DEPARTMENT OF MATHEMATICS
MATHS 761 Worksheet 3 - Invariant manifolds in linear systems

1. Find E^s, E^u and E^c for the origin in the system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 6 & 2 \\ -3 & -1 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} -1 & 0 & 0 \\ -2 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -3 & 0 \end{pmatrix}$$

2. Describe the possible types of behaviour on a centre manifold for a linear system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$. If $\mathbf{x} \in E^c$, can $\phi(\mathbf{x}, t) \to 0$ as $t \to \infty$? Can $\phi(\mathbf{x}, t) \to \infty$ as $t \to \infty$? Can $\phi(\mathbf{x}, t) \to -\infty$ as $t \to \infty$? Hint: You might consider the cases:

$$\mathbf{A} = \begin{pmatrix} 0 & -2 \\ 4 & 0 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

to help you answer this question.

3. (Glendinning Exercise 3, #8)
Consider the system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ for $\mathbf{x} \in \mathbf{R}^7$ and

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Find:

(a) $E^s(0)$

(b) $E^c(0)$

(c) $E^u(0)$

(d) the invariant manifold associated with the eigenvalue 1.