Department of Mathematics

MATHS 190/190G

Great Ideas Shaping our World

Mid-semester Test
29 April 2010

Instructions

- This test contains **FIVE** questions. Attempt **ALL** questions.
- Show **ALL** your working.
- You have 1 hour to do the test. Total marks = 40.

1. (8 marks) You have 30 socks in a drawer, 10 red, 16 black and 4 blue. You are removing socks from the drawer in the dark, so you cannot see the colours. How many socks do you need to take out to be certain you have:

   (a) A matching pair of socks.

   (b) Two matching pairs of socks.

   (c) Two matching pairs of *different* colours.

   (d) Two blue socks.

2. (6 marks) ISBN numbers have 10 digits $d_1d_2d_3d_4d_5d_6d_7d_8d_9d_{10}$. If the ISBN is correct, then the following number

$$d_1 + 2d_2 + 3d_3 + 4d_4 + 5d_5 + 6d_6 + 7d_7 + 8d_8 + 9d_9 + 10d_{10}$$

   should be equivalent to 0 mod 11.

   The following ISBN numbers each have one digit covered up by a black square. Work out what that digit should be.

   (a) 0-219-60512-■

   (b) 0-■01-54199-8

3. (6 marks)

   (a) What is a rational number?

   (b) Let a and b be two rational numbers. Can you always find a third rational number which is between a and b? Explain your answer carefully.

   (c) Suppose the number p is irrational. Show that $p + 3$ is also irrational.

4. (10 marks)

   (a) State the definition of a regular solid (alternative names: regular polyhedra or platonic solid).

   (b) How many regular solids are there?

   (c) The *tetrahedron* is the regular solid which has 4 faces, all equilateral triangles. If you slice off a small part of one of the 4 corners, what shape is the cross-section?

   (d) Write V for the number of vertices (i.e., corners), E for the number of edges, F for the number of faces, p for the number of edges of each face and q for the number of faces which meet at each vertex.
   Explain why $pF = 2E$ and why $qV = 2E$.

5. (10 marks)

   (a) Suppose set A and set B are infinite sets. What does it mean to say that A and B have the same *cardinality*?

   (b) Let M be the set of all integers which contain the digit 3, so

   $$M = \{\cdots -32, -31, -23, -13, -3, 3, 13, 23, 31, 32, \dots\}$$

   Carefully show that M has the same cardinality as the set of natural numbers $N = \{1, 2, 3, \dots\}$.

   (c) Suppose you have infinitely many coins, one for each natural number. You toss each coin, and it comes down either heads (H) or tails (T). Let C be the set of all possible sequences of heads and tails. Carefully show that the cardinality of the set C is greater than the cardinality of the set of natural numbers.