Maths 190 Lecture 10

Topic for today:

Infinity in Geometrical Objects

Question of the day:

Are there more points inside a square of side length 1 or on a circle of radius 1?

Line segments

- Which line segment has more points on it, S or B?

S 

B 

Line segments and the real line

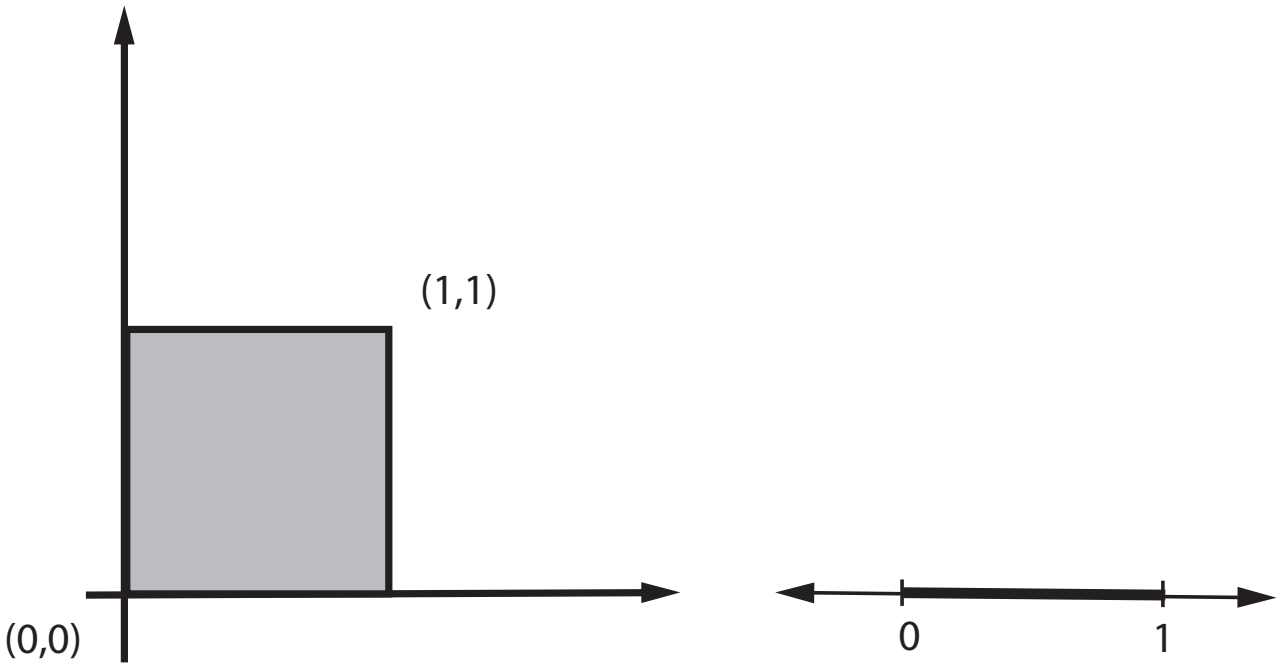
- Does a line segment or the real line contain more points?

S  (end points missing)

R 

Lines and squares

- Are there more points in a square or on the line segment from 0 to 1?



Summary

We have explored the cardinality of sets of points in geometric objects:

- We can define a one-to-one correspondence between sets geometrically (i.e., with a picture).
- We know from last lecture that the interval [0,1] has greater cardinality than the set $\{1, 2, 3, \dots\}$
- All line segments have the same cardinality as [0,1]
- The real line has the same cardinality as [0,1].
- A solid square has the same cardinality as [0,1].

We have seen that there is an infinity bigger than the cardinality of the set $\{1, 2, 3, \dots\}$. But:

- Is there an infinity greater than the infinity of $\{1, 2, 3, \dots\}$ but less than the infinity of [0,1]?
- Is there an infinity greater than the infinity of [0,1]?
- Are there infinitely many different sizes of infinity?
- Is there a largest infinity - one that encompasses all the others?

Important ideas from today:

- Many geometrical objects have the same cardinality as each other. 2D space is as big as 3D space.
- One-to-one correspondences can sometimes be defined geometrically (i.e., with a picture).
- Understanding issues of infinity is an intellectual triumph. These ideas are challenging but they allow us to analyze concepts that at first seem beyond human comprehension.

For next time

- Try some Mindscapes at the end of §3.5 of the textbook.
- Read §4.5 in the textbook.