Maths 190 Lecture 8

- **Topic for today:** More on Infinity

- **Question of the day**:
  How much bigger is the set of rational numbers than the set $\{1,2,3,\ldots\}$?

Recall from last lecture

Two sets are of the same size if and only if there is a one-to-one correspondence between members of one set and members of the other set.

Examples

- $\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\}$

  $\{2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$

- $\{1,\ 2,\ 3, \dots\}$

  $\{2,\ 3,\ 4, \dots\}$

- $\{\ 1,\ \ 2,\ \ 3,\ \ 4,\ \ 5,\ \ 6,\ \ 7\}$

  $\{-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3\}$

- $\{\ 1,\ \ 2,\ \ 3,\ \ 4,\ \ 5,\ \ 6,\ \ 7, \dots\}$

  $\{\dots, -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3, \dots\}$

One-to-one pairings are frequently shifts or shuffles.

The ping-pong ball conundrum

We do a thought experiment lasting exactly 60 seconds.

Start with a large empty barrel and a long line of ping-pong balls, numbered in order 1,2,3,. . . . Start the clock.

- In the first 30 seconds, put the first 10 balls into the barrel (numbers 1-10), find number 1 and throw it out.
- In half the remaining time (15 s), put the next 10 balls into the barrel (numbers 11-20), find number 2 and throw it out.
- In half the remaining time (7.5 s), put the next 10 balls into the barrel (numbers 21-30), find number 3 and throw it out.
- Continue in this way until 60 seconds has passed, then stop.

How many balls are now in the barrel?

Ping-pong ball variation

- What is the outcome of the ping-pong ball experiment if the balls are not numbered? Discuss this in pairs.
- (You could imagine that the balls are numbered with invisible ink that only the experimenter can see.)

Are there any sets bigger than $\{1, 2, 3, \dots\}$?

- Is $\{\dots, -2, -1, 0, 1, 2, \dots\}$ bigger than $\{1, 2, 3, \dots\}$?

Is the set of all rational numbers (fractions) bigger than $\{1, 2, 3, \ldots\}$?

Important ideas from today:

- Ideas about infinity are often counter-intuitive - intuition based on finite sets does not always work for infinite sets.
- The idea of one-to-one correspondence developed with finite sets does carry over to infinite sets and is our main tool for comparing the size of infinite sets.
- The set of all fractions (rational numbers) is the same size as the set $\{1, 2, 3, \dots.\}$ (natural numbers).

For next time

- Read §3.2 in the textbook.
- Try some Mindscapes at the end of §3.2 of the textbook.
- Review the game of Dodgeball from Lecture 1 (Story 5 in section 1.1 of the textbook).
- Try to explain the ping-pong ball conundrum to a friend who is not in Maths 190.