Department of Mathematics

Maths 190 Assignment 1

July 27, 2010 Due: 4pm, 4th August, 2010

- Your completed assignment should be handed in to the appropriate box on the ground floor of the Mathematics/Physics Building **before** 4pm on the date due.
- Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.
- Late assignments or assignments placed in the wrong box **will not be accepted**.
- You must show all your working, and write careful, clear explanations of your solutions, stating all your assumptions.
- You may discuss your attempts to answer the questions with other members of the class. However, **you should write up your answers yourself** (**do not copy** from another person or allow another person to copy from you). If you worked with someone on your answers, state who you worked with.

1. (6 marks) I have a packet of M&M's. M&M's come in six possible colours: red, green, blue, orange, yellow and brown. How many M&M's need to be in the packet so that I can be sure I have:

   (a) At least five M&M's of one colour.

   (b) At least nine M&M's of one colour.

   (c) At least five red M&M's.

   (d) At least two M&M's of one colour, and at least two M&M's of a second colour.

   Note: answer each question separately, and if you don't think a particular case is possible for any number of M&M's, explain why. Remember to show all your working.

2. (6 marks) Put the following numbers in order from smallest to largest (show all your working, and state all assumptions you make):

   - The number of grains of sand on Ninety Mile Beach.
   - The number of tennis balls you could fit inside your bedroom.
   - The total number of cars in New Zealand.

3. (5 marks)

   (a) How many (natural) numbers less than 100 contain a 3? (Note: 13, 35 and 73 all contain a 3 but 42, 65 and 88 do not). What proportion is this?

   (b) Approximately what proportion of numbers less than 1000 contain a 3?

   (c) Explain why almost all million-digit numbers contain at least one 3.

4. (5 marks) The Lucas sequence is generated using the same rule as the Fibonacci sequence, except the first two terms are $L_1 = 1$ and $L_2 = 3$.

   (a) Write down the first 15 Lucas numbers.

   (b) Compute $L_n + L_{n+2}$ for $n = 1, \ldots, 13$, that is, the sum of a Lucas number and the Lucas number that comes after the next one. Look for a pattern and find a formula for this expression (or explain it in words).

   (c) Find the ratio L_n/L_{n-1} of successive Lucas numbers, for $n = 1, \ldots, 14$. What do you notice about the ratios?

5. (8 marks) The following is a proof that there are infinitely many prime numbers, but some parts of some of the steps are missing. Write out clearly what should be inserted in the missing steps (b), (d), (e) and (f).

   (a) First assume that there are finitely many primes, and label them $p_1, p_2, \ldots, p_m$. Thus p_m is the largest prime.

   (b) Construct the number $q =$ ________________

   (c) By the prime factorization theorem, q is either prime, or is a product of primes.

   (d) But none of the prime numbers $p_1, \ldots, p_m$ are a factor of q, because ____ and so q must be prime.

   (e) But q cannot be prime, because ____

   (f) Hence q is both prime and not prime, which is a ____________

   (g) Thus our original assumption, that there were finitely many primes, must be false.

Tutorial write up: Remember to hand in with your assignment your written solutions to question 4 on tutorial 1 (5 marks) and question 5 on tutorial 2 (5 marks).