Example 7.2.2:	
Instructions	Screen Shot
First enter the augmented matrix, here	F1+ F2+ F3+ F4+ F5 F6+ ToolsAlgebraCalcOtherPrgmIDClean Up
into m1.	-1 -1 2 1
Note: To remove a variable used for a	■DelVar m1 Done
matrix or other calculation use	[1 -1 1 -5]
Ω Delvar (variable name).	■m1 3 1 -1 25
	[2 1 3 20]
	MAIN BAD EXACT FUNC
Then use the command [MATH] menu ([2nd]	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCalcOtherPr9miDClean UP
5) select 4: Matrix and 3. ref for row	2 1 3 20
echelon form,	■ref(m1)
	MAIN RAD EXACT FUNC 5/30
or 4. rret for reduced echelon form.	ToolsA19ebra Calcather Pr9mill Clean Up
	0 1 -1 10
	rref(m1)
	MAIN RAD EXACT FUNC 6/30

7.3 Systems which contain unknown coefficients

Example 7.3.1:

For what values of k does the following system of linear equations have no solution, a unique solution, or infinitely many solutions?

$$x_1 - x_2 + x_3 = 2$$

$$3x_1 + x_2 - x_3 = 2$$

$$2x_1 + x_2 - x_3 = k$$

Instructions	Screen Shots
First enter the augmented matrix, here into m, then continue as follows:	F1+ F2+ F3+ F4+ F5 F6+ Too1s A13ebra Ca1c Other Pr3ml0 C1ean Up
	$\bullet m \qquad \begin{bmatrix} 1 & -1 & 1 & 2 \\ 3 & 1 & -1 & 2 \\ 2 & 1 & -1 & k \end{bmatrix}$
PA 494 94	MAIN RAD AUTO FUNC 1/30
$R2 - 3R1 \rightarrow R2$	Tools Algebra Calc Other Pramin Clean Up
mRowAdd(-3, M, 1, 2) STON M	2 1 -1 k
multiply R1 by -3 , add the product to R2 and store it in row 2.	■ mRowAdd(-3, m, 1, 2) → m $\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 4 & -4 & -4 \\ 2 & 1 & -1 & k \end{bmatrix}$
	mRowAdd(-3,m,1,2)≯m
	MAIN RAD AUTO FUNC 2/30

	F1+ F2+ F3+ F4+ F5 F6+
$\mathbf{D}_{\mathbf{A}} = \mathbf{A} + $	Too is in second conclusiver in residual chean and
mRowAdd(-2, M, 1, 3) SION M	[2 1 ⁻ 1 k]
	■mRowAdd(-2,m,1,3)→m
multiply R1 by -2 add the product to R3	[1 -1 1 2]
and store it in P3	0 4 -4 -4
and store it in R5.	0 3 -3 k-4
	mRowAdd(-2,m,1,3)→m
	MAIN RAD AUTO FUNC 3/30
_R2	Tools Algebra Calc Other PrgmID Clean Up
mRow(1/4, M, 2) STON M	0 3 -3 4-4
	[0 3 -3 k = 4] ■ nPou(1 (4 n 2) > n
multiply DO by	$\Gamma_{1} = \Gamma_{1} = \Gamma_{1$
multiply K2 by _	
	[0 3 -3 k-4]
	MAIN BODAUTO FUNC 5/30
R1+R2→R1	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCalcOtherPr9mIOClean Up
rowAdd(M, 2, 1) STOF M	0 3 -3 k-4
	■rowAdd(m,2,1) → m
add R1 to R2 and store it in R1	Γ1 0 0 1]
add K1 to K2 and store it in K1.	0 1 -1 -1
	0 7 7 4-4
	$\frac{10 3 3 k - 4}{100}$
	MAIN RAD AUTO FUNC 5/30
R3-3R2→R3	F1+ F2+ F3+ F4+ F5 F6+ Too1s A19ebra Ca1c Other Pr9m10 C1ean Up
mRowAdd(-3 M 2 3) STON M	
multiply D2 by 2 add the product to	[0 3 -3 k - 4]
multiply K_2 by -5 , and the product to	■ mRowHad(-3, m, 2, 3) → m
R3, store it in R3.	
	0 1 -1 -1
	[0 0 0 k−1]
	mRowAdd(3, m, 2, 3)→m
	MININ RADIAUTO FUNC 6730

Of course we can get the last stage directly using rref(m). However, note that the full working will be required for the examination.

We see that the coefficients of the variables have all become zero on the bottom row, R3, so we can not have a unique solution.

Thus if k - 1 = 0 i.e. k = 1 then there will be an infinite number of solutions.

If $k \neq 1$ then we get an inconsistent set of equations since R3 is false and there are no solutions. There is never a unique solution.

When k = 1and $x_2 - x_3 = -1$ so $x_2 = x_3 - 1$ $x_1 = 1$

so for any real number parameter *t*

 $(x_1, x_2, x_3) = (1, t - 1, t)$ a straight line.

8. Matrices

8.1 Matrix arithmetic

Instruction	Screen Shot
The order of the matrix is	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIOClean UP
determined by the number of rows	
and columns it contains. Example Determine 3.2×3	
matrix and represent it as A	_ [3 -2 5] _ _ [3 -2 5]
[2nd][1]3, -2, 5; 2, 4,	[2 4 -7] ² 2 4 -7]
_7, 2nd [] STO► A	MAIN RAD AUTO FUNC 1/40
Column vector	F1+ F2+ F3+ F4+ F5 ToolsAlgebraCalcatherPrgmlaClean UP
$(e.g 2 \times 1 \text{ matrix})$	
D. (MAIN RAD AUTO FUNC 1/40
Row vector $(2, 3, 1, 2, 3, matrix)$	Tools Algebra Calc Other Pr9mill Clean Up
(e.g I × 5 maulx)	
	•[1 3 5] [1 3 5] [1,3,5]
Company on station	MAIN RAD AUTO FUNC 1/40
Square matrix $(e, g, 3, \chi, 3, matrix)$	τόοτε Ατθέδη α ζάτζ αξήση Ρηθήμα Οτεάη με
(0.g 5 × 5 matrix)	
	[135] [135]
	[3, 0, 1] $[3, 0, 1]$ $[1, 3, 5; 2, 4, 6; 3, 0, 1]$
Addition of Matrices	MAIN RAD AUTO FUNC 1/40 (F1+) F2+) F3+) F4+) F5) F6+)
Matrices are added by adding	Too1s A19ebra Ca1c Other Pr9ml0 Clean Up
elements in corresponding	[-2 3] [-2 3]
positions. \star Matrices can only be	
added if they are of the same order.	
lo perform matrix arithmetic simply	[3, -1; -4, 2] +b
Then find the sum or the difference	
Then find the sum of the difference.	Tools A19ebra Calc Other Pr9ml0 Clean UP = → a =
	[3 -1] $[3 -1]$
	[-4 2] ^{≠ b} [-4 2]
	■a+b
	a+b MAIN RADIAUTO FUNC 3/40

8.2 Multiplying matrices

Instructions	Screen Shot
Multiplication of a Matrix by a Matrix	$\begin{bmatrix} F1_{-1} & F2_{-1} & F3_{-1} & F4_{-1} & F5_{-1} & F6_{-1} & F6$
We need the multiplication sign (*) in AB and BA: A*B and B*A	$ \begin{array}{c c} \hline F1_{7} & F2_{7} \\ \hline Tools A13ebra Calc Dther Pr3miD Clean Up \\ \hline \hline \\ 0 & -2 \\ 3 & 2 \\ \end{array} \rightarrow b \\ \hline \\ \bullet & \bullet \\ \hline \\ \bullet & \bullet \\ \hline \\ \bullet & \bullet \\ \hline \\ \hline \\ \bullet & \bullet \\ \hline \\ \hline \\ \bullet & \bullet \\ \hline \\$
	D#2 MAIN RAD AUTO FUNC 9/40

8.3 Identity Matrix

Instructions	Screen Shot
Identity matrix	HIT Algebra Calc Other PromIO Clean Up
This is defined as that matrix <i>I</i> for which	[2 3] [2 3]
AI = IA = A	■ [-4 2] → a [-4. 2.]
	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \neq i \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
	• a·i
	a*i
	-4 2
	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \neq i \qquad \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
	-4. 2.
	• i · a
	IXa. MAIN RAD APPROX FUNC 4/30
On the TI-89 this is obtained by :	
[2nd] [MATH] {see 5} 4: Matrix 6: identity (1:Number
then type n) for an <i>nxn</i> identity.	2:Angle) 3:List)
51 / 5	4:Matrix ► 5:Complex ►
	6:Statistics → 7:Probability →
	84Test
	MAIN RAD AUTO FUNC 0740

	F1- F2- F3- F4- F5- F6- To MATH DC1can Up 1:T D 2:det(D 3:ref(D 4:rref(D 5:simult(D 6:identitu(D 7:augment(D 8-diag(MAIN MAIN RAD AUTO
Example:	F1: F2: F3: F4: F5: F6: F

8.4 The Transpose of a Matrix

Instructions	Screen Shot
On the TI-89 the transpose A^{T} of a	F17700 F2▼ → H1gebra Calc Other PrgmIO Clean Up
matrix A is obtained by : $[2, 3; -4, 2]$ STO a	[2 3]
[ENTER] a [2nd] [MATH] 4: Matrix ► 1: 1 [ENTER]	[-4 2] T [2 -4]
	• a` [3 2]
	CAT)T MAIN RAD AUTO FUNC 3/30
	F1770 Algebra Calc Other PrgmIO Clean Up
	[-1 2 3] [-1 2 3]
	■ 4 -3 2 → b 4 -3 2 5 5 -2 5 5 -2
	■ bT [-1 4 5]
	3 2 -2
	MAIN RAD AUTO FUNC 2/30
	F177700 F2 F3 F3 F4 F5 F5 F6 UP
	[¹ ¹ ¹ ¹ ¹
	■b' 2 -3 5 3 2 -2
	$\begin{bmatrix} -1 & 2 & 3 \\ 4 & -3 & 2 \end{bmatrix}$
	5 5 -2
	MAIN RAD AUTO FUNC 3/30
Example:	F17700 F27 HigebraCalcOtherPrgmIOClean Up
	[-1 2]
	[3 4] ^{7 a} [3 4] T [-1 3]
	• a' 2 4
	$ \begin{bmatrix} 3 & 12 \\ 1 & 3 \end{bmatrix} \Rightarrow b \begin{bmatrix} 3 & 12 \\ 1 & 3 \end{bmatrix} $
	[3, -2;1,3]→b MAIN RAD AUTO FUNC 3/30

F17700 F2▼ F17700 F2 F17700	Other PrgmIO Clean Up	
• b ^T	$\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$	
■b ^T ·a ^T	[-1 13 [8 6] [-1 13]	
■(a+b) (a+b) MAIN RAD AUTO		

_

Example 8.4.1:

Consider the following matrices.

$\mathbf{A} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 2\\0 \end{bmatrix}$ E	$\mathbf{B} = \begin{bmatrix} 9 & 1 \\ 0 & -3 \end{bmatrix}$	$C = \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ -2 & 0 \end{bmatrix}$	$\begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$			
$\mathbf{D} = \begin{bmatrix} 1\\1\\7\\2 \end{bmatrix}$	3 1 0 -6 0 1 2 1	$\mathbf{E} = \begin{bmatrix} 1\\ 2\\ 1 \end{bmatrix}$		$F = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$	2 -3 -3 1	3 0 2 4	2 2 0 3
If possib a) d) g)	ble, compute A + B A + C E.C	ute each of th b) e) h)	the following. 3A - 4B D.E $D.E + F^2$	c) f)		A.B E.D)

_

Here are the results.

F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCa1cOtherPr9mlOClean UP	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCa1cOtherPr9mlOClean Up	F1+ F2+ F3+ F4+ F5 ToolsAl3ebraCalcOtherPr3mIDClean UP
■a+b [10 3 [-1 -3]		Dimension mismatch
■ 3·a - 4·b [-33 2 -3 12]	■a·b [9 -5] -9 -1]	
38-46 Main Rad Auto Func 2/30	AMAIN BAD AUTO FUNC 1/30	A+C MAIN BAD AUTO FUNC 0/30
F1+ F2+ F3+ F4+ F5 ToolsAl3ebraCalcOtherPr3miDClean UP	F1+ F2+ F3+ F4+ F5 Too1s A19ebra Ca1c Dther Pr9milD C1ean Up	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCa1c0therPr9mi0Clean Up
■a+c _Error: Dimension mismatch	■d·e $\begin{bmatrix} 8 & 8 & 14 & 10 \\ -5 & -1 & -12 & -17 \\ 8 & -7 & 2 & 10 \\ 7 & 4 & 10 & 9 \end{bmatrix}$	■e·d [2 5 8 37 10 -10 21 9 6
a+C Main Repairts Func 1/30	d*e Main Bablauth Fund 1/30	E*6 MAIN RADIAUTO FUNC 1/30
F1+ F2+ F3+ F4+ F5 ToolsAl9ebra[Calc]Other[Pr9mil]Clean Up	F1+ F2+ F3+ F4+ F5 F6+ Too15A13ebra[Cate]BtherPr3ml0[Clean UP]	
	■ d·e+f ² [14 -3 31 22] -7 12 -1 -15 8 -2 9 6	
■e·c Error: Dimension		
e*C Main rad auto func 1/30	MAIN RAD AUTO FUNC 1/30	

8.5 The inverse of a matrix

Example 8.5.1:

Instruction	Screen Shot
To find the inverse of a matrix enter the	F1+ F2+ F3+ F4+ F5 F6+
required matrix The augmenting is	
done with the following commands:	
done with the following communes.	Г1 0 1 T
	■a 210
	0 1 -1
	8
	MAIN RAD AUTO FUNC 1/30 (F1+) F2+ (F3+) F4+ (F5) F6+ ()
[Ord [MATH] (ass 5) 4. Matrix 7. anomant(Too1s A19ebra Ca1c Other Pr9ml0 Clean Up
[2nd] [MATH] {see 5} 4. Matrix 7. augment	LO 1 -1]
then $[2nd]$ [MATH] {see 5} 4: Matrix 6:	■augment(a,identity(3))→a
identity(
then type 3)) and store the result in A	
using STO► A.	LU 1 -1 U U 1 augment(a identitu(3))ta
	MAIN RAD AUTO FUNC 5/30
	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIDClean Up
We would then add -2 lots of row 1 to	
row 2 (R2+ $(-2)xR1$) and store the result	■ mRow8dd(:2,a,1,2) → a
in A using	[1 0 1 1 0 0]
[2nd] [MATH] {see 5} 4: Matrix J: Row ops	0 1 -2 -2 1 0
4: mRowAdd(
mRowAdd(-2 A 1 2) STON A	mRowAdd(2, a, 1, 2)≯a
Continue as above remembering to edit	
the last command each time to save time	TooisiHisebralcaiclutherirrsmilliciean up
the last command each time to save time.	
	■MROWHOO(-1,a,2,3)≠a [1 0 1 1 0 0]
	0 1 -2 -2 1 0
	0 0 1 2 -1 1
	mRowAdd(⁻1,a,2,3)→a
	MAIN RAD AUTO FUNC 7/30
	Too1s A19ebra Ca1c Other Pr9ml0 C1ean Up
	■mRowAdd(2,a,3,2)→a [1 0 1 1 0 0]
	0 0 1 2 -1 1
	mRowAdd(2,a,3,2)→a
II	MAIN RAD AUTO FUNC 8/30
riele is the inverse of the matrix.	Tööls Al9ebra Calc Other Pr9mIO Clean UP
	0012-11
	■mRowAdd(-1,a,3,1)→a
	$\begin{bmatrix} 0 & 0 & 1 & 2 & -1 & 1 \end{bmatrix}$
	MACOWINESS 1, 3, 3, 1773

The TI-89 will give us the inverse of matrix A directly as follows, if it exists.	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCa1cOtherPr9miDClean Up
NB remember to use the (-) not the minus sign!	$\bullet a^{-1} \begin{bmatrix} -1 & 1 & -1 \\ 2 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$
	a^-1 MAIN RAD AUTO FUNC 1/30

Instructions **Screen Shot** F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOther Pr9mIOClean UP Not all square matrices have an inverse (such matrices are called singular). If we 1 -2 -11 -1 try to obtain the inverse of a matrix which ∎ a 5 6 is not invertible on the TI-89, an error 5 5 -4 ∎ a ⁻¹ results. Error: Singular matrix RAD AUTO Solving systems of equations using F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIOClean Us inverses. We can solve an equation of the AX = Bform by multiplying both sides on the left by 1 01 A^{-1} ∎ a 2 1 0 $A^{-l}AX = A^{-l}B$ Θ 1 a Main RAD AUTO 1/30 FUNC $IX = A^{-1}B$ $X = A^{-1}B$ So the solution comes from calculating $A^{-1}B$. Actually this is not an efficient way to solve equations, but it is an alternative, provided the inverse exists. Using the TI-89 here we first enter the matrix A and the vector B. F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIOClean Us ∎ a 2 1 Θ 0 1 -1 1 ∎b 3 b MAIN <u>rad auto</u> FUN Then simply calculate $A^{-1}B$. F1+ F2+ F3+ F4+ ToolsAl9ebraCalcOther F5 F6+ Pr9mIOClean Up ∎ h 3 -1 3 -1. L ∎ a -3 -2 a^-1b Main

8.6 Inverses and systems of Equations

3730

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Application of systems of linear equations Example 8.6.1:

A baby food is to be manufactured from the ingredients carrot, cereal, and chicken. The amounts of three vitamins, measured in mg per ounce are as shown below:

	Vitamin A	Thiamine	Riboflavin
Carrot	1	0.02	0.01
Cereal	0	0.10	0.05
Chicken	0	0.02	0.05

Calculate in what proportion these ingredients should be mixed in order to produce a mixture with the three vitamins in the ratio 5:4:3.

Solution:

Matrices	TI-89
In matrix form we have:	On the TI-89 this can be approached directly,
$(1 \ 0 \ 0)(x) \ (5)$	by multiplying by A^{-1} . Note the <i>K</i> is put inside
$\begin{vmatrix} 0.02 & 0.1 & 0.02 \end{vmatrix} \begin{vmatrix} y \end{vmatrix} = k \begin{vmatrix} 4 \end{vmatrix}$	B.
$\begin{pmatrix} 0.01 & 0.05 & 0.05 \end{pmatrix} \begin{pmatrix} z \\ z \end{pmatrix}$ (3)	Tools Algebra Calc Other Pr9mil Clean Up
$k \ge 0$ and solving by row reduction to echelon	
form we use:	F1 0 0 1
(1 0 0 5k)	• a .02 .1 .02
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.01 .05 .05
$(0.01 \ 0.05 \ 0.05 \ 3k)$	
Note we can actually work without the ks.	Tools Algebra Calc Other Prgm10 Clean Up
R2x100; R3x100 (1 0 0 51)	• a .02 .1 .02
$\begin{pmatrix} 1 & 0 & 0 & 5k \end{pmatrix}$	[.01 .05 .05] [5.4]
$\begin{bmatrix} 2 & 10 & 2 & 400k \end{bmatrix}$	■b 4·k
$\begin{pmatrix} 1 & 5 & 5 & 300k \end{pmatrix}$	 [3·k]
R2–2R1; R3–R1	D MAIN BAD AUTO FUNC 2/30
$(1 \ 0 \ 5k)$	
$\begin{bmatrix} 0 & 10 & 2 & 390k \end{bmatrix}$	
$\begin{pmatrix} 0 & 5 & 5 & 295k \end{pmatrix}$	
R3⇔R2; R3–2R2; R2/5	F1+ F2+ F3+ F4+ F5 ToolsA13ebra[Ca]cl0ther/Pr3mi0[Clean UP]
$(1 \ 0 \ 0 \ 5k)$	■b 4·k
$\begin{bmatrix} 0 & 1 & 1 & 59k \end{bmatrix}$	[3·k]
$\begin{pmatrix} 0 & 1 & 1 & 200k \\ 0 & 0 & 8 & 200k \end{pmatrix}$	[5·k]
$(0 \ 0 \ -8 \ -200k)$	■a *·b 34.·k
(1, 0, 0, 5k)	a^-1*b
	MAIN RAD AUTO FUNC 3/30
$\begin{bmatrix} 0 & 1 & 0 & 34k \end{bmatrix}$	The required ratio is $5.34.25$
$(0 \ 0 \ 1 \ 25k)$	
(x) (5)	
so $ y = k 34 $ and the	
$\begin{pmatrix} z \end{pmatrix}$ $\begin{pmatrix} 25 \end{pmatrix}$	
required ratio is 5:34:25.	
A	

8.7 Determinants

In show stings	Savaar Shata
The determinant of a 2×2 matrix is obtained on the TI-89 by :	Firm Firm Fire Fire Fire Fire Fire Fire Fire Fire
[2nd [MATH] 4: Matrix ► 2: det ([a, b ; c , d]) ENTER	• $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ • det $\begin{pmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $a \cdot d - b \cdot c$ det ([a, b; c, d]) MAIN RAD AUTO FUNC 2/30
For example:	$ \begin{array}{c} \begin{array}{c} f1 & \hline \\ \hline & \hline \\ \hline$
The determinant of a 3×3 matrix is obtained on the TI-89 by : [3, 2, -1; 1, 6, 3; 2, -4, 0] STOP a ENTER 2nd [MATH] 4: Matrix \triangleright 2: det (a) ENTER	$ \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix} $ $ = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix} $ $ = \det(a) $ $ = \det(a) $ $ = \det(b) $

Determinant can be used at the start of a problem on simultaneous equations to check for consistency.

Example 8.7.1:

Solve the following sets of simultaneous equations.

Equations	
a) $x+y = 4$	F17770 F2★ F3★ F4★ F5 ★
2x+2y=6	
	• solve($x + y = 4$ and $2 \cdot x + 2 \cdot y = 6$, ($x = y$))
	$\frac{1}{100} = \frac{1}{100} = \frac{1}$
	F17700 F2+ F3 ↓ ↓ Zoom Trace Regraph Math Draw ↓ ↓ ↓ ↓ ↓
	^{₽LOTS} √y1=4 − ×
	$y_{22} = \frac{6 - 2 \cdot x}{2}$
	93= 94= 95=
	96= 97=
	MAIN RAD AUTO FUNC

No Solution.	🖓 🚰 Algébra Cálc Other Prim IO Cleán Up
	■solve(x+y=4 and 2·x+2·y=6,(x y))
	false $det \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ 0
	(12 21) det (1,1;2,2]) MAIN RAD RUTD FUNC 2/30
b) $2x - y = 3$	[1] 700) F2▼ → ∰ Algebra[Calc]Other PrgmIO Clean Up]
4x-2y=0	
	■solve(2·x - y = 3 and 4·x - 2·y = 6, (x y)
	$x = \frac{02 + 3}{2} \text{ and } y = 02$ 2*x-y=3 and 4*x-2*y=6,{x,y}}
	Minik Rab Auto FUK 1/300 F1 7780) F2→ F1 + + 2000m Trace Regraph Math Draw + 2 11 11
	Artu2 Artu2
	<u> </u>
Infinitaly many ashtions	
infinitely many solutions.	▼ <u>f</u> ~~ Algebra Calc Other PrgmIO Clean Up
	■ solve(2·x - y = 3 and 4·x - 2·y = 6, (x _ y) @1 + ₹
	$\times = \frac{G_{1} + G_{2}}{2} \text{ and } y = 01$
	det<[2,-1];4,-2])
c) $4x - 3y = 12$	F17700 F27 - Algebra Calc Other PrgmIO Clean Up
x - 2y = -2	
	■ solve(4·x - 3·y = 12 and x - 2·y = -2, (x) x = 6 and y = 4 solve(4*x-3y=12 and x-2*y=-2
	Main BAD BUTD FUUC 1/200 F1™™ F2× F3 F4 F6× F6× F7 F1 ★ F1000 F1× F6× F6× F7 F1 F1
	$\frac{\varphi_{LOTS}}{\sqrt{y_1=\frac{4\cdot x-12}{3}}}$
	$\begin{array}{c} \sqrt{22} = \frac{x+2}{2} \\ y_{3}^{2} = 2 \end{array}$
	95= 96= 97= 97= xci6, t
Unique solution: $x = 6$ and $y = 4$	₩ f[™] Algebra Cálc Other PrgmIO Cleán Up
	\blacksquare solue(4.v - 3.u = 12 and v - 2.u = -2. (v)
	$det \begin{bmatrix} 4 & -3 \\ -5 \end{bmatrix} = -5$
	U_1 -2JJ det([4,-3;1,-2]) MAIN RAD AUTO FUNC 2/30

9. Vectors

9.1 Eigenvalues and Eigenvectors

Example 9.1.1:

Find the eigenvalues and eigenvectors for the following matrices:

		Г 1	2	21		1	2	3	2
[2 -3]	\mathbf{p} $\begin{bmatrix} 2 & 1 \end{bmatrix}$	\mathbf{C}	2	-3	D _	1	-3	0	2
$A = \begin{vmatrix} 1 & -2 \end{vmatrix}$	$\mathbf{B} = \begin{bmatrix} -2 & 0 \end{bmatrix}$	$\mathbf{C} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	2		D=	1	-3	2	0
		[-2	0	3		0	1	4	3

Solution:

Instructions	Screen Shot
Use the define option to enter the matrices as a , b , c , and d , respectively. To find the eigenvalues of the matrix A , use Math 4 9 a) ENTER or type eigvl(a) and ENTER . To find the eigenvectors of the matrix a use Math 4 A a) ENTER or type eigvc(a) . The figure shows the eigenvalues and eigenvectors of the matrix A .	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIDClean UP eigV1(a) (-1. 1.) eigV2(a) [.707107 948683] .707107 948683] .707107 eigUc(a) [.707107 316228] eigUc(a) MAIN RAD AUTO DE 2/30

Note: The first column of the result of eigenvector is an eigenvector corresponding to the eigenvalue which is listed in **eigv1(a)**, similarly for the second. Note that TI 89 is normalizing the vectors, that is the eigenvectors are unit vectors. For easier notations, it is convenient to rewrite the eigenvectors with integer entries. One possible method is to replace the smallest number in the columns by 1 and divide the other entries in that column by the smallest value you just replaced. Use the command **eigvc(a)[j,k]** to refer to the j-k entry of the matrix **eigvc(a)**. It is clear that the entries in the first column are equal. Thus for an eigenvector corresponding to the eigenvalue -1, we may take $\begin{bmatrix} 1\\1\\1 \end{bmatrix}$. The second one may not be clear so we replace -.316228 by 1. Note then that -.96683/-.316228 is 3.062. Thus it is highly recommended that you compute **eigvc(a)[2,1]/ eigvc(a)[2,2]**. We find that this is 3. Thus we may take $\begin{bmatrix} 3\\1\\1 \end{bmatrix}$ as the second eigenvector.

Instructions	Screen Shot
If we are evaluating eigvl(b) , we will get the message Non-real result . This means that the characteristic equation of the matrix B has complex roots. Note that by using the command cSolve(det(b - x*identity(2))=0,x) ENTER , we get the complex eigenvalues, namely, $x=1+i$ or $x=1-i$ as shown.	Fit F2+ F3- F4+ F5 ToolsAlgebra Calc ather Pr3m10 Clean UP = eigV1(b) Error: Non-real result = cSolve(det(b - \times ·identity() $\times = 1 + i$ or $\times = 1 - i$ et(b- \times *identity(2))=0, \times) MAIN RAD AUTO DE 2/30