DEPARTMENT OF MATHEMATICS
MATHS 761 Laboratory 2: Notes

In this laboratory session you will use the software package XPP to investigate solutions to two systems of differential equations. The main purpose of the session is to help you become familiar with use of XPP, so play around with the various options in XPP as well as working through the examples below.

Some hints for some parts of these questions can be found on the handout 'Laboratory 2: Hints'.

1. Investigate solutions to the following system of differential equations:

$$\begin{aligned}\frac{dx}{dt} &= y,\\ \frac{dy}{dt} &= \mu + y + x^2 + xy,\end{aligned}$$

where μ is a real parameter.

(a) Write a `.ode` file that contains the equations above.

(b) Consider the case $\mu = -1.5$.

i. Use XPP to find all stationary solutions and determine their types and stabilities.

ii. Find all periodic orbits and determine their stabilities.

iii. Plot a nice phase portrait, showing stationary and periodic solutions, stable and unstable manifolds of any saddle-type stationary solutions, and a few other representative orbits.

iv. Save this picture in a file and also print a copy.

(c) Now consider the case that μ is allowed to vary. Use XPP to find the phase portraits corresponding to various values of μ. For instance, you might see what the phase portraits look like for ten different values of μ in the interval $[-10, 10]$.

(d) The original equations have a saddle-type stationary solution at $(x, y) = (\sqrt{-\mu}, 0)$ when $\mu < 0$.

i. Use XPP to determine the value of μ, correct to three decimal places, at which $W^u(\sqrt{-\mu}, 0)$ is coincident with $W^s(\sqrt{-\mu}, 0)$. Call this value of the parameter μ^*.

ii. Use the information you get from XPP to draw, by hand, phase portraits illustrating the main qualitative differences between the dynamics for the cases $\mu < \mu^*$ and $\mu > \mu*$.

2. (a) Write a `.ode` file that contains the Lorenz equations:

$$\frac{dx}{dt} = 10(y - x),$$
$$\frac{dy}{dt} = Rx - y - xz,$$
$$\frac{dz}{dt} = -8z/3 + xy.$$

Use a default R value of 0.

(b) i. Start XPP using these equations.

ii. Change the plot window so that it shows the projection of solutions onto the $x - y$ plane.

iii. Create a second plot window showing the x and z coordinates.

iv. Plot a solution in the new window using the options menu as usual.

v. Plot the same solution in the other window by clicking in that window then selecting 'Restore' from the main menu on the original window. Remember that changes you make with the main menu will apply to whichever window has a small white dot in the top left corner.

(c) You may notice that the solution curve plotted in the previous step has jagged edges. Fix this by changing the stepsize for the integration using the 'nUmerics' menu.

(d) Investigate how the dynamics of the Lorenz equations changes as the parameter R is varied in the interval $[0, 28]$. As well as plotting solutions starting from various initial conditions, use XPP to find all the equilibria and their stabilities. Identify some values of R at which the dynamics changes qualitatively. You should be able to find at least four qualitatively different types of flow for this system.

(e) If you have time, export your data to Matlab and make some nicer pictures using the Matlab plot functions.