

# Maths 190      Assignment 2

August 17, 2010

Due: 4pm, August 25th, 2010

- Your completed assignment should be handed in to the appropriate box on the ground floor of the Mathematics/Physics Building **before** 4pm on the date due.
- Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.
- Late assignments or assignments placed in the wrong box **will not be accepted**.
- You must show all your working, and write careful, clear explanations of your solutions, stating all your assumptions.
- You may discuss your attempts to answer the questions with other members of the class. However, **you should write up your answers yourself (do not copy from another person or allow another person to copy from you)**. If you worked with someone on your answers, state who you worked with.

1. (4 marks) Write the following infinite decimal expansions as rational numbers  $m/n$ , where  $m$  and  $n$  are integers to be found. Show all your working

(a)  $0.27272727\dots$

(b)  $1.23012012012012\dots$

2. (4 marks) A bank allocates 9-digit account numbers using a check digit system. If the account number is  $n_1n_2n_3n_4n_5n_6n_7n_8n_9$  then the number

$$7n_1 + 3n_2 + 9n_3 + 7n_4 + 3n_5 + 9n_6 + 7n_7 + 3n_8 + 9n_9$$

is equal to  $0 \pmod{10}$ . For each of the following bank account numbers, one digit has been covered by a black square. Calculate what the correct digit should be. Show all your working.

(a)  $21187294\blacksquare$

(b)  $1\blacksquare3288019$

3. (3 marks) Is it possible to construct an irrational number whose decimal expansion contains only the digits 1 and 2? If so, explain how, if not, explain why not.

4. (12 marks) Let  $N$  be the set of natural numbers, i.e.,

$$N = \{1, 2, 3, 4, \dots\}.$$

For each of the following sets described below ( $B$ ,  $F$ ,  $L$  and  $P$ ), state whether the set has cardinality that is less than the cardinality of  $N$  or equal to the cardinality of  $N$  or greater than the cardinality of  $N$ .

In each case, prove that the cardinality of the set is as you claim. In your answers you may assume that the cardinality of the set of points in the interval  $[0, 1]$  has greater cardinality than  $N$ .

- (a) Vaughan started emptying the Auckland harbour using a 10L bucket. He took a full bucket of water out of the harbour and then emptied it into his XXL drink bottle he was carrying, which interestingly can hold an infinite volume of liquid. Vaughan recorded what bucket number he was up to on a different piece of paper after each bucket. Set  $B$  is the set of numbers on these pieces of paper that represent the numbered buckets of water he can remove from the Harbour until it is empty.
- (b) Claire wanted to know how big the set  $F$  was, where  $F$  was the set of all possible fractions formed using the digits 2, 3, 4 and 5. For example  $\frac{5}{2}$  is in the set, but  $\frac{1}{5}$  is not.
- (c) James likes interesting trousers. In fact, he likes them so much, he owns an infinite number of pairs similar to those you saw in lecture 1 (strangely, he has them labeled  $T_1, T_2, T_3, \dots$ ). The set  $L$  is the set of all the legs of these trousers. You can assume each pair of trousers has only two legs.
- (d) Phillip described the members of set  $P$  as all the set of points around the perimeter of a regular pentagon, which has sides of length 2cm each.
5. (5 marks) Downtown Auckland was extremely busy one morning. An infinite number of cruise ships turned up (call them  $S_1, S_2, S_3, \dots$ ), each ship having an infinite number of rooms (all numbered  $R_1, R_2, R_3, \dots$ ) and each room containing space and comfort for an infinite number of passengers. Unfortunately for Ivo (the local tour organiser), each ship has been promised busses to take their passengers to see the All Blacks training session at Eden Park. Explain to Ivo, the minimum number of infinite buses he needs to organise so that all the passengers from all the rooms from all the ships have a seat. An infinite bus is one with an infinite number of seats all individually numbered, plus the driver of course! You must use an appropriate argument in order to gain full marks. An answer with no explanation will score zero.

**Tutorial write up:** Remember to hand in with your assignment your written solutions to question 5 on tutorial 3 (5 marks), question 7 on tutorial 4 (5 marks) and question 6 on tutorial 5 (5 marks).