DEPARTMENT OF MATHEMATICS
MATHS 761 Worksheet 7 - Local bifurcations in maps

This worksheet gives you some ideas about how to locate and identify local bifurcations in maps.

For the map $x_{n+1} = f(x_n; \mu)$ for each of the following choices of f, determine the type of bifurcation which occurs at the given μ value and sketch the bifurcation diagram.

- $f(x;\mu) = \mu - x^2, \quad \mu = -0.25$
- $f(x;\mu) = \mu \sin x, \quad \mu = 1$
- $f(x;\mu) = \mu \sin x, \quad \mu = -1$
- $f(x;\mu) = \mu x(1-x), \quad \mu = 1$

Ideas for getting started

1. At a local bifurcation, the number and/or type of fixed points or periodic orbits changes. Therefore, to locate a local bifurcation, you could find the fixed points and determine their stabilities. If possible, calculate your results two ways: by direct calculation of numbers and types of fixed points *and* by using the bifurcation theorems.

   There are two main ways to find fixed points:

   - either solve the fixed point equation explicitly (remember, solve $x = f(x)$, not $f(x) = 0$),
   - or find the fixed points graphically (i.e., plot $y = x$ and $y = f(x)$ on the same set of axes. Fixed points for f will be points of intersection of the two graphs.)

   You should have a reasonable idea of the bifurcation involved once you know how many fixed points there are and how this number changes at the bifurcation.

2. The types of the fixed points can be determined via linearization (if you found the fixed points explicitly in Step 1) or from inspection of the slope of the graph at the fixed points (if you found the fixed points graphically in Step 1).

3. Sometimes the quickest way to identify a bifurcation is to use the bifurcation theorems directly. To do this, you may need to change coordinates so that the non-hyperbolic equilibrium is at the origin of both phase and parameter space before you use the theorem.