DEPARTMENT OF MATHEMATICS

MATHS 190 **Lecture 14 Summary**

In this lecture, we considered the question of how many guards are required to keep an eye on an art gallery.

More precisely, we defined a **polygonal closed curve** to be any figure made up of straight line pieces that are connected end-to-end to form a loop, and called the corners where the walls meet **vertices**. We restricted attention to art galleries whose floor plans are polygonal closed curves, with no interior walls or partitions, and allowed guards to stand only at vertices.

We showed that for an art gallery of this type with v vertices, it is always possible for $v/3$ guards to view the whole gallery. If $v/3$ is not an integer, then the number of guards needed is equal to *the biggest integer less than* $v/3$—we also write $\lfloor v/3 \rfloor$.

This result means that **at most** $v/3$ guards are needed in a gallery with v vertices. However, for a particular gallery it may be possible for fewer than $v/3$ guards to view the whole gallery. For instance, one guard standing at a vertex will be able to see all points of a regular octagonal gallery (where $v = 8$ and $v/3 = 2.666...$ and so the Art Gallery theorem predicts at most 2 guards are needed).

Our discussion of this example illustrates a useful process in problem solving.

- First we looked at **lots of examples** and looked for patterns.
- Then we tackled the **complex cases** (polygonal curves with many vertices) by dividing them into many **simpler pieces** (triangles). This is called "divide and conquer".

This process is useful in many areas of mathematics and elsewhere.

Before you come to the next lecture: You should spend an hour or two reviewing the material from today's lecture. You should also:

- Read §4.2 in the textbook.
- Try some of the Mindscapes at the end of §4.2 in the textbook.

Other activities you could do if you have time are:

- Draw a polygonal art gallery floor plan for a friend and ask them how many guards should be placed at corners in order to keep an eye on all parts of the gallery. Tell them about the Art Gallery theorem and try to explain why it is true.