

Maths 761 Notes for Laboratory 11

Use XPPAUT and any other methods you like to find the bifurcations in the following system, paying particular attention to the appearance and disappearance of periodic orbits. Sketch the bifurcation set and some relevant phase portraits.

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= \lambda x + \mu y - x^3 - x^2 y.\end{aligned}$$

Hints:

1. Find the local bifurcations analytically and check your results with XPP.
2. You may need to use XPPAUT to check whether any Hopf bifurcations are supercritical or subcritical.
3. You should use XPPAUT to determine what happens to any periodic orbits created in the Hopf bifurcation(s) as you make a circuit around the origin in parameter space.
4. You can use XPPAUT to follow local bifurcations as two parameters vary. For instance, to find the locus of a Hopf bifurcation in the $\lambda - \mu$ -plane:
 - (a) Grab a point at which there is a Hopf bifurcation.
 - (b) Select Axes/Two par. Change the ranges for the axes to something appropriate (e.g., $[-10, 10]$).
 - (c) Use Run/Two Param to follow the Hopf bifurcation in two parameters. Use both signs of DS.
(You can also use XPPAUT to follow global bifurcations in two parameters, but that is more complicated. Ask your lecturer or tutor for help if you want to do this.)