shapes

Can either rubber band be distorted into the letters P, R, E, F or H?

Equivalence by distortion

We can

1. stretch
2. shrink
3. bend
4. twist (this one only makes sense when discussing higher dimensions, such as surfaces in space)

but not

1. cut
2. glue

This is an important notion in mathematics which underlies many theories and applications.

Equivalence by distortion

Group the following symbols into sets where all members are equivalent by distortion.

A B e f g P R T X 2 3 6 8 @ +

Properties preserved by distortion

To show that two objects are equivalent involves an explicit description of the necessary distortions.

To show that two objects are **not** equivalent, find a property of one of the objects which is not shared by the other and which is preserved by distortions.

For example, why are 4 and 8 not equivalent by distortion?

From lines to surfaces

Now imagine a sheet of rubber or stretchy cloth which can be stretched, shrunk, bent and twisted.

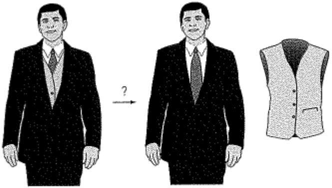
We can ask when shapes formed from such a material are equivalent by distortion.

For example, folding a sheet of paper is an equivalence by distortion, as long as you don't glue or cut.

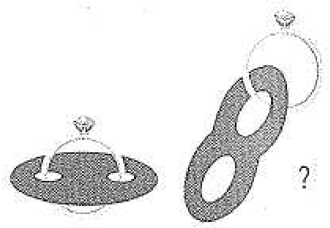
A topologist cannot tell her coffee cup from her doughnut!

Questions

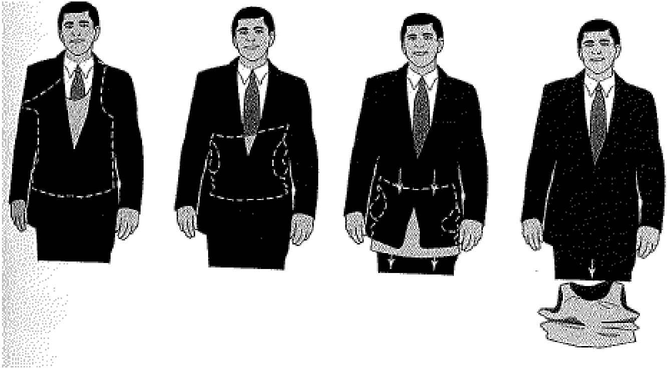
1. Can you remove your vest without removing your jacket?



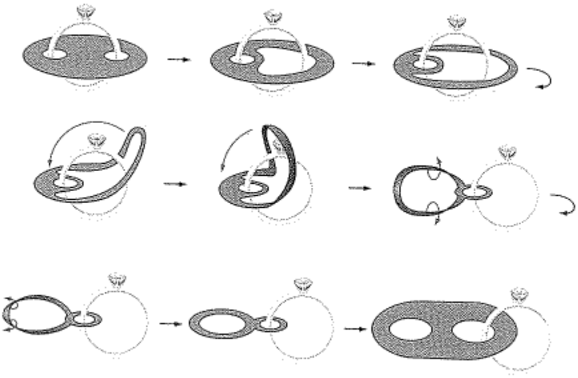
2. Can you take make the ring pass through only one hole?



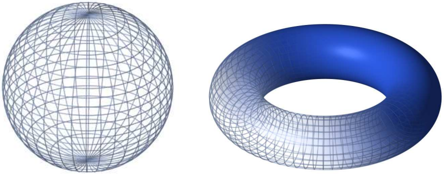
How to remove your vest



The problem of the rings



Is a torus equivalent to a sphere?



Stress that these are surfaces, not solids.

Properties preserved by distortion

What are some features of a shape which are preserved by distortion?

Is there such a feature which is different for the sphere and the torus?

Poincaré conjecture

The sphere has the property that every loop on it can be "shrunk to a point".

The torus does not have this property.

It is a theorem that every connected, bounded surface (2-manifold) with no boundary, such that every loop can be shrunk to a point, is equivalent by distortion to the sphere.

Poincaré asked the question, in around 1900, of whether the same holds in "higher dimensions". This was proved by Smale in 1961 for dimension > 4. In 1982, Freedman proved it in dimension 4. The case of dimension 3 remained open until 2003.

Important ideas from today

We tried to get an understanding of the intrinsic shape or geometry of objects. This branch of mathematics is called **topology**.

To show that two objects (e.g., the sphere and the torus/doughnut) were not equivalent we found a property which is preserved by distortion.

Read Sections 5.1 and 5.2 of the book.