DEPARTMENT OF MATHEMATICS

MATHS 190 **Lecture 21 Summary**

We learned how the Königsberg bridges problem motivated Euler to discover graph theory. We learned that abstraction is a powerful tool for solving problems.

A **graph** is a picture made from dots (called vertices) and lines between dots (called edges). A **path** is a list of edges, each starting from the end point of the previous one. A graph is **connected** if there is a path between any two vertices. A **circuit** is a path which returns to its starting point. The **degree** of a vertex in a graph is the number of edges which meet that vertex. An **Euler circuit** in a connected graph is a path which starts and ends at a vertex and which crosses every edge exactly once.

We observed that a graph with an Euler circuit has the property that every vertex has even degree. Furthermore, the following is true.

Theorem: A connected graph has an Euler circuit if and only if the degree of every vertex is even.

We proved this theorem as follows: For any graph such that all vertices have even degree we can start at any vertex v and make a path where no edge is used twice. Such a path must eventually lead back to v (in other words, it becomes a circuit). If this circuit covers every edge then we are finished. If not, then it can be extended by taking a new circuit starting from a vertex in the circuit with an un-used edge.

Before you come to the next lecture: Make sure you are familiar with the words graph, vertex, edge, path, circuit, degree etc.

- Read §5.3 and §5.4 in the textbook.
- Try some of the Mindscapes at the end of §5.3 in the textbook.