Maths 761 Notes for Laboratory 7: Using AUTO within XPP

In this laboratory session you will use a set of programs called AUTO, called from within XPP, to follow solutions and find bifurcations as parameters vary in a system of equations. The XPP online manual provides more information about using AUTO and you can refer to it for help if you need it.

1. Write an .ode file for the differential equation

$$\dot{x} = \mu - x^2$$

   Set the initial condition to $x = 0$ and the initial parameter value to $\mu = 1$. Then do the following tasks:

   (a) Use Initialconds/Go, then Initialconds/Last to converge to an equilibrium solution.

   (b) Click on File/Auto to start AUTO. A new window appears.

   (c) Click on Run/Steady State in the new window to follow the solution as μ varies. AUTO will start to plot a bifurcation diagram: one curve segment appears in the bifurcation diagram, starting from the initial μ value specified in the .ode file and extending out to the maximum parameter value specified in AUTO.

   (d) The bifurcation diagram so far has been plotted for μ increasing only. The case μ decreasing is also interesting. To plot the bifurcation diagram with μ decreasing, first pick a point to start from: click on Grab, then press the tab key until the cross on the bifurcation diagram is at label 1, then press the Enter key to select this label. You can tell if it is at label 1 by looking in the rectangular box below the bifurcation diagram. The number under the word 'Lab' tells you the label of the point.

   Now change the direction of μ. To do this, use Numerics, and change DS to -0.02. Now use Run again. A bifurcation appears at label 3.

   (e) The eigenvalue for the equilibrium solution is shown schematically in the AUTO window. Look at the little box in the bottom left of the AUTO window. There is a small circle in the box which represents the unit circle in the complex plane. The cross shows e^{λ}, where λ is the eigenvalue. Thus, if the eigenvalue is negative, the cross will be inside the circle, while if the eigenvalue is positive the cross will be outside the unit circle. (For systems where there is more than one eigenvalue, there will be as many crosses as eigenvalues. Complex eigenvalues will correspond to crosses which lie off the real axis.)

   Using Grab and the Tab key, and then the arrow keys, see what happens to the eigenvalues of the Jacobian as you move along the various branches of the bifurcation diagram. Work out why some parts of the bifurcation diagram are plotted with a thick (bold) line style and other parts are plotted with a thin line style.

(f) Your bifurcation diagram may have sharp corners instead of smooth curves, especially near the bifurcation value. In this case your step size is probably too large. Try changing DSMAX in the Numerics menu to a smaller value, then rerun AUTO to get a smoother curve.

2. (a) Use XPP and AUTO and the procedure outlined in steps (a)-(d) above to compute a bifurcation diagram for the equation

$$\dot{x} = \mu x + x^3$$

Notice that AUTO automatically computes both branches passing through the bifurcation at $\mu = 0$.

(b) Now modify your .ode file for this example so that default values for some of the AUTO constants are set in the .ode file. Work out how to change the limits used when the bifurcation diagram is drawn. Hint: Including the line

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@ dsmin=0.00001,dsmax=0.05,ds=0.02,parmin=0,parmax=500
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in your .ode file will ensure that the initial AUTO step size for computing the bifurcation diagram is ds=0.02 with minimum and maximum step sizes of dsmin=0.00001 and dsmax=0.05, and with the parameter allowed to range between 0 and 500. More details about these are other options (such as how to set the limits for drawing the bifurcation diagram) are in the online manual.

These constants can also be set once you have started XPP or AUTO but it is sometimes more convenient to set the values within the .ode file.

3. The equations from Worksheet 5 were:

$$\begin{aligned} \dot{x} &= y - x - x^2, \\ \dot{y} &= \mu x - y - y^2. \end{aligned}$$

(a) Show (by hand!) that $x = y = 0$ is an equilibrium solution for all μ.

(b) Write a .ode file for this system, setting the default values of x, y and μ to zero.

(c) Start XPP and then straight away start AUTO. Since explicit values of the variables and parameter at an equilibrium are known it is not necessary to use Initialconds/Go etc. as above to find the equilibrium before using AUTO; just start AUTO with the equilibrium values as the default values.

Follow the trivial solution (i.e., the solution with $x = y = 0$) with AUTO, both for positive and negative DS. You should obtain a bifurcation diagram similar to the one calculated on Worksheet 5.

(d) Using axes/hi, plot bifurcation diagrams with both μ versus x and μ versus y. Use these to identify any bifurcations that occur.