VERSION 1

MATHS 190 and 190G

THE UNIVERSITY OF AUCKLAND

SEMESTER TWO, 2008
Campus: City

MATHEMATICS

Great Ideas Shaping Our World

(Time allowed: TWO hours)

NOTE:

- Attempt **ALL** questions.
- Section A contains 30 questions, each worth 2 marks, making a total of 60 marks.
  Section B contains 6 questions, each worth 10 marks, making a total of 60 marks.
- For Section A write your answers on the coloured answer sheet attached as Page 13.
- For Section B write your answers in the answer book provided.

  All working must be shown.
  Start each question on a new page.
- Please do not use **red ink** in your solutions.
- Calculators are allowed for this exam.
  Stored memory on programmable calculators must be cleared before the exam starts.

SECTION A: MULTI-CHOICE

Detach the coloured answer sheet and write your name and ID number on it.
Mark your answers with crosses in the grid provided.
Do not write your answers to Section A in the answer book.

1. Mary came back from a birthday party with 31 candies, of three different flavours. We can be sure that she has at least □ candies of one flavour (answer with the largest possible number).

(a) 1 (b) 10 (c) 11 (d) 29

2. Mary decides to divide all her 31 candies between herself and her two siblings. We can be sure that every kid has at least □ candies.

(a) 1 (b) 10 (c) 11 (d) 29

3. Suppose that we have infinitely many candies, but only a finite number of flavours. Which one of the following statements is ALWAYS TRUE?

(a) There are infinitely many candies of each flavour.

(b) There are infinitely many candies of one flavour.

(c) There are finitely many candies of one flavour.

(d) That cannot happen, if there are infinitely many candies, there have to be infinitely many flavours.

4. How many positive divisors does 10 have?

(a) 1 (b) 2 (c) 3 (d) 4

5. In how many different ways can we write 3^4 as the sum of even numbers?

(a) 0 (b) 1 (c) 2 (d) 3

6. In how many different ways can we write 81 as the product of primes?

(a) 0 (b) 1 (c) 2 (d) 4

7. Recall that a 13 digit bar-code $d_1d_2d_3d_4d_5d_6d_7d_8d_9d_{10}d_{11}d_{12}d_{13}$ is valid if the following holds:

$$d_1 + 3d_2 + d_3 + 3d_4 + d_5 + 3d_6 + d_7 + 3d_8 + d_9 + 3d_{10} + d_{11} + 3d_{12} + d_{13} \equiv 0 \mod 10$$

What should the last digit of the bar-code 9 40055064620□ be?

(a) 4 (b) 5 (c) 6 (d) 7

8. Let S be a set of numbers, and let x be a number that is not in S. Let $T = S \cup \{x\}$. Then $|\mathcal{P}(T)| = \square|\mathcal{P}(S)|$. What should $\square$ equal, to this to be TRUE?

(a) 1

(b) 2

(c) 4

(d) None of the above.

9. Let S be a set of numbers, and let x be a number that is not in S. Let $T = S \cup \{x\}$. Which of the following statements is TRUE?

(a) The cardinality of S is smaller than the cardinality of T.

(b) The cardinality of S is the same as the cardinality of T.

(c) The cardinality of S might be smaller than the cardinality of T.

(d) The cardinality of S is bigger than the cardinality of T.

10. Which of the following statements is TRUE?
Statement 1: The cardinality of the natural numbers is smaller than the cardinality of the rational numbers.
Statement 2: The cardinality of the rational numbers is smaller than the cardinality of the real numbers.

(a) Statement 1 only.

(b) Statement 2 only.

(c) Neither statement.

(d) Both statements.

11. Which of the following sets has a different cardinality than the rest?

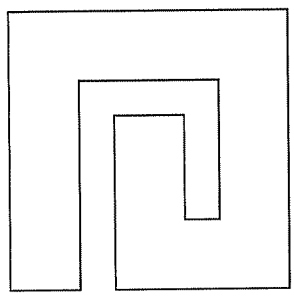
(a) The set of all positive integers.

(b) The set of all even numbers.

(c) The set of all negative integers

(d) None of them, they all have the same cardinality.

12. How many guards does the following gallery need to be guarded?



(a) 2 (b) 3 (c) 4 (d) 5

13. An art gallery with 10 walls is triangulated and the corners properly coloured red, green and blue. What is the largest number of red corners which is possible?

(a) 1 (b) 3 (c) 4 (d) 5

14. What is the smallest number of guards that we need to make sure that an art gallery with 10 walls is always guarded?

(a) 1 (b) 3 (c) 4 (d) 5

15. Consider the Pinwheel Pattern. Which of the following statements is true?

(a) The Pinwheel Pattern has symmetry of scale and rigid symmetry.

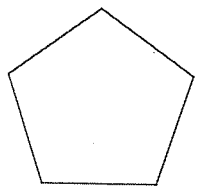
(b) The Pinwheel Pattern has symmetry of scale but no rigid symmetry.

(c) The Pinwheel Pattern has no symmetry of scale but it has rigid symmetry.

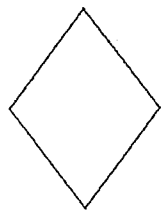
(d) The Pinwheel Pattern has no symmetry of scale and no rigid symmetry.

16. Which one of the following shapes does NOT have symmetry of scale?

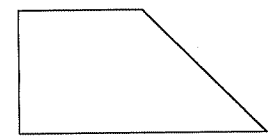
(a)



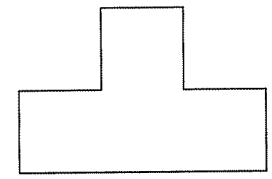
(b)



(c)

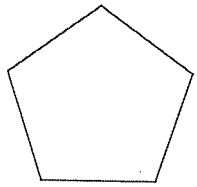


(d)

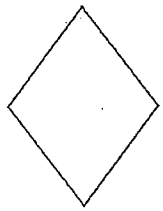


17. Which one of the following patterns exhibits NO non-trivial rigid symmetry?

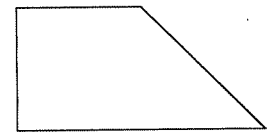
(a)



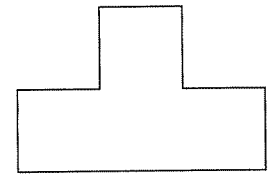
(b)



(c)



(d)



18. Which of the following pairs of objects are equivalent by distortion?

(a) A sphere and a torus.

(b) A torus and a cube.

(c) A sphere and a cube.

(d) None of the above.

19. Which of the following pairs of letters are equivalent by distortion?

(a) Z, Y. (b) Y, F. (c) X, F. (d) Y, X.

20. Which of the following numbers is NOT equivalent by distortion to the letter Z?

(a) 5 (b) 2 (c) 4 (d) 7

21. Consider a Möbius band, and cut it along a line through the middle of the band. What would you obtain?

(a) Two Möbuis bands, linked.

(b) A Möbius band and a band with two twists, linked.

(c) A long Möbuis band.

(d) A long band with two twists.

22. Consider a Möbius band, and cut it along a line very close to the edge of the band. What would you obtain?

(a) Two Möbuis bands, linked.

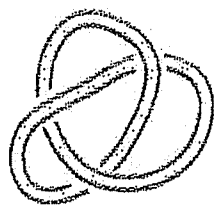
(b) A Möbius band and a band with two twists, linked.

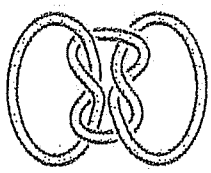
(c) A long Möbuis band.

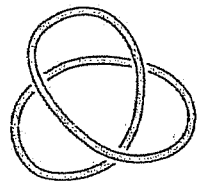
(d) A long band with two twists.

23. Consider the following four knots.

I 

III 

II 

IV 

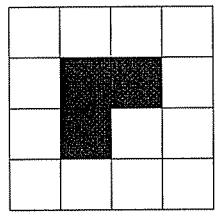
Which of them are drawn as alternating knots?

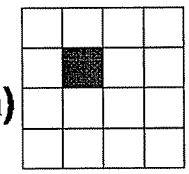
(a) I and IV (b) II and III (c) I, II and III (d) only IV

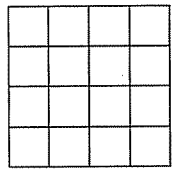
24. From the knots of question 23, which of them represents the unknot?

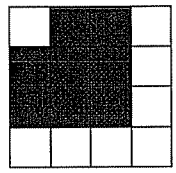
(a) III (b) III and IV (c) IV (d) None of them.

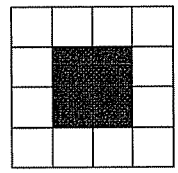
25. Consider the below population in the Game of Life. Which answer is the state of the system in the next time period?



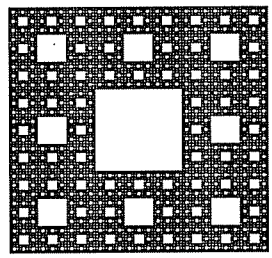
(a) 

(b) 

(c) 

(d) 

26. What is the fractal dimension of this object?



(a) $\ln(2)/\ln(3)$ (b) 1 (c) $\ln(8)/\ln(3)$ (d) $\ln(9)/\ln(8)$

27. What is the length of the Koch curve.

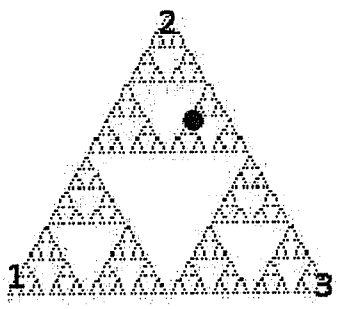
(a) 1 (b) 4/3 (c) infinity (d) $(4/3)^n$

28. What is the period of the value $x = 2/9$ when iterating the function

$$y = \begin{cases} 2x & 0 \leq x < 0.5 \\ 2 - 2x & 0.5 \leq x \leq 1 \end{cases}$$

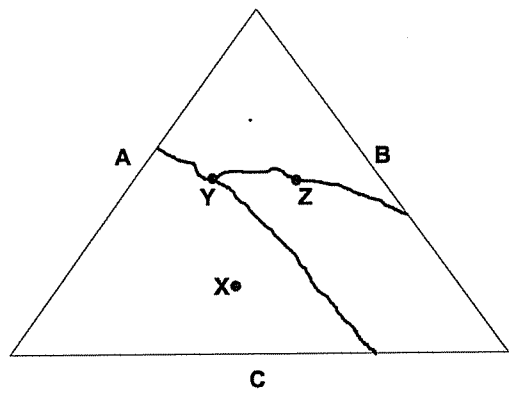
(a) 2 (b) 3 (c) 4 (d) 5

29. Suppose the chaos game is used to generate the Sierpinski triangle starting from vertex 1. Which sequence of dice throws leads to the marked point on the below triangle.



(a) 2, 3, 2 (b) 3, 3, 2 (c) 3, 2, 2 (d) 2, 3, 3

30. The diagram below shows Bob's preference for a triangular cake. Which of the following statements is true?



(a) If the cake is cut at X he will choose the piece labelled C.

(b) If the cake is cut at Y or Z he will always choose the piece labelled B.

(c) If the cake is cut at Z he will choose either of the pieces labelled A or C.

(d) If the cake is cut at Z he will choose the piece labelled A.

SECTION B: LONG ANSWER

Answer each question in this section in the answer book provided.
Start your answer to each question on a new page.

31. (a) Write 190 as the sum of non-consecutive Fibonacci numbers. (3 marks)

(b) Write 190 as the sum of different Fibonacci numbers, in a different way than in (a). (You may use consecutive Fibonacci numbers this time.) (3 marks)

(c) Find a third way of writing 190 as the sum of different Fibonacci numbers. (2 marks)

(d) Write 190 as the product of primes. (2 marks)

32. (a) What is $(23 + 5^4) \bmod 4$? Show all your work. (4 marks)

(b) What is the last digit of 3^{99}? Show all your work. (6 marks)

33. (a) Explain what does it mean for two sets to have the same cardinality. (4 marks)

(b) If A and B are two infinite sets, do they always have the same cardinality? If so, explain why; if not, find two infinite sets A and B that have different cardinalities. (2 marks)

(c) Let $S = \{a, b, c\}$.

(i) Find its power set, $\mathcal{P}(S)$. (2 marks)

(ii) What is the cardinality of $\mathcal{P}(\mathcal{P}(S))$? (2 marks)

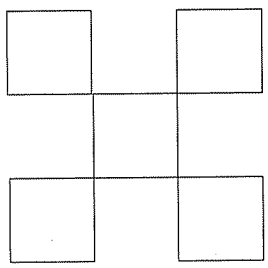
34. (a) What is a regular or Platonic solid? (1 marks)

(b) How many Platonic solids are there? Name them all. (2 marks)

(c) Draw and identify the dual of each of the Platonic Solids. (3 marks)

(d) Check that each Platonic Solid satisfies Euler's Theorem. (4 marks)

35. The following picture describes the procedure for making a fractal using the collage method.



(a) Sketch the resulting fractal. (6 marks)

(b) What is the dimension of this object? (4 marks)

36. This question is about iterating the function $y = 4x(1 - x)$.

(a) Determine the fixed points of this iteration. (3 marks)

(b) Draw a cobweb plot for the first 4 iterations starting from $x = 0.25$. (4 marks)

(c) Which fixed points are stable and which are unstable? (3 marks)

VERSION 1

QUESTION/ANSWER SHEET – 13 – **MATHS 190 and 190G**

Candidate's Name: _____ ID No: _____

Answer Section A by putting a cross in the appropriate box for each question.
Marks will not be deducted for incorrect answers.

	(a)	(b)	(c)	(d)
1.				
2.				
3.				
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	(a)	(b)	(c)	(d)
16.				
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TIE THIS ANSWER SHEET TO YOUR SCRIPT BOOK