DEPARTMENT OF MATHEMATICS

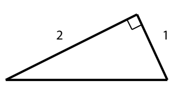
MATHS 190 **Lecture 13 Summary**

In this lecture, we looked at symmetries of tilings of the plane. By looking at some simple examples we found there were two types of symmetry:

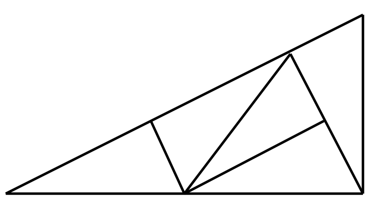
- A **rigid symmetry** of a pattern in the plane is a motion of the plane that preserves the pattern and does not shrink, stretch, or otherwise distort the pattern. Examples of rigid symmetries are shifts, rotations and flips (i.e., reflections).
- A pattern in the plane has a **symmetry of scale** if the tiles that make up the pattern can be grouped into super-tiles that still cover the plane and, if scaled down, can be rigidly moved to coincide with the original pattern.

Note that these definitions refer to the tiling pattern, not to the individual tiles. Some tilings have rigid symmetries and symmetry of scale (e.g., tilings using square tiles or equilateral triangle tiles). Other tilings have rigid symmetries but no symmetry of scale (e.g., tilings using hexagonal tiles). Some tilings can even have symmetry of scale but no rigid symmetries - we looked in some detail at a Pinwheel Pattern, an example of this last type of tiling.

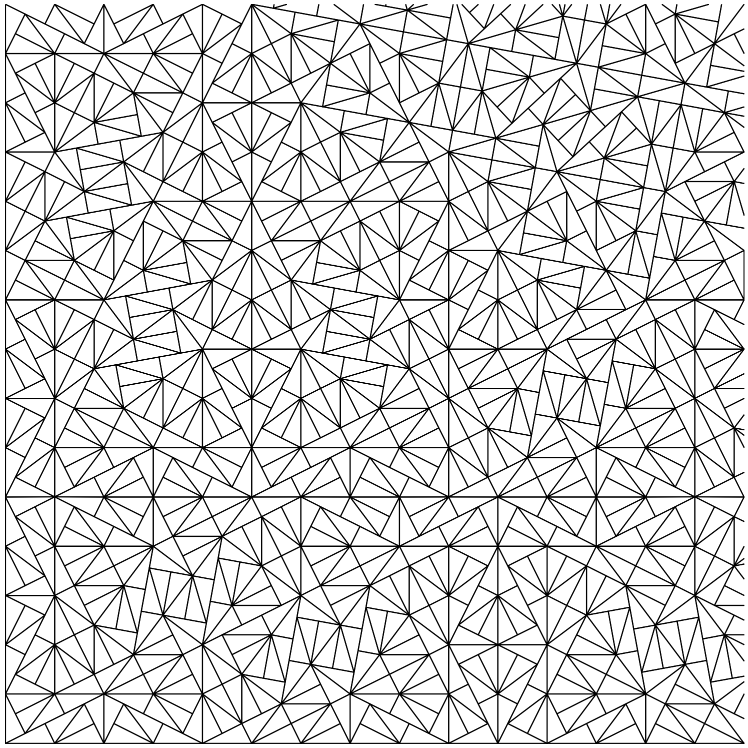
The Pinwheel Pattern we looked at uses a triangular tile: a right angle triangle with sides of length 1, 2 and $\sqrt{5}$ units.



Five pinwheel triangles are combined to make one super-tile.



Five pinwheel super-tiles can be combined to make one super-super-tile, and so on forever .... Part of a pinwheel pattern is shown over the page. This method of construction of the Pinwheel Pattern ensures that it has symmetry of scale and we noticed that there appeared to be no rigid symmetries in the pattern (a proof of this is given in the book).



Before you come to the next lecture: You should spend an hour or two reviewing the material from todays lecture. You should also

- Read §4.4 in the textbook.
- Try some of the Mindscapes at the end of §4.4 in the textbook.

Other activities you could do if you have time are:

- Show the Pinwheel Pattern to a friend who is not doing Maths 190. Show them that the Pinwheel Pattern has symmetries of scale but no rigid symmetries. Marvel with them over this observation.
- If you found the tiling pattern shown on the last slide beautiful consider taking a look at the blog "This Week's Finds in Mathematical Physics" by John Baez, weeks 221, 267 and 281:
  `http://math.ucr.edu/home/baez/week221.html`
  `http://math.ucr.edu/home/baez/week267.html`
  `http://math.ucr.edu/home/baez/week281.html`
- Search the internet for "Escher tessellations" to find more tilings by the Dutch artist M. C. Escher (1898-1972), check out
  `http://en.wikipedia.org/wiki/M._C._Escher` and `http://www.mcescher.com/`
  if you are interested in the artist.