Maths 190 Lecture 5

- **Topic for today: Cyclical arithmetic**

- **Vitally important question of the day**:

  What do clocks and bar codes have in common?

What time will it be in 50 hours time?

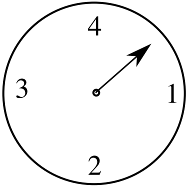
Clock arithmetic

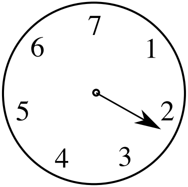
- $2 + 4 =$
- $10 + 4 =$
- $7 + 7 =$
- $3 - 5 =$

- Suppose it takes you 13 hours to do one problem from your Maths 190 assignment. If you start at 1 o'clock and there are 5 problems, what time will it be when you are finished (assuming you work straight through)?

Crazy clocks

- There is no good reason why our clock is divided into 12 hours.
- Do the following sums on the 4 hour clock and the 7 hour clock:





- $2 + 4 =$
- $3 - 4 =$
- $7 + 7 =$
- $3 - 5 =$
- $1 + 13 \times 5 =$

Modular arithmetic

- '$m \mod n$' means 'the remainder when I divide m by n'.
- We do modular arithmetic when we calculate the time.
- We define equivalences

$$a \equiv b \mod n$$

Calculations mod 10

- Doing calculations means using equivalences:

$$10 \equiv 0 \mod 10$$

- We can simplify expressions as we go along:

$$\begin{aligned} & 12 + 34 + 67 \mod 10 \\ = & 2 + 4 + 7 \mod 10 \\ = & 13 \mod 10 \\ = & 3 \mod 10 \end{aligned}$$

- The same thing works for multiplication

$$
\begin{aligned}
& 37 \times 16 \mod 11 \\
=& (3 \times 11 + 4) \times (11 + 5) \mod 11 \\
=& 4 \times 5 \mod 11 \\
=& 20 \mod 11 \\
=& 9 \mod 11
\end{aligned}
$$

- And for taking powers:

$$
\begin{aligned}
& 33^5 \mod 8 \\
=& (4 \times 8 + 1)^5 \mod 8 \\
=& 1^5 \mod 8 \\
=& 1 \mod 8
\end{aligned}
$$

Check digits

- Find something with a bar code
- There should be 13 digits; if there are only 12, imagine there's a zero in front.
- This book: 9 781931 914413
- But not every 13 digit number is a bar code.

A check sum

- Call the digits d_1 to d_{13}
- Multiply the even-numbered digits by 3, and then add them all together:

$$d_1+3d_2+d_3+3d_4+d_5+3d_6+d_7+3d_8+d_9+3d_{10}+d_{11}+3d_{12}+d_{13}$$

- For my example, I get:

$$9+(3\times7)+8+(3\times1)+9+(3\times3)+1+(3\times9)+1+(3\times4)+4+(3\times1)+3$$

- Then compute the result $\mod 10$.
- How can you tell if there's a mistake?

Try it!

- Get into pairs.
- Taller student: find a 13 digit bar code.
- If your birthday is on an even date, write down the bar code *with one digit incorrect*.
- If your birthday is on an odd date, write down the bar code *correctly*.
- Shorter student: is your partner's birthday on an odd or even date?

ISBN numbers

- 10-digit ISBN numbers use mod 11 (using $X = 10$)
- $d_1 + 2d_2 + 3d_3 + 4d_4 + 5d_5 + 6d_6 + 7d_7 + 8d_8 + 9d_9 + 10d_{10} = 0 \mod 11$
- This book: ISBN 1-931914-41-9

Fermat's little theorem

- If p is a prime number and n is any integer which does not have p as a factor, then

$$n^{p-1} = 1 \mod p$$

- Try it!
- $p = 5$, try $n = 3$, $n = 7$, $n = 9$.

Important ideas from today:

- When we compute the time, we do modular arithemetic.
- Modular arithemetic is also used in bar codes and ISBN numbers (among other things).

For next time

- Read Section 2.6 of the textbook