Maths 761 Notes for Laboratory 10

This lab looks at the Bogdanov-Takens system of equations:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \lambda + \mu y + x^2 + xy. \end{aligned}$$

You will get more detail about the behaviour of the periodic orbit created in the Hopf bifurcation.

Your results from this lab will be useful for Assignment 4, so make sure you look at the lab before Friday, do as much as you can in advance, and come to the lab ready to ask questions.

1. Find the local bifurcations in the system.

2. Use XPP to check whether the Hopf bifurcation is supercritical or subcritical and then locate the approximate position of the homoclinic bifurcation curve.

3. Find a path in the $\lambda - \mu$ plane that cuts both the Hopf bifurcation curve and the homoclinic bifurcation curve. Something like μ =constant would be best.

4. Use XPP to get the bifurcation diagram along this path, including following the periodic orbit created in the Hopf bifurcation until it has high period (say a period of about 200).

5. For the value of μ you used in the drawing the bifurcation diagram above, plot a graph of the period of the periodic orbit vs. the natural log of $|\lambda - \lambda^*|$, where λ^* is the value of λ at which the homoclinic bifurcation occurs. You should get a straight line, at least for small enough $|\lambda - \lambda^*|$. Compare the slope of this line with the prediction from the theory we developed in class.

   Note: to do this, you will need to export the data to Matlab. Use the File/Write points option in AUTO to save a file containing the data points from the periodi vs¿ parameter bifurcation diagram, read this file into Matlab, then plot period vs. $\log|\lambda - \lambda^*|$. You may need to remove some points very close to the homoclinic bifurcation from your data file before plotting so that round-off error is not a problem.