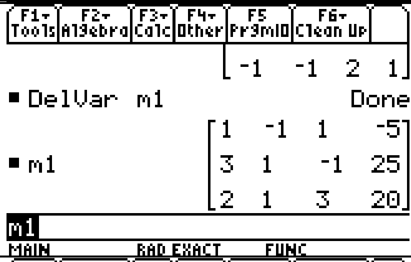
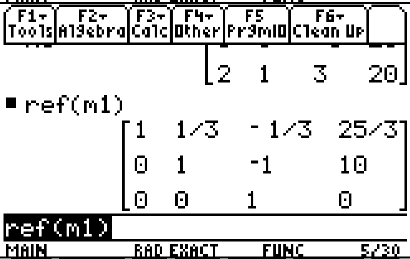
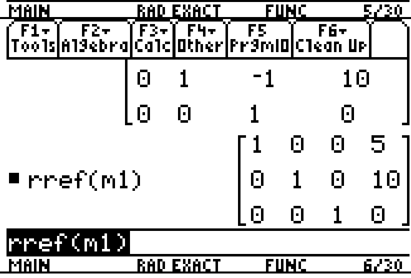
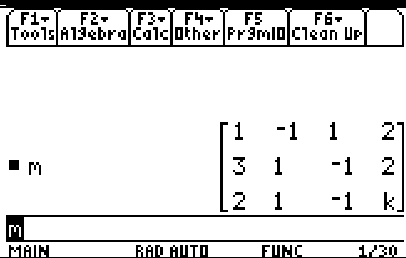
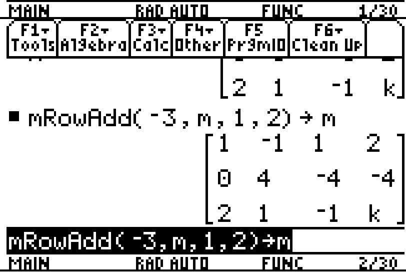
Example 7.2.2:

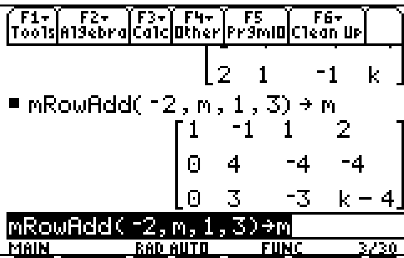
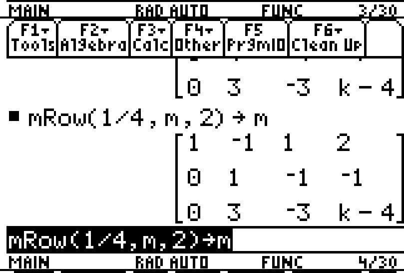
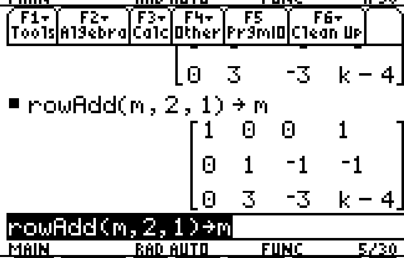
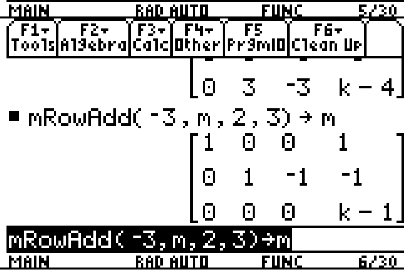
Instructions	Screen Shot
First enter the augmented matrix, here into m1. **Note:** To remove a variable used for a matrix or other calculation use Ω Delvar (variable name).	
Then use the command [MATH] menu (2nd 5) select 4: Matrix and 3. ref for row echelon form,	
or 4. rref for reduced echelon form.	

7.3 Systems which contain unknown coefficients

Example 7.3.1:
For what values of *k* does the following system of linear equations have no solution, a unique solution, or infinitely many solutions?

$$x_1 - x_2 + x_3 = 2$$
$$3x_1 + x_2 - x_3 = 2$$
$$2x_1 + x_2 - x_3 = k$$

Instructions	Screen Shots
First enter the augmented matrix, here into m, then continue as follows:	
R2 –3R1→R2 mRowAdd(–3, M, 1, 2) STO▸ M multiply R1 by –3, add the product to R2 and store it in row 2.	

mRowAdd(–2, M, 1, 3) STO▸ M multiply R1 by –2, add the product to R3 and store it in R3.	
_ R2 mRow(1/4, M, 2) STO▸ M multiply R2 by _	
R1+R2→R1 rowAdd(M, 2, 1) STO▸ M add R1 to R2 and store it in R1.	
R3-3R2→R3 mRowAdd(–3, M, 2, 3) STO▸ M multiply R2 by –3, add the product to R3, store it in R3.	

Of course we can get the last stage directly using rref(m). However, note that the full working will be required for the examination.

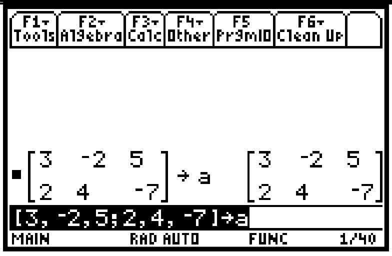
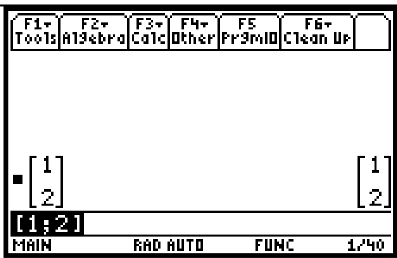
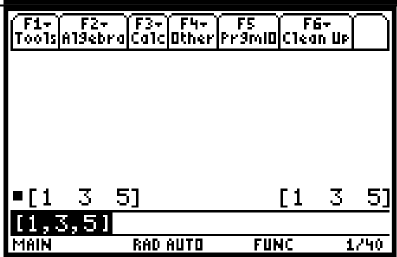
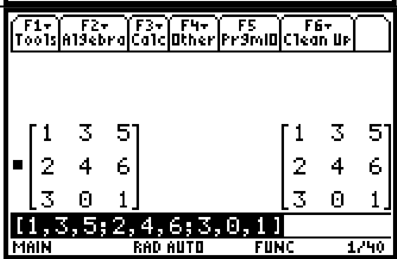
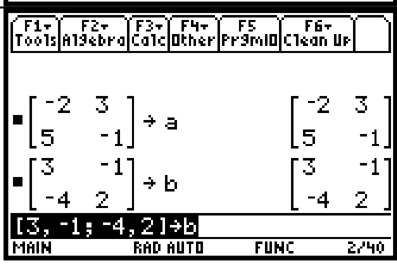
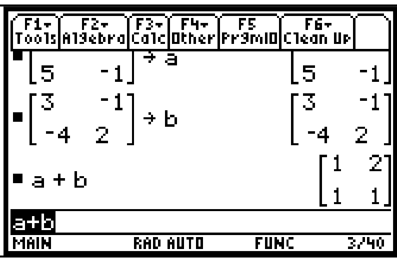
We see that the coefficients of the variables have all become zero on the bottom row, R3, so we can not have a unique solution.
Thus if $k - 1 = 0$ i.e $k = 1$ then there will be an infinite number of solutions.
If $k \neq 1$ then we get an inconsistent set of equations since R3 is false and there are no solutions.
There is never a unique solution.
When $k = 1$ $\quad x_2 - x_3 = -1$ so $x_2 = x_3 - 1$
and $\quad x_1 = 1$

so for any real number parameter t

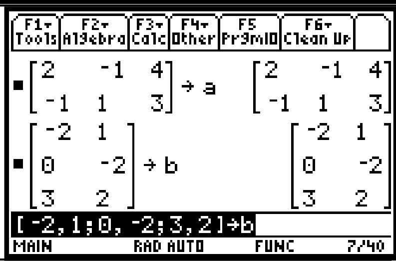
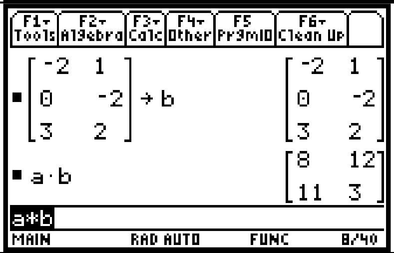
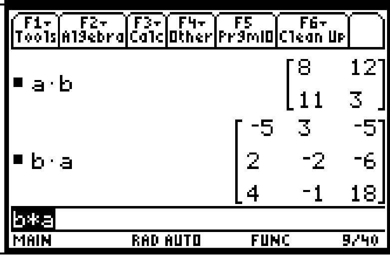
$(x_1, x_2, x_3) = (1, t - 1, t)$ a straight line.

8. Matrices

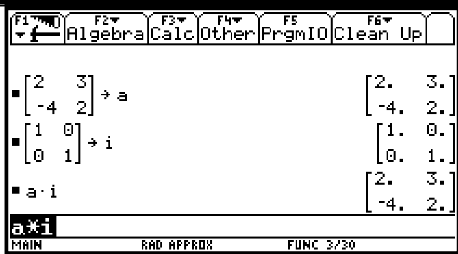
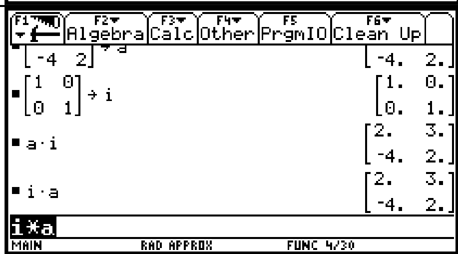
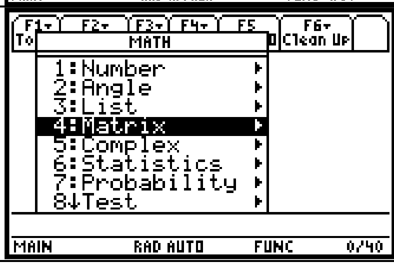
8.1 Matrix arithmetic

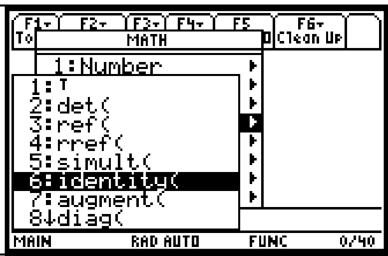
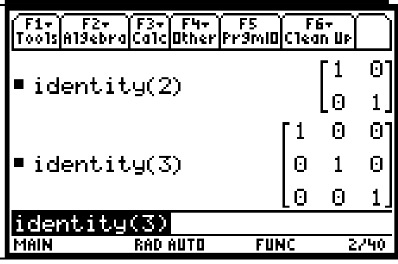
Instruction	Screen Shot
The order of the matrix is determined by the number of rows and columns it contains. **Example**. Determine a 2 × 3 matrix and represent it as A. [2nd] [[] 3 [,] –2 [,] 5 ; 2 [,] 4 [,] –7 [,] [2nd] []] [STO▸] A	
Column vector (e.g 2 × 1 matrix)	
Row vector (e.g 1 × 3 matrix)	
Square matrix (e.g 3 × 3 matrix)	
Addition of Matrices Matrices are added by adding elements in corresponding positions. ★ Matrices can only be added if they are of the same order. To perform matrix arithmetic simply define the matrices.	
Then find the sum or the difference.	

8.2 Multiplying matrices

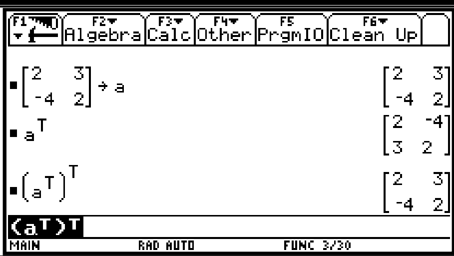
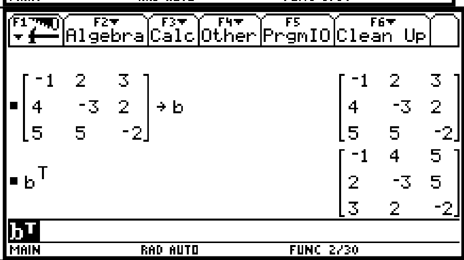
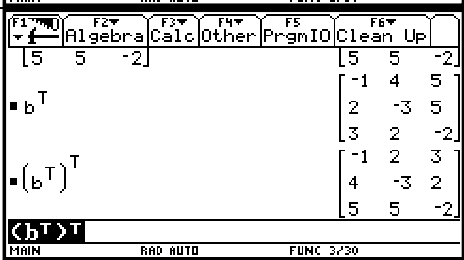
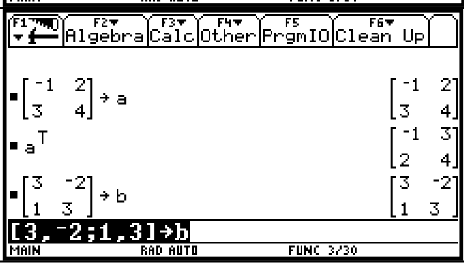
Instructions	Screen Shot
Multiplication of a Matrix by a Matrix	
	
We need the multiplication sign (*) in AB and BA: A*B and B*A	

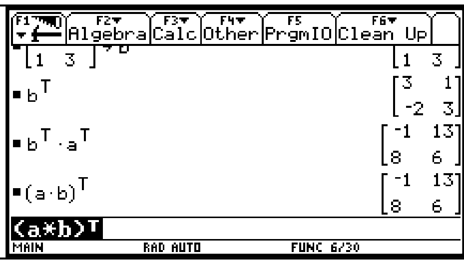
8.3 Identity Matrix

Instructions	Screen Shot
Identity matrix This is defined as that matrix *I* for which $AI = IA = A$	
	
On the TI-89 this is obtained by : [2nd] [MATH] {see 5} 4: Matrix 6: identity (then type n) for an *nxn* identity.	

	
Example:	

8.4 The Transpose of a Matrix

Instructions	Screen Shot
On the TI-89 the transpose A^T of a matrix A is obtained by : [2, 3; –4, 2] [STO▶] a [ENTER] a [2nd] [MATH] 4: Matrix ▶ 1: ^{T} [ENTER]	
	
	
Example:	



Example 8.4.1:

Consider the following matrices.

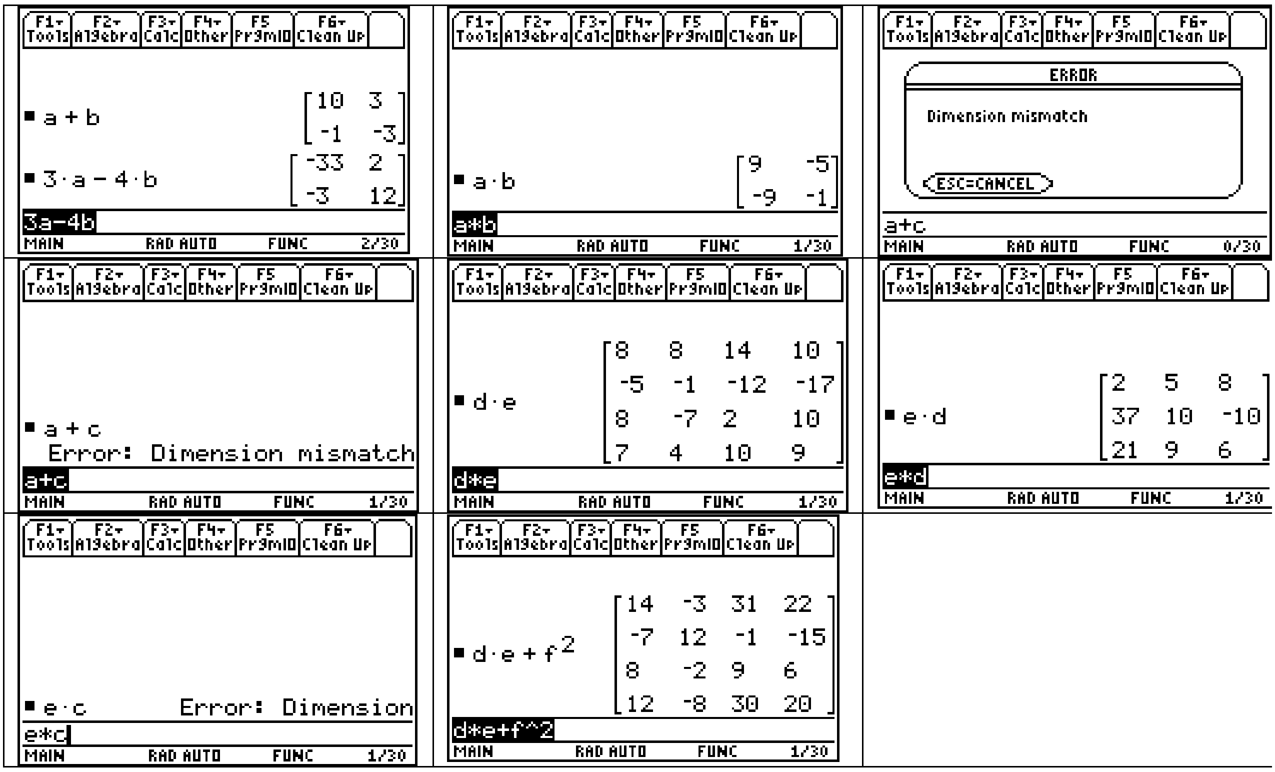
$$A=\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix} \qquad B=\begin{bmatrix}9 & 1\\ 0 & -3\end{bmatrix} \qquad C=\begin{bmatrix}1 & 2 & -3\\ 0 & 2 & 1\\ -2 & 0 & 3\end{bmatrix}$$

$$D=\begin{bmatrix}1 & 3 & 1\\ 1 & 0 & -6\\ 7 & 0 & 1\\ 2 & 2 & 1\end{bmatrix} \qquad E=\begin{bmatrix}1 & -1 & 0 & 1\\ 2 & 3 & 4 & 2\\ 1 & 0 & 2 & 3\end{bmatrix} \qquad F=\begin{bmatrix}1 & 2 & 3 & 2\\ 1 & -3 & 0 & 2\\ 1 & -3 & 2 & 0\\ 0 & 1 & 4 & 3\end{bmatrix}$$

If possible, compute each of the following.

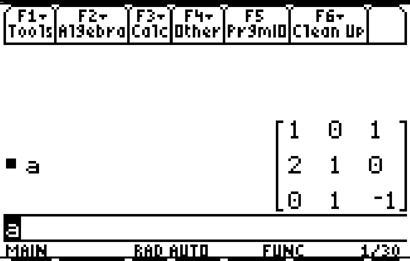
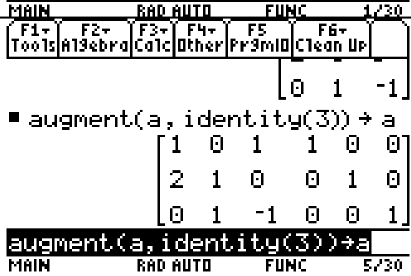
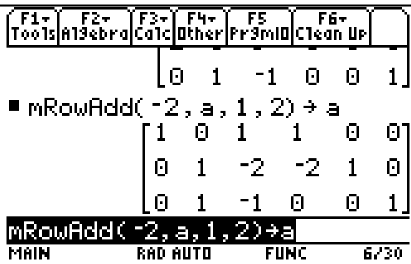
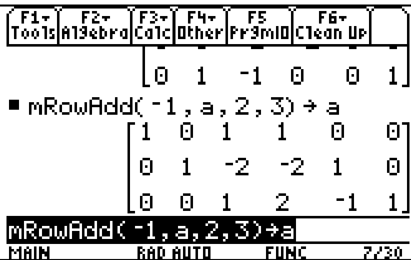
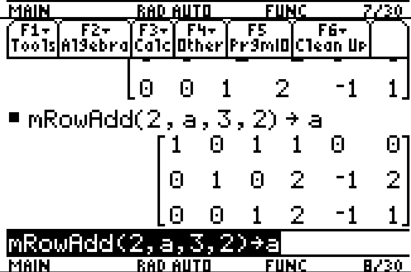
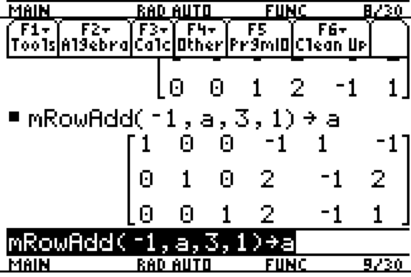
a) A + B b) 3A - 4B c) A.B
d) A + C e) D.E f) E.D
g) E.C h) D.E + F^2

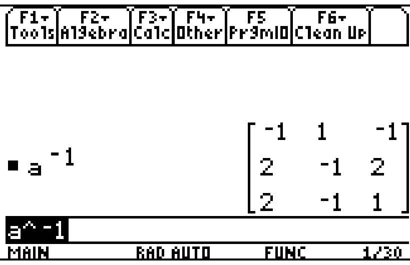
Here are the results.



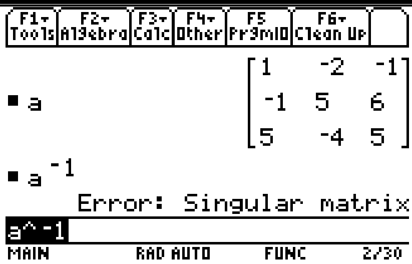
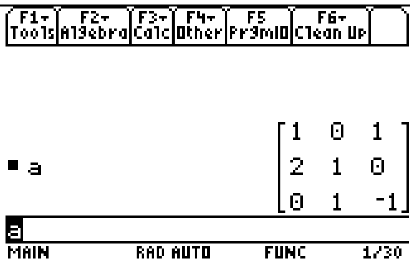
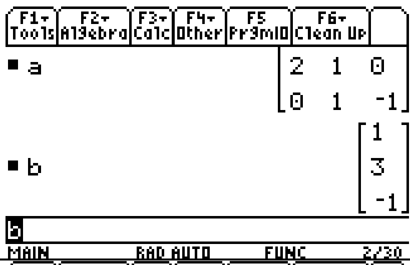
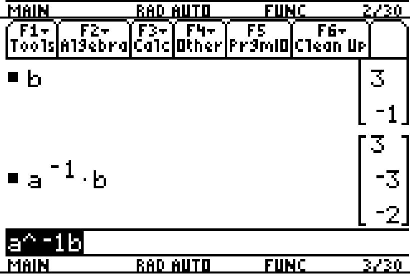
8.5 The inverse of a matrix

Example 8.5.1:

Instruction	Screen Shot
To find the inverse of a matrix enter the required matrix. The augmenting is done with the following commands:	
[2nd] [MATH] {see 5} 4: Matrix 7: augment(then [2nd] [MATH] {see 5} 4: Matrix 6: identity(then type 3)) and store the result in A using [STO▶] A.	
We would then add –2 lots of row 1 to row 2 (R2+ (–2)xR1) and store the result in A using [2nd] [MATH] {see 5} 4: Matrix J: Row ops 4: mRowAdd(mRowAdd(–2, A, 1, 2) [STO▶] A	
Continue as above, remembering to edit the last command each time to save time.	
	
Here is the inverse of the matrix.	

The TI-89 will give us the inverse of matrix A directly as follows, if it exists. j A ^ (-) 1 **NB remember to use the (-) not the minus sign!**	

8.6 Inverses and systems of Equations

Instructions	Screen Shot
Not all square matrices have an inverse (such matrices are called singular). If we try to obtain the inverse of a matrix which is not invertible on the TI-89, an error results.	
Solving systems of equations using inverses. We can solve an equation of the form $AX = B$ by multiplying both sides on the left by A^{-1} $A^{-1}AX = A^{-1}B$ $IX = A^{-1}B$ $X = A^{-1}B$ So the solution comes from calculating $A^{-1}B$. Actually this is not an efficient way to solve equations, but it is an alternative, provided the inverse exists.	
Using the TI-89 here we first enter the matrix A and the vector B.	
Then simply calculate $A^{-1}B$.	

Application of systems of linear equations

Example 8.6.1:

A baby food is to be manufactured from the ingredients carrot, cereal, and chicken. The amounts of three vitamins, measured in mg per ounce are as shown below:

	Vitamin A	Thiamine	Riboflavin
Carrot	1	0.02	0.01
Cereal	0	0.10	0.05
Chicken	0	0.02	0.05

Calculate in what proportion these ingredients should be mixed in order to produce a mixture with the three vitamins in the ratio 5:4:3.

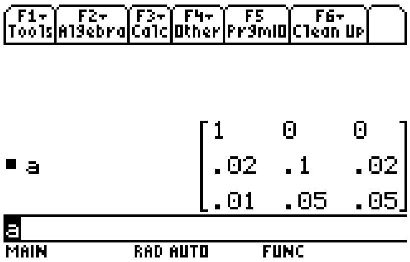
Solution:

<table>
<tr><th>Matrices</th><th>TI-89</th></tr>
<tr><td>
In matrix form we have:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0.02 & 0.1 & 0.02 \\ 0.01 & 0.05 & 0.05 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = k\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

$k>0$, and solving by row reduction to echelon form we use:

$$\begin{pmatrix} 1 & 0 & 0 & 5k \\ 0.02 & 0.1 & 0.02 & 4k \\ 0.01 & 0.05 & 0.05 & 3k \end{pmatrix}$$
</td><td>
On the TI-89 this can be approached directly, by multiplying by A^{-1}. Note the K is put inside B.

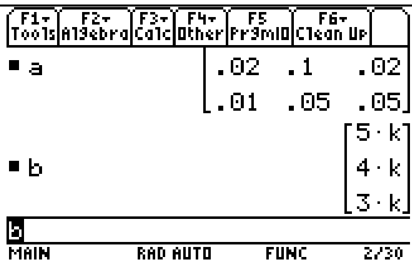


</td></tr>
<tr><td>
Note we can actually work without the *ks*.

R2x100; R3x100

$$\begin{pmatrix} 1 & 0 & 0 & 5k \\ 2 & 10 & 2 & 400k \\ 1 & 5 & 5 & 300k \end{pmatrix}$$

R2–2R1; R3–R1

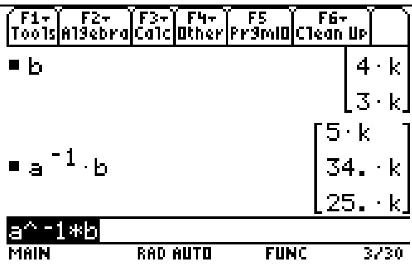
$$\begin{pmatrix} 1 & 0 & 0 & 5k \\ 0 & 10 & 2 & 390k \\ 0 & 5 & 5 & 295k \end{pmatrix}$$
</td><td>


</td></tr>
<tr><td>
R3↔R2; R3–2R2; R2/5

$$\begin{pmatrix} 1 & 0 & 0 & 5k \\ 0 & 1 & 1 & 59k \\ 0 & 0 & -8 & -200k \end{pmatrix}$$

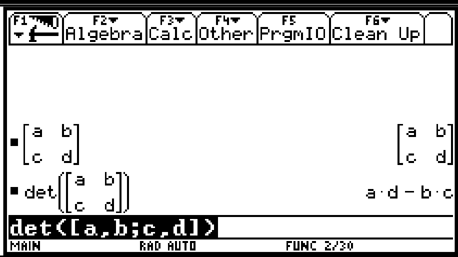
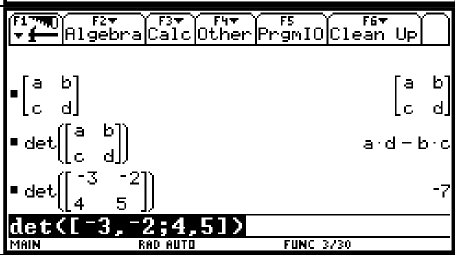
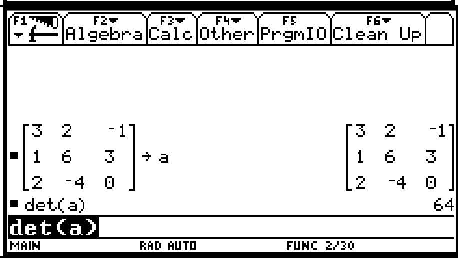
R3/–8; R2–R3

$$\begin{pmatrix} 1 & 0 & 0 & 5k \\ 0 & 1 & 0 & 34k \\ 0 & 0 & 1 & 25k \end{pmatrix}$$

so $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = k\begin{pmatrix} 5 \\ 34 \\ 25 \end{pmatrix}$ and the required ratio is 5:34:25.
</td><td>


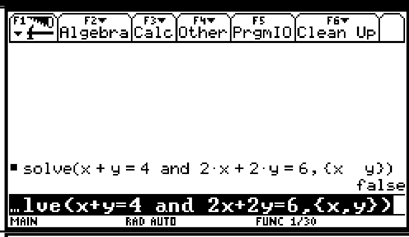
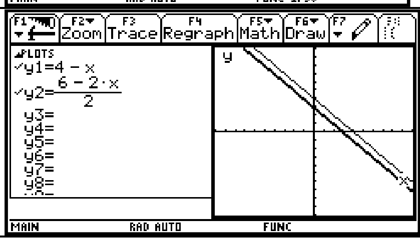
The required ratio is 5:34:25.
</td></tr>
</table>

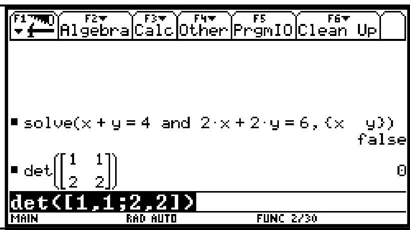
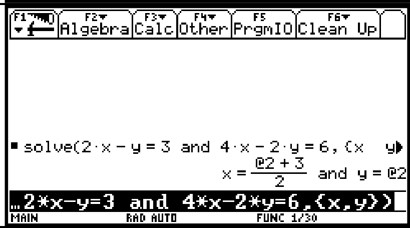
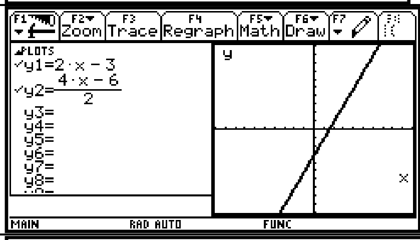
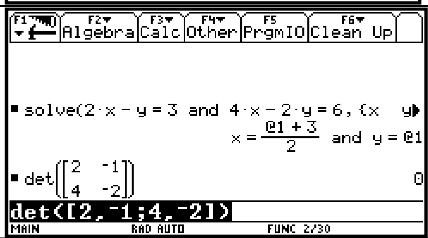
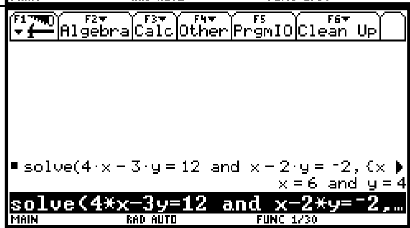
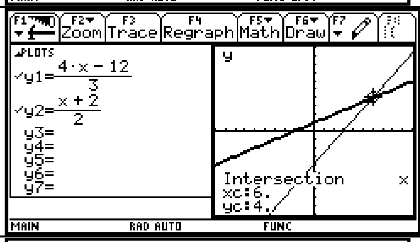
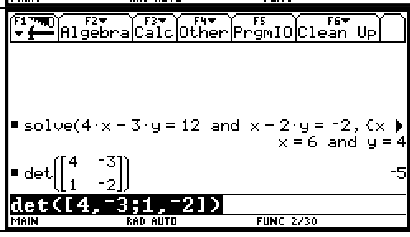
8.7 Determinants

Instructions	Screen Shots
The determinant of a 2×2 matrix is obtained **on the TI-89** by : [2nd] [MATH] 4: Matrix ▶ 2: det ([a, b ; c , d] [)] [ENTER]	
For example:	
The determinant of a 3×3 matrix is obtained **on the TI-89** by : [3, 2, –1 ; 1, 6, 3; 2, –4, 0] [STO▶] a [ENTER] [2nd] [MATH] 4: Matrix ▶ 2: det (a [)] [ENTER]	

Determinant can be used at the start of a problem on simultaneous equations to check for consistency.

Example 8.7.1:
Solve the following sets of simultaneous equations.

Equations	
a) $x+y\ = 4$ $2x+2y= 6$	
	

No Solution.		
b) 2x- y = 3 4x-2y = 6		
		
Infinitely many solutions.		
c) $4x - 3y = 12$ $x - 2y = -2$		
		
Unique solution: $x = 6$ and $y = 4$		

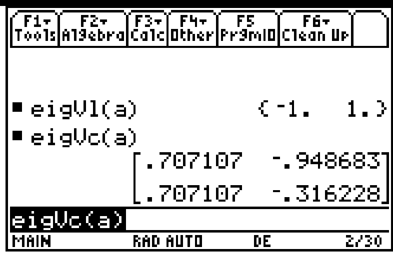
9. Vectors

9.1 Eigenvalues and Eigenvectors

Example 9.1.1:
Find the eigenvalues and eigenvectors for the following matrices:

$$\mathbf{A}=\begin{bmatrix}2 & -3\\ 1 & -2\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}2 & 1\\ -2 & 0\end{bmatrix}\quad \mathbf{C}=\begin{bmatrix}1 & 2 & -3\\ 0 & 2 & 1\\ -2 & 0 & 3\end{bmatrix}\quad \mathbf{D}=\begin{bmatrix}1 & 2 & 3 & 2\\ 1 & -3 & 0 & 2\\ 1 & -3 & 2 & 0\\ 0 & 1 & 4 & 3\end{bmatrix}$$

Solution:

Instructions	Screen Shot
Use the define option to enter the matrices as **a, b, c,** and **d**, respectively. To find the eigenvalues of the matrix **A,** use **Math 4 9 a) ENTER** or type **eigvl(a)** and **ENTER**. To find the eigenvectors of the matrix **a** use **Math 4 A a) ENTER** or type **eigvc(a).** The figure shows the eigenvalues and eigenvectors of the matrix **A**.	

Note: The first column of the result of eigenvector is an eigenvector corresponding to the eigenvalue which is listed in **eigv1(a)**, similarly for the second. Note that TI 89 is normalizing the vectors, that is the eigenvectors are unit vectors. For easier notations, it is convenient to rewrite the eigenvectors with integer entries. One possible method is to replace the smallest number in the columns by 1 and divide the other entries in that column by the smallest value you just replaced. Use the command **eigvc(a)[j,k]** to refer to the j-k entry of the matrix **eigvc(a)**. It is clear that the entries in the first column are equal.

Thus for an eigenvector corresponding to the eigenvalue -1, we may take $\begin{bmatrix}1\\1\end{bmatrix}$

The second one may not be clear so we replace -.316228 by 1. Note then that -.96683/-.316228 is 3.062. Thus it is highly recommended that you compute

eigvc(a)[2,1]/ eigvc(a)[2,2]. We find that this is 3. Thus we may take $\begin{bmatrix}3\\1\end{bmatrix}$ as the second eigenvector.

Instructions	Screen Shot
If we are evaluating **eigvl(b)**, we will get the message **Non-real result**. This means that the characteristic equation of the matrix **B** has complex roots. Note that by using the command **cSolve(det(b – x*identity(2))=0,x) ENTER**, we get the complex eigenvalues, namely, x=1+i or x=1-i as shown.