Maths 761 Lecture 7 Summary

Topic for today

Dynamics in maps

Recommended problems

Glendinning: §6: 1,2

Reading for today's lecture

Glendinning: §6.1, §6.5

Reading for next lecture

Glendinning: §6.1, §6.5

Dynamics in maps

Given a function $f : \mathbb{R}^m \to \mathbb{R}^m$, we can define a *discrete dynamical system* or *discrete map* by

$$x_{n+1} = f(x_n),$$

for $x_i \in \mathbb{R}^m$. The phase space of such a system is $\mathbb{R}^m$. The *solution* (or *orbit* or *trajectory*) of this equation started from the initial condition $x = x_0$ is the sequence of discrete points in the phase space, $x_0, f(x_0), f(f(x_0)), \ldots$.

A *fixed point* or *stationary solution* for the map associated with function f is a value $x = \bar{x}$ such that $f(\bar{x}) = \bar{x}$.

A *periodic orbit* (of least period n) is a set of points $x_0, x_1, \ldots, x_{n-1}$ such that $f(x_i) = x_{i+1}$ for $i = 0, 1, \ldots, n-2$, $f(x_{n-1}) = x_0$ and $f(x_{i+k}) \neq f(x_i)$ for any k with $0 < k < n$.

Cobweb plots

Cobweb plots are a useful way of examining the dynamics of one-dimensional maps.