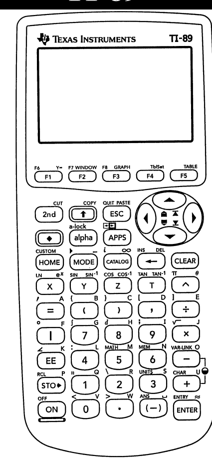
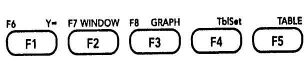
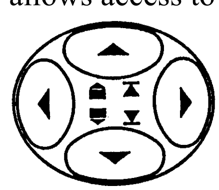
Introduction to Using the TI-89 Calculator

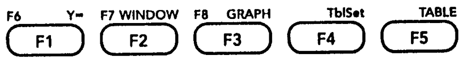
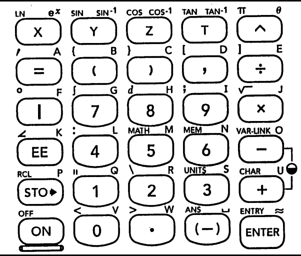
Note: If this is the first time that you have used the TI-89 computer algebra system (CAS) calculator then you should first work through the Introduction to Using the TI-89. Some of the information found there in terms of key presses, menus, etc. will be assumed in what follows. While much of the information provided here describes how to carry out many mathematical procedures on the TI-89, using technology is not simply about finding answers, and we would encourage you NOT to see this calculator as primarily a quick way to get answers. Research by the authors, and others, shows that those who do this tend to come to rely on the calculator to the detriment of their mathematical understanding. Instead learn to see the TI-89 as a problem-solving, and investigative tool that will help you to understand concepts by providing different ways of looking at problems, thus helping you reflect on the underlying mathematics.

1. Saying 'Hello' to your graphics calculator

Instructions	TI-89
You will use the following keys. a) Press [ON] The calculator cursor should be in the Home Screen (see the black cursor flashing in the bottom left hand corner). • Press [2nd] [ON] The calculator should turn off. • If you can't see the screen use [♦] [+] (darker) or [♦] [−] (lighter) to change screen contrast. • [HOME] displays the Home Screen, where you perform most calculations.	

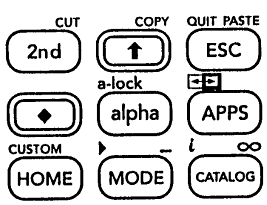
Basic Facilities of the TI-89

Function Keys	Cursor Pad
[F1] through [F8] function keys let you select toolbar menus. 	The cursor is controlled by the large blue circle on the top right hand side of the calculator. This allows access to any part of  an expression.

Application Short Keys	Calculator Keypad
Used with the ◆ key to let you select commonly used applications: [Y=] [WINDOW] [GRAPH] [TblSet] [TABLE] 	Performs a variety of mathematical and scientific operations. 

2nd ◆ ↑ and j modify the action of other keys:

Modifier	Description
2nd (Second)	Accesses the second function of the next key you press
◆ (Diamond)	Activates "shortcut" keys that selects applications and certain menu items directly from the keyboard.
↑ (Shift)	Types an uppercase character for the next letter key you press.
j	Used to type alphabetic letters, including a space character. On the keyboard, these are printed in the same colour as the j key. 2nd j used to type alphabetic letters.

	Key	Description
	APPS	Displays a menu that lists all the applications available on the TI-89.
	ESC	Cancels any menu or dialogue box.
	ENTER	Evaluates an expression, executes an instruction, selects a menu item, etc.
	MODE	Displays a list of the TI-89's current mode settings, which determine how numbers and graphs are interpreted, calculated, and displayed.
	CLEAR	Clears (erases) the entry line.
	[CATALOG]	Press b or c to move the d indicator to the function or instruction. (You can move quickly down the list by typing the first letter of the item you need.) Press ENTER. Your selection is pasted on the home screen.

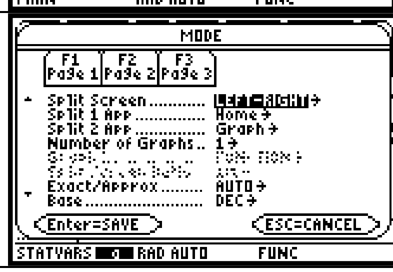
Application	Lets you:
[Home]	Enter expressions and instructions, and performs calculations
[Y=]	Define, edit, and select functions or equations for graphing
[Window]	Set window dimensions for viewing a graph
[Graph]	Display graph
[Table]	Display a table of variable values that correspond to an entered function

Press:	To display
F1, F2, etc.	A toolbar menu– Drops down from the toolbar at the top of most application screens. Lets you select operations useful for that application
2nd [CHAR]	CHAR menu– Lets you select from categories of special characters (Greek, math, etc.)
2nd [MATH]	MATH menu– Lets you select from categories of mathematical operations

Press	To perform
2nd [F6]	Clean Up to start a new problem:
Clear a–z	Clears (deletes) all single-character variable names in the current folder. If any of the variables have already been assigned a value, your calculation may produce misleading results.

Problem?	Try this!
If you make a typing error	If you make a typing error use ← to undo one character at a time If necessary, also press M to clear the complete line.
If you want to clear the home screen completely	Press F1 n

Mode Settings

Instructions	Screen Shot
Press MODE, which shows the modes and their current settings	
If you press **F2** then 'Split Screen' specifies how the parts are arranged: FULL (no split screen), TOP-BOTTOM, or LEFT-RIGHT	

(a) Entering a Negative Number

Instructions	Examples
Use | for subtraction and use ∑ for negation. To enter a negative number, press ∑ followed by the number.	To enter the number –7, press (-) 7. 9 × (-) 7 = –63, 9 × − 7 = displays an error message To calculate –3 – 4, press (-) 3 − 4 ENTER

(b) Implied Multiplication

If you enter:	The TI-89 interprets it as:
2*a*	2**a*
xy	Single variable named *xy*; TI-89 does not read as *x***y*

(c) Substitution

Instructions	Examples
Using [2nd] [|]key	eg) [(-)] *x*^2+2 | *x*=3 −7 This calculates the value of $-x^2 + 2$ when $x = 3$
Using 'STORE' key: [STO▸]	eg) Find *f*(2) if $f(x) = -x^3 + 2$ [(-)] *x*^3+2 [STO▸] *f*(*x*) $-x^3 + 2 \rightarrow f(x)$ *f*(2) -6

(d) Rational Function Entry

Instructions	Example
$\frac{f(x)}{g(x)} = \frac{(f(x))}{(g(x))}$ = **(numerator)** [÷] **(denominator)**	$\frac{x+1}{2x-1} \rightarrow (x+1)$ [÷] $(2x-1)$

(e) Operators

addition: + subtraction : – multiplication: × division: ÷ Exponent: ^

(f) Elementary Functions

Exponential: e^(*x*)	Trigonometric:
natural logarithm: ln(*x*) square root: √ absolute value: abs(*x*)	sin(*x*), cos(*x*), tan(*x*), $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$ If you want sec(*x*) then put 1/cos(*x*) , cosec(*x*) is 1/sin(*x*). Note: The trigonometric functions in TI-89 angles are available in both degrees and radians. If you want degrees (180°) or radians (π) change using the [MODE] key previously discussed.

(g) Constants

To find:	Work
i : imaginary number	**with** [2nd] **key**
π : Pi	**with** [2nd] **key**
∞ : infinity	**with** [♦] **key**

(h) Recalling the last answer

Instructions	Example	
[2nd] [ANS]	ans(1)	Contains the last answer
	ans(2)	Contains the next-to-last answer

(i) Cutting, Copying and Pasting

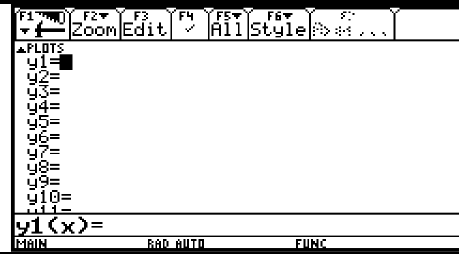
Press:	To:
[↑] [►] or [↑] [◄]	highlight the expression.
[♦] 5, [♦] 6 and [♦] 7	cut, copy and paste.
[2nd] [ENTRY]	replace the contents of the entry line with any previous entry.

(j) When differentiating with respect to *x*

To find:	Type:
Limit $\lim_{x \to a} f(x)$:	**lim(*f(x)*, *x* , *a*)**
Indefinite Integral $\int f(x)dx$:	**∫ (*f(x)* , *x*, *c*)**
Definite integral $\int_a^b f(x)dx$:	**∫ (*f(x)*, *x*, *a*, *b*)**
Area between *f(x)* and *g(x)* on the interval [*a*, *b*]:	$\int_a^b \lvert f(x) - g(x) \rvert \, dx$
Differentiation $\frac{d}{dx} f(x)$:	*d*(*f(x)* , *x*)

2. [Y=] and [Table]

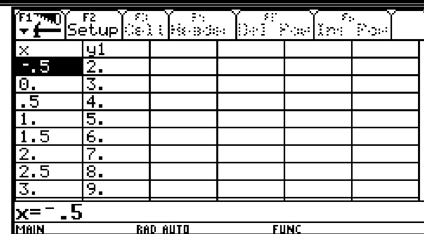
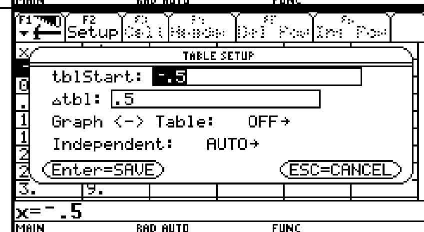
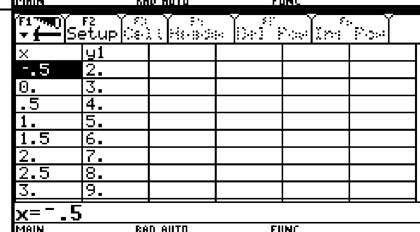
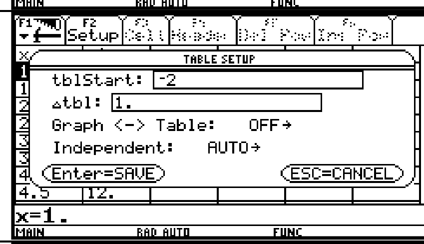
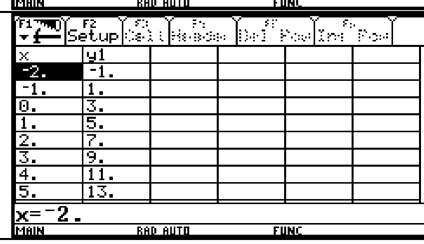
(a) The [Y=] menu

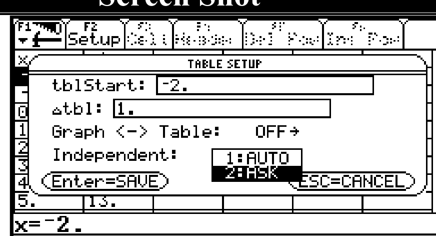
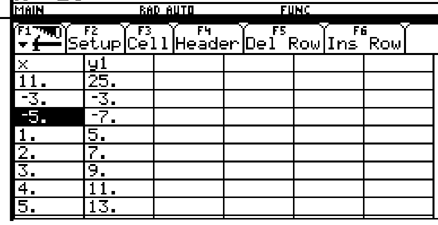
Instructions	Screen Shot
Press [♦] [Y=] to see the following:	

If there are any functions to the right of any of these eight equal signs, place the cursor on them (using the arrow keys) and press [CLEAR]
Place the cursor just to the right of y1= and follow the sequence below.

Press	See	Explanation
2*x* ´ 3	y1(*x*) = 2*x* + 3	You have entered y1 = 2*x* + 3
[HOME]		This returns you to a blank Home Screen.
y1(*x*) ∏	y1(*x*) 2*x* + 3	This pastes y1 on the Home Screen.
y1(4) ∏	y1(4) 11	This finds the value of y1 when *x* = 4.

(b) Table

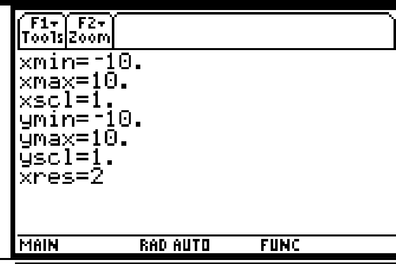
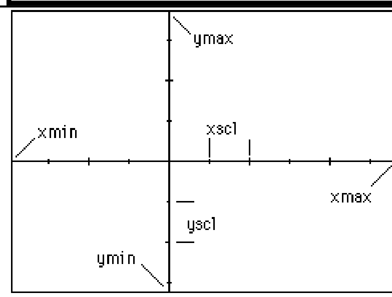
Instructions	Screen Shot
Press [♦] [TABLE] to see the table of values for $2x + 3$, as shown below:	
Press [♦] [TblSet], try change the settings and see the effect in [TABLE].	
	
	
	

Instructions	Screen Shot
By changing [TblSet] from [1. AUTO] to [2.ASK], complete the table below: Remember: y1 is still set to $2x + 3$	
(see table below)	

x	y1
11	?
-3	?
-5	?

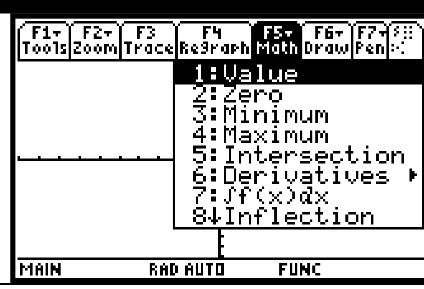
3. Graphing

(a) Displaying Window Variable in the Window Editor

Instructions	Screen Shot
Press [♦] **[WINDOW]** or [APPS] 3 to display the Window Editor.	
	

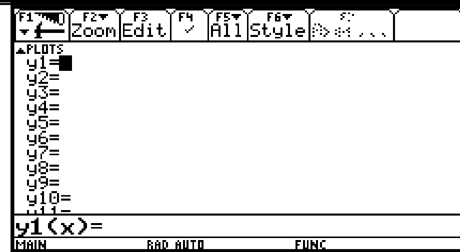
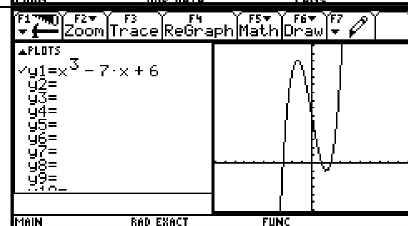
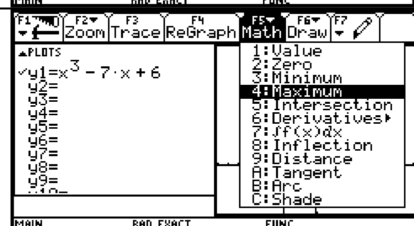
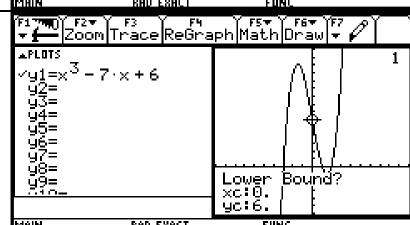
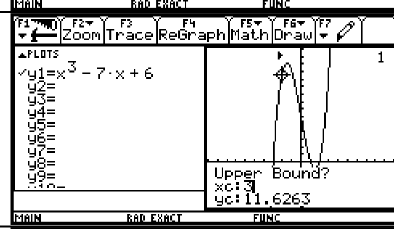
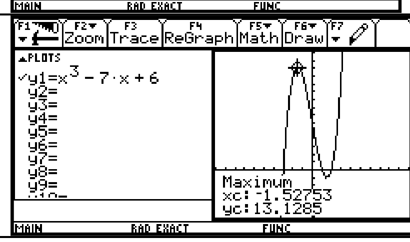
Variables	Description
*x*min, *x*max, *y*min, *y*max	Boundaries of the viewing window.
*x*scl, *y*scl	These *x* and *y* scales set the distance between tick marks on the *x* and *y* axes (see above right)..
*x*res	Sets pixel resolution (1 through 10) for function graphs. The default is 2.

(b) Overview of the Math Menu

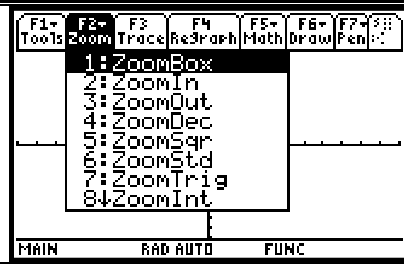
Press **F5** from the Graph screen	

Math Tool	Description
Value	Evaluates a selected $y(x)$ function at a specified x value
Zero, Minimum, Maximum	Finds a zero (x-intercept), minimum, or maximum point within an interval.
Intersection	Finds the intersection of two functions.
Derivatives	Finds the derivative (slope) at a point.
$\int f(x)dx$	Finds the approximate numerical integral over an interval.
A:Tangent	Draws a tangent line at a point and displays its equation

(b) **Finding the Maximum & Minimum Values of a Function from its Graph**

Instructions	Screen Shot
1. Display the **Y=Editor**.	
2. Enter the function	
3. Enter graph mode (◆ F3). Open the Math Menu **F5**, and select **4: Maximum**.	
4. Set the lower bound.	
5. Set the upper bound.	
6. Find the maximum point on the graph between the lower and upper bounds.	
7. Transfer the result to the Home screen, and then display the Home screen. [**Home**]	

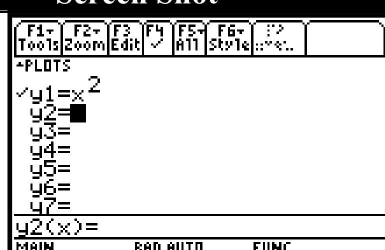
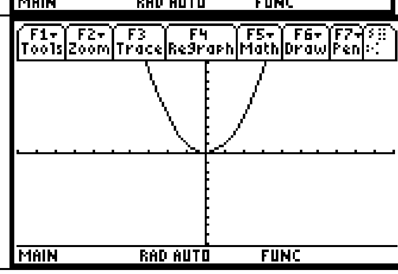
(c) **Overview of the Zoom Menu**

Instructions	Screen Shot
Press **F2** from ***y*=Editor**, window Editor, or Graph screen	

Zoom tool	Description
1:ZoomBox	Lets you draw a box and zoom in on that box.
2:ZoomIn 3:ZoomOut	Lets you select a point and zoom in or out by an amount defined by **SetFactors**.
4:ZoomDec	Sets Δx and Δy to 0.1, and centres the origin.
6:ZoomStd	Sets Window variables to their default values. $x$min= –10, $x$max= 10, $x$scl=1, $y$min= –10, $y$max= 10, $y$scl= 1, $x$res= 2

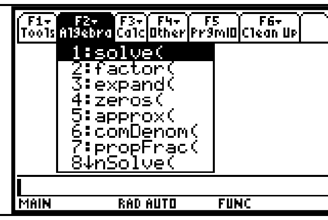
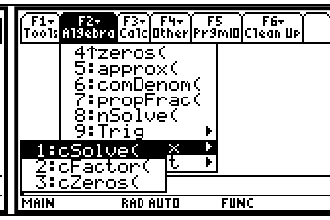
***Note*:** To get out of the graphing mode use 2 K.
This will not work while the **BUSY** icon is flashing in the bottom right hand corner.
Adjust your graph by selecting **F2** and choosing **2:ZoomIn**, **3:ZoomOut**, or **A:ZoomFit**

Example: Graph $y = x^2$ by following these instructions.

Instructions	Screen Shot
◆ [Y=] x ^ 2 [ENTER]	
◆ [GRAPH]	

To draw a new graph go to **[*y*=]** and change the formula in the ***y*1** position using the cursor to move up to it to delete it. This effectively clears the previous graph as well. Alternatively, using y2 will add the new graph to $y = x^2$.
[HOME] returns you to the Home screen.

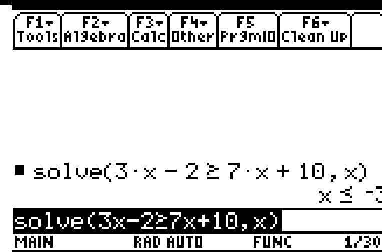
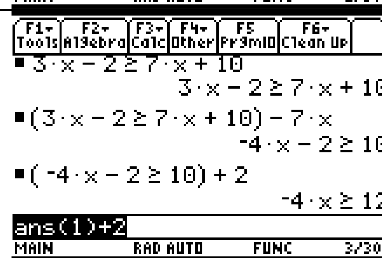
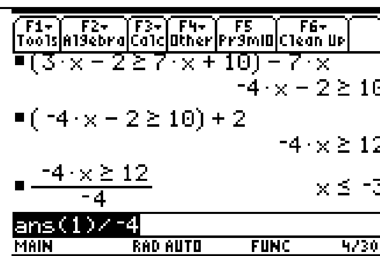
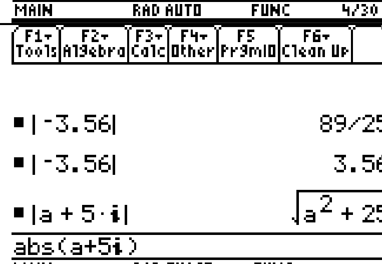
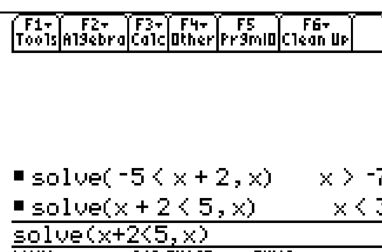
4. The Algebra Menu

Menu Item	Description F2 MENU  
1: solve	Solves an expression for a specified variable. This returns solutions only, regardless of the Complex Format mode setting (For complex solutions, select A:Complex from the algebra menu).
2: factor	Factorises an expression with respect to all its variables or with respect to only a specified variable.
3: expand	Expands an expression with respect to all its variables or with respect to only a specified variable.
4: zeros	Determines the values of a specified variable that make an expression equal to zero.
5: approx	Evaluates an expression using floating-point arithmetic, where possible.
6: comDenom	Calculates a common denominator for all terms in an expression and transforms the expression into a reduced ratio of a numerator and denominator.
7: propFrac	Returns an expression as a proper fraction expression.

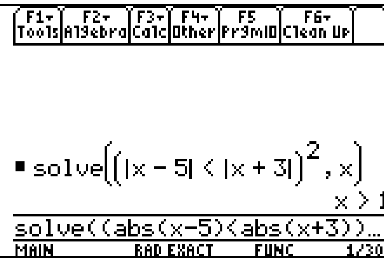
Using the TI-89 in Mathematics

Topic 0 Preliminaries

0.2 Inequalities and the absolute value

Instructions	Screen Shot
Inequalities We can directly solve these, for example $3x - 2 \ge 7x + 10$ F2 3*x* [−] 2 [♦] [>] 7*x* [+] 10 [,] *x* [)] [ENTER]	
We can also transform an inequality into the form *x* ≥ or *x* ≤ by performing the same operation on both sides. For example we can solve the inequality $3x - 2 \ge 7x + 10$ [ENTER] by adding $-7x$ to both sides of the equation, then adding 2 [2nd] [ANS] [−] 7*x* [ENTER] [2nd] [ANS] [+] 2 [ENTER]	
and dividing by –4 gives the answer. [2nd] [ANS] [÷] [(-)] 4 [ENTER] Note that the CAS reverses the inequality when dividing by the –ve quantity.	
The absolute value function is found in the [MATH] menu (press [2nd] 5), select 1: Number, select 2: abs((or Press D and [ENTER]) and press [ENTER]. (This function also gives the modulus of a complex number.). To switch from **exact** to **approximate** mode we press [♦] [ENTER]	
Inequalities with absolute values can be solved when they are broken down into single inequalities,	

<table>
<tr>
<td>or sometimes by squaring both sides of the inequality (note the unusual notation for this).

F2 1: Solve(([2nd] 5 1: Number 2: abs(x–5)< ([2nd] 5 1: Number 2: abs(x+3))^2, x)</td>
<td>
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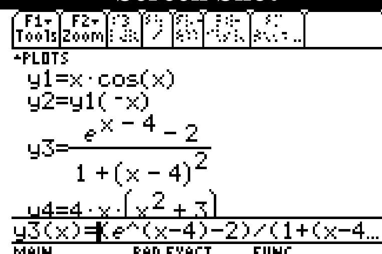
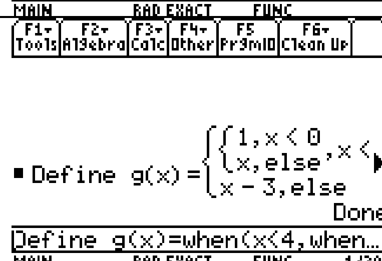
Note: The use of the [◆] key to switch between exact and approximate modes (the TI-89 tries to use fractions in exact mode).

0.3 Domain, range and graph of a function

<table>
<tr><th>Instructions</th><th>Screen Shot</th></tr>
<tr>
<td>We can use the [Y=] menu obtained by pressing [◆] [Y=] to draw graphs. Place the cursor just to the right of y1= and enter the function required. Note that we can use previously defined functions in later ones.</td>
<td>
</td>
</tr>
<tr>
<td>To enter a split domain function we use the when ( ) function and nest them if there are more than two parts to the piecewise function. This has been done by defining a function g and using y1= g.</td>
<td>
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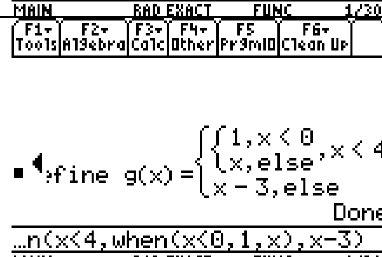
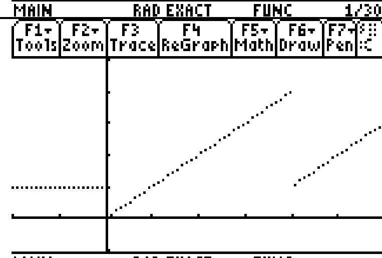
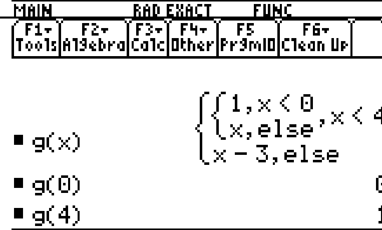
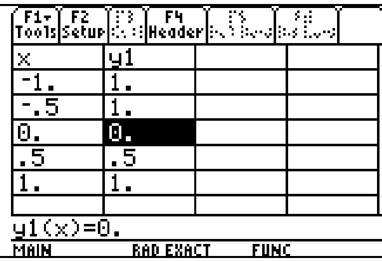
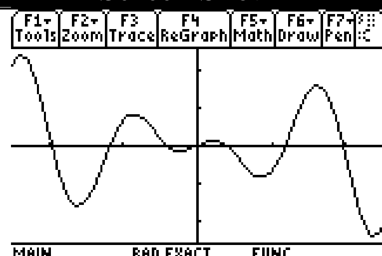
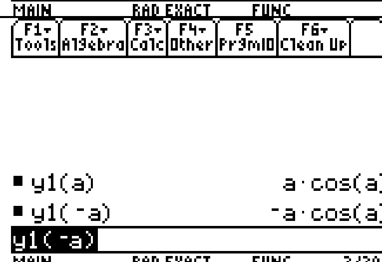
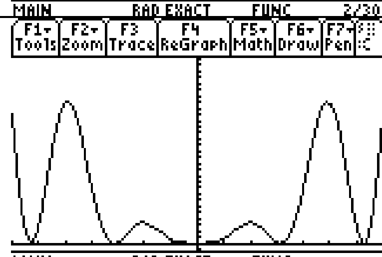
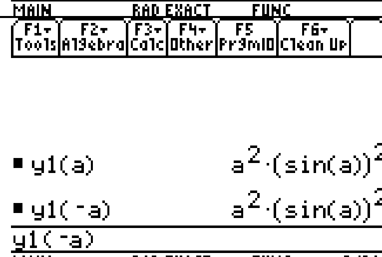
<tr>
<td>We can use g a number of times this way.</td>
<td>
</td>
</tr>
<tr>
<td>Note that these graphs look better when plotted in the dot style. This found on the [Y=] screen under F6 Style, 2: Dot.</td>
<td>
</td>
</tr>
<tr>
<td>We can test the value of the function g at the points x=0, and x=4 on the [HOME] screen, as shown.</td>
<td>
</td>
</tr>
</table>

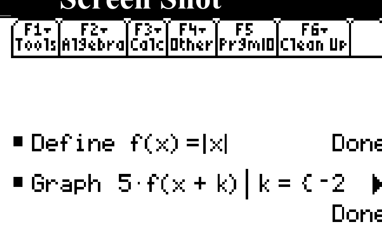
Or we could use a table of values.	

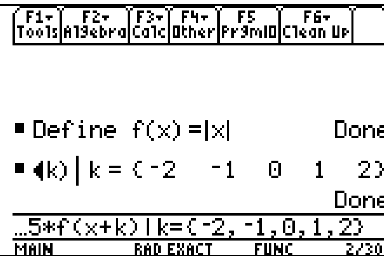
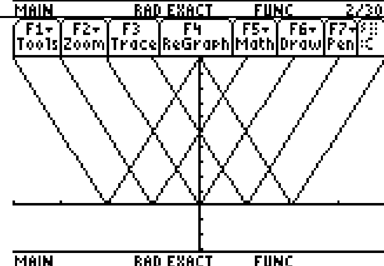
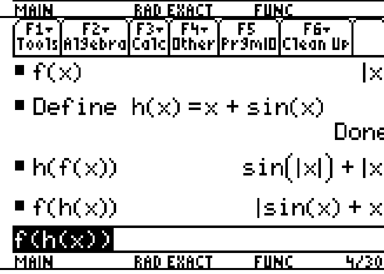
Note: Use [♦] [TblSet] to zoom in on the table values.

0.4 Trigonometric functions

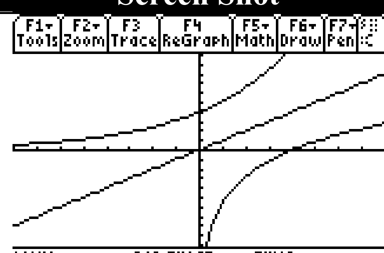
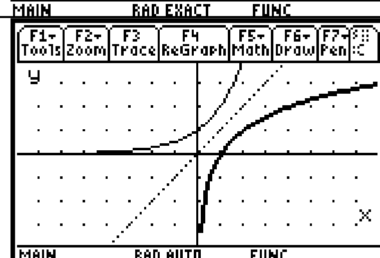
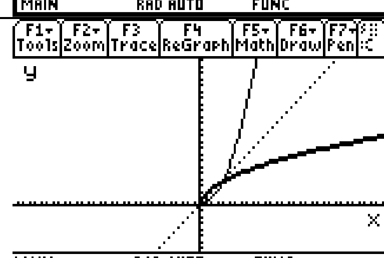
Instructions	Screen Shot
The graphs of the functions *f(x)=x*cos(*x*) and $f(x)=x^2\sin^2(x)$ (entered as sin(*x*)^2) are shown on the TI-89. We can verify that one is an odd function and the other even, by checking *f(a)* against *f(–a)* on the [HOME] screen.	
It's an odd function.	
Graph of the function $f(x)=x^2\sin^2(x)$	
It's an even function.	

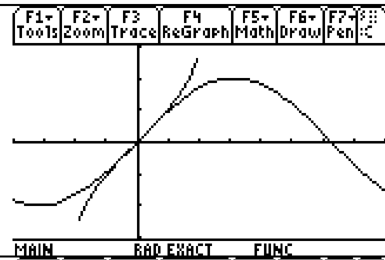
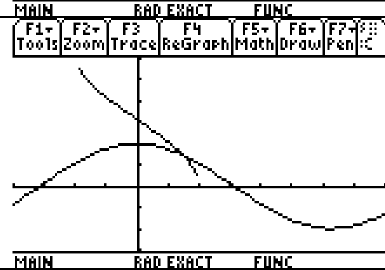
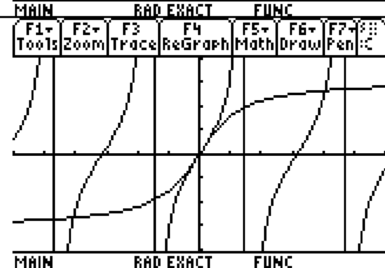
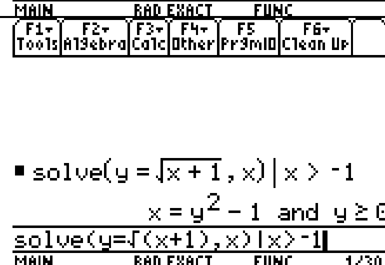
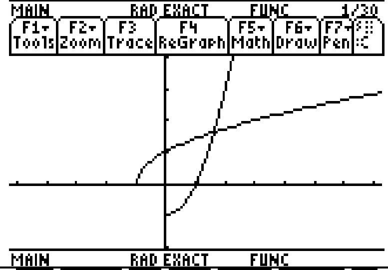
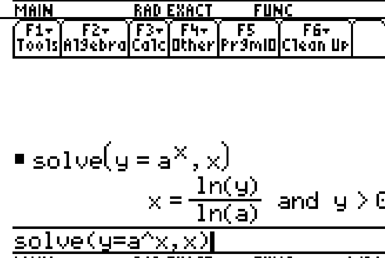
0.5 Translations and compositions of functions

Instructions	Screen Shot
We can check the effect of a transformation by looking at multiple graphs of a function, using the | command to set values of a variable (which can be read as 'when'). Enter F4 1: Define i F(x)= [2nd] 5 1:	

Number 2: abs(x)	
	
The graph function is at F4 Other 2: Graph	
Composite functions can be obtained from previously defined functions using the notation $f(g(x))$.	

0.6 0ne-to-one and inverse functions

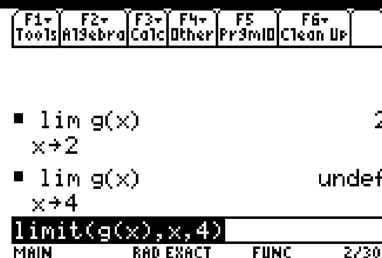
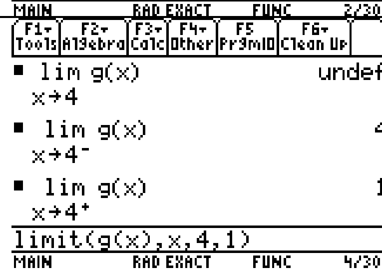
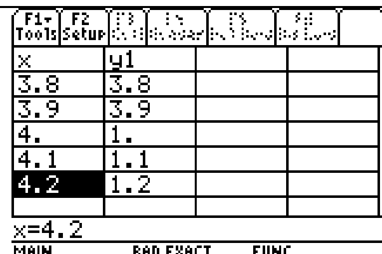
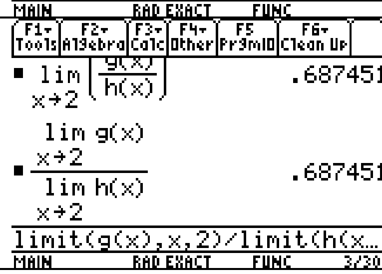
Instructions	Screen Shot
Graphs of functions which are inverses, such as *exp* and *ln* will not look like reflections in $y=x$ on the TI-89 unless the same scale can be used on each axis.	
This can be done using F2 Zoom 5: Zoom Sqr (as shown in these graphs).	
Note there is also a function in graph mode F6 Draw 3; DrawInv to draw an inverse function's graph	

The **inverse trig functions** are provided on the TI-89 and their domains are known by the calculator. $\sin^{-1}$ is at [♦] Y.	
$\cos^{-1}$ is at [♦] Z.	
$\tan^{-1}$ is at [♦] T.	
Other inverse functions can be found (if they exist) by solving for *x* *y*=the function and replacing *y* by *x*. Note that a limited domain where the function is 1–1 can be entered. Here we enter: F2 1: Solve(*y*=[2nd] [×] *x*+1), *x*) | *x*>–1 Graphing the function and its inverse and using ZoomSqr gives support for the answer.	 
Note that the inverse of a^x is given in terms of natural logarithms, instead of $\log_a x$.	

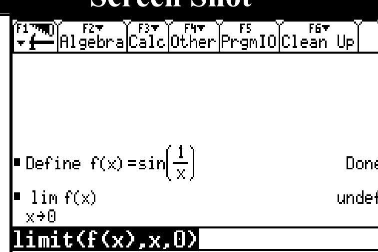
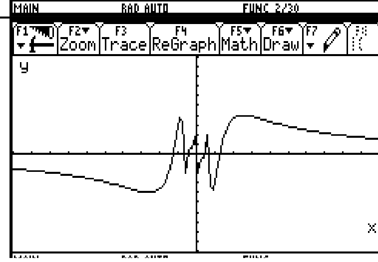
Note: We can not use *f(x)*^–1 for inverse functions. This gives the reciprocal of the function.

Topic 1 Limits

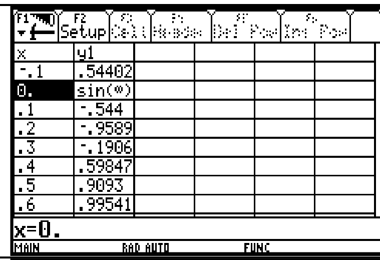
1.1 Limits of a function

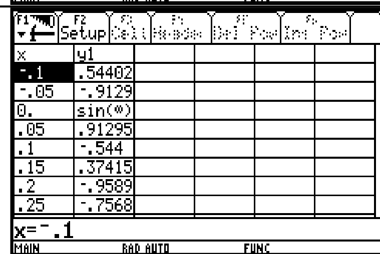
<table>
<tr><th>Instructions</th><th>Screen Shot</th></tr>
<tr><td>Use F3 3: Limit(to find limits. The order is Limit(function, variable, value approached).</td><td>
</td></tr>
<tr><td>We can also find one-sided limits by writing 1 or –1 before closing the bracket for right and left limits respectively (NB do NOT enter +1, only 1). If the limit does not exist we are given the answer undef(ined). Checking the right and left limits may help us see why this is so.</td><td>
</td></tr>
<tr><td>Checking a table of values can also be useful.</td><td>
</td></tr>
<tr><td>While not proving them, we can verify limit laws for some examples. You should check them with some functions of your own. Note the use of the calculator’s approximate mode here.</td><td>
</td></tr>
</table>

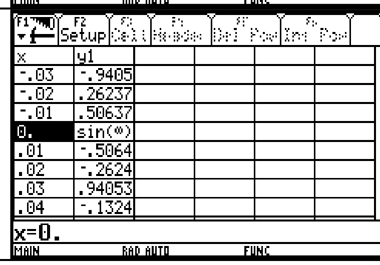
Example 1.1

<table>
<tr><th>Instructions</th><th>Screen Shot</th></tr>
<tr><td>Example $\lim_{x \to 0} \sin \frac{1}{x}$
Some limits do not exist. We can build an understanding of some reasons for this.</td><td>
</td></tr>
<tr><td>We can plot the graph and zoom in on $x = 0$.</td><td>
</td></tr>
</table>

Or from the table we can see that no matter how much we zoom in on $x = 0$ values do not tend towards the same number (left and right limits do not exist).





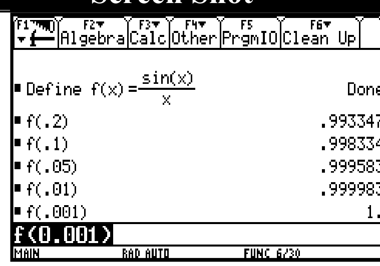


Example 1.2

Instructions	Screen Shot

Example: Find $\lim_{x \to 0} \frac{\sin x}{x}$

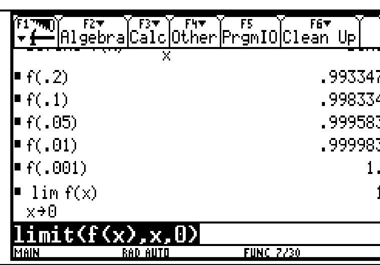
This is an important limit, but one that cannot be found by putting $x = 0$, since the function is undefined for $x = 0$. Enter F3 [3] *f(x)* = [SIN] *x* [)] [÷] *x* [,] *x* [,] [0] [)] [ENTER]



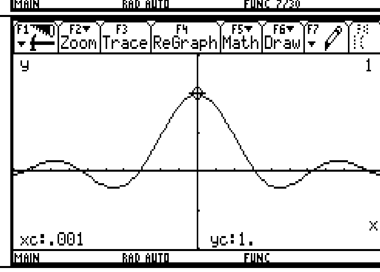
If we change the value of *x,* taking steps closer to 0 then the value of *f*(*x*) gets closer to 1.

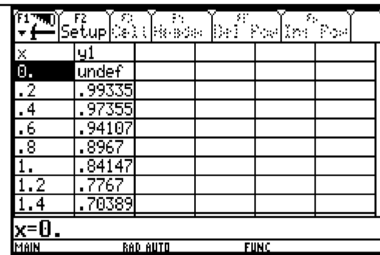
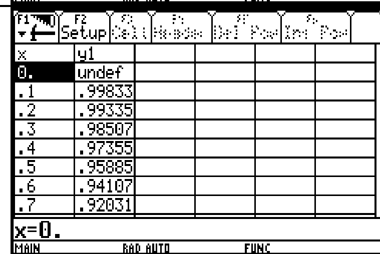
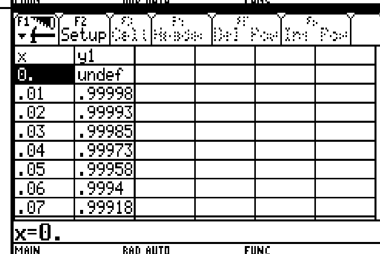
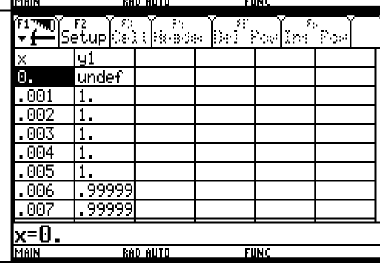
F3 [3] *f(x)* = [SIN] *x* [)] [÷] *x* [,] *x* [,] [0] [)] [ENTER]

F3 [3] *f(x)* [,] *x* [,] [0] [)] [ENTER]

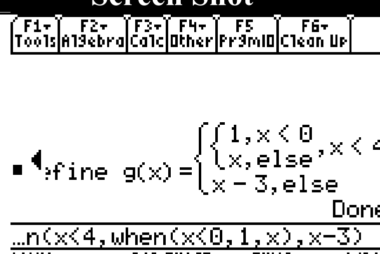
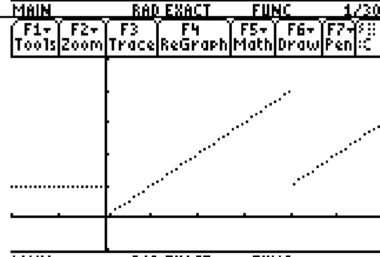


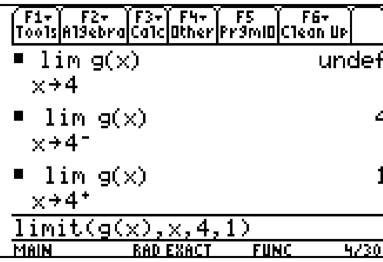
Looking at the graph can help with what the limit might be.



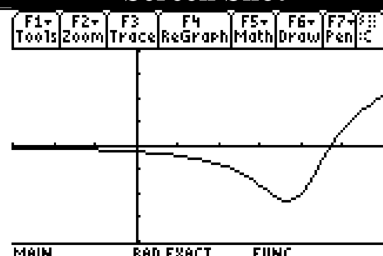
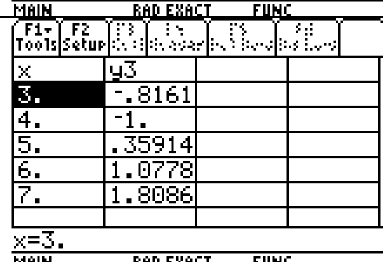
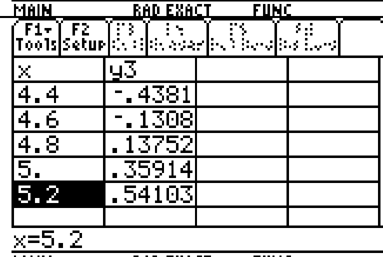
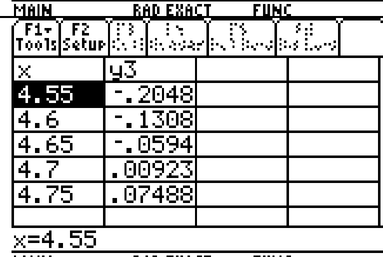
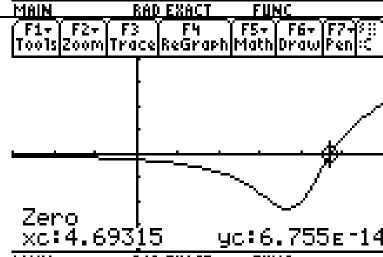
<table>
<tr><td>We can use a table…</td><td>
</td></tr>
<tr><td></td><td>
</td></tr>
<tr><td></td><td>
</td></tr>
<tr><td>We find that:
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$</td><td>
</td></tr>
</table>

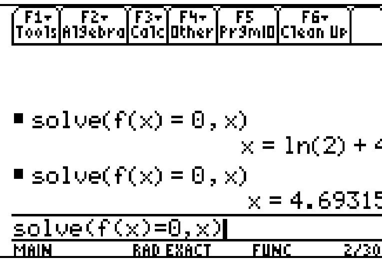
1.3 Continuity

<table>
<tr><th>Instructions</th><th>Screen Shot</th></tr>
<tr><td>The function g used in 0.3 is discontinuous at x=0 and x=4. The limits at x=4 were calculated in Example 1.1.</td><td>
</td></tr>
<tr><td></td><td>
</td></tr>
</table>

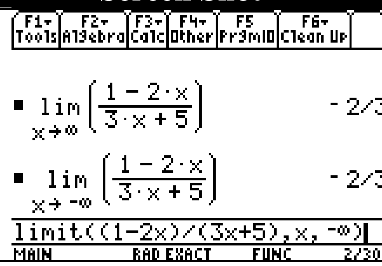
To be continuous at x=4 we would need the right limit to equal the left.	

1.3.2 The intermediate value theorem

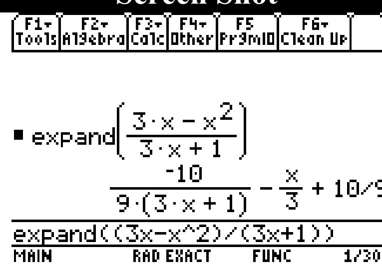
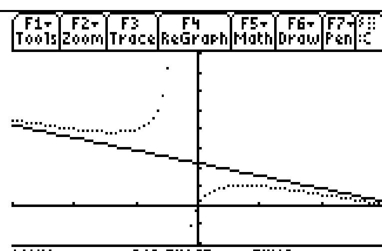
Instructions	Screen Shot
This is very useful for showing that there is a root of $f(x)$=0 between two domain values. If $f(a)<0$ and $f(b)>0$ (or vice versa) then there is a zero of f between a and b. Using the table of values in a graph we can then zoom in on the root.	
For the function below we see from the table that $f(4)<0$ and $f(5)>0$, so we zoom in to find the root, using the Intermediate Value Theorem.	
This table shows it is between 4.6 and 4.8.	
This table shows it is between 4.65 and 4.7. This can be continued to the required accuracy.	
We could of course get the TI-89 to find the root directly from the [Graph] or [Home] screens, but we need to understand that this theorem is one basis for finding it. For the graph use F5 Math, 2: Zero, enter the lower and upper bounds (4 and 5 from the theorem) and we get 4.69315 for the root of $f(x)$=0 or the zero of f.	

In the [Home] screen we use F2 1: Solve (and enter *f*(*x*)=0, *x*). We need approximate mode (holding down [♦] when pressing [ENTER]) to get the decimal answer.	

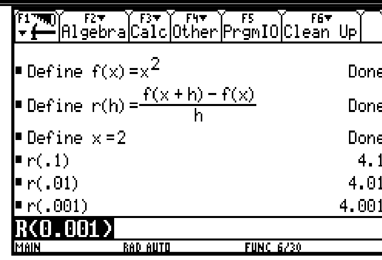
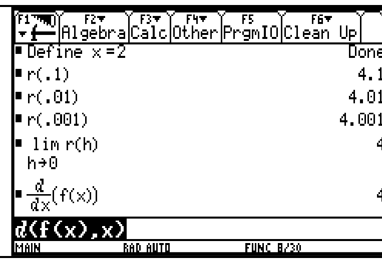
1.4 Limits involving infinity

Instructions	Screen Shot
Limits involving infinity are entered as before but using [∞] key ([♦] [CATALOG])as if it is the value approached.	

1.4.3 Asymptotes

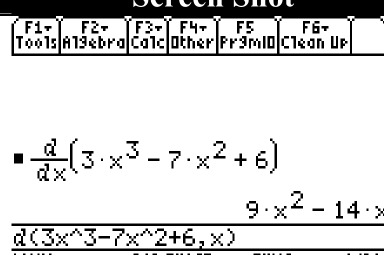
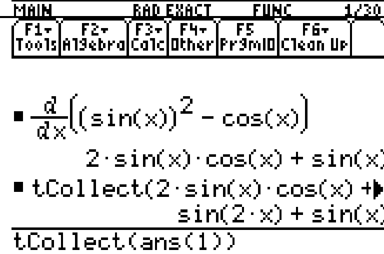
Instructions	Screen Shot
Use the limits to find the horizontal asymptotes. For sloping asymptotes we can use the TI-89 to divide the numerator of a function by its denominator, using F2, 3: Expand (the function F2, 7: propFrac(will give the same result here). The asymptote here is $y=-x/3 + 10/9$ since as $x \to \infty$ the remainder of the expansion approaches 0.	
The answer can be checked by drawing both graphs.	

2.1 Tangents and rate of change

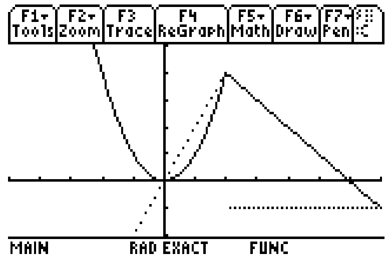
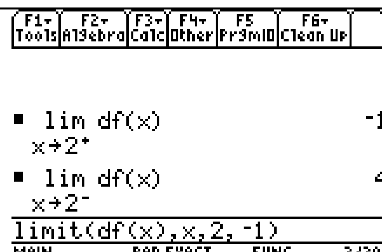
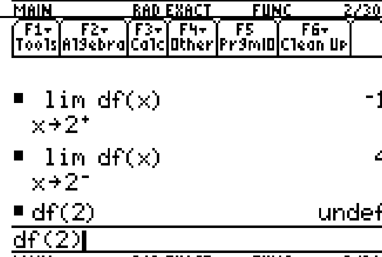
Instructions	Screen Shot
Consider the function $y=x^2$. We can define a rate of change function r to be the gradient of a chord of length h. That is: $r(h)=\dfrac{f(x+h)-f(x)}{h}$ (NB there is a built-in numeric derivative function at F3 A: nDeriv which could be used but looks rather different). We can then use this function r at a point, for example, $x = 2$. Whenever we change h taking steps of h closer to 0 then the value of r is getting closer to 4.	
We can confirm this by asking for the limit of r as h approaches 0.	

Note: $f'(x)=\lim_{h\to 0} r(h)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ (where the limits exist). Thus here the rate of change at $x = 2$: $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=f'(2)=4$.

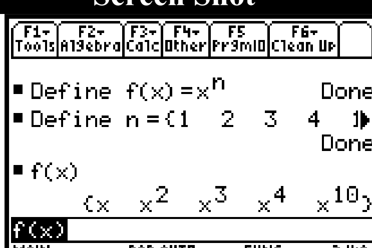
2.2 The derivative as a function

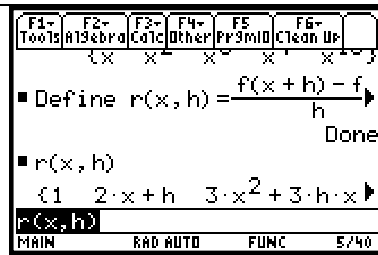
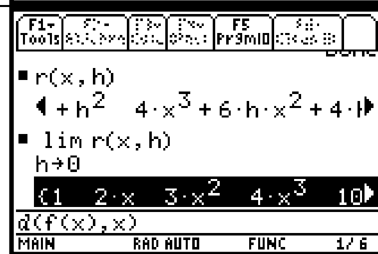
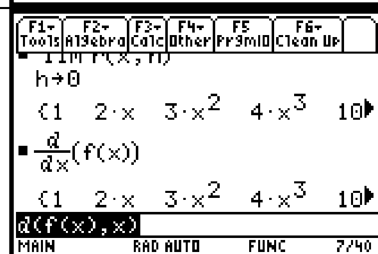
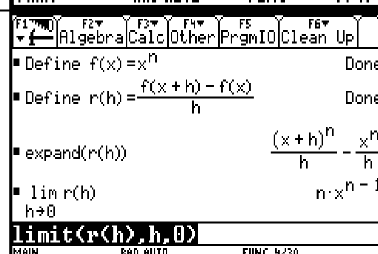
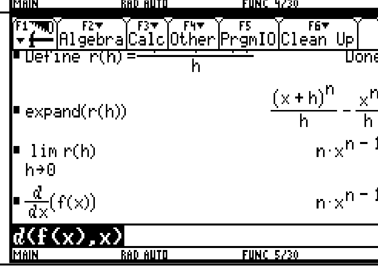
Instructions	Screen Shot
To differentiate on the TI-89 we use the F3 Calc, 1: *d*(differentiate command, which is also found at [2nd] [8]. The format is *d*(function, variable to differentiate with respect to).	
In the second example we can use the function Trig collect, found in F2, 9: Trig, 2: tCollect to simplify the answer. Use [2nd] [(-)] for ANS, the previous answer.	

Example 2.1

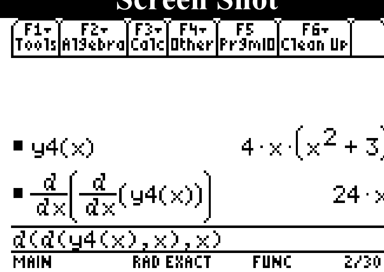
Instructions	Screen Shot
Find out whether the function $f(x) = \begin{cases} x^2 & for\ \ x < 2 \\ 6 - x & for\ \ x \geq 2 \end{cases}$ is differentiable at x=2. Define the piecewise functions by using the following instructions. F4 [1] *f(x)* = when [(] *x* [2nd] [<] 2 [,] *x* [^] 2 [,] 6 [−] *x* [)] [ENTER] Then we graph the function *f* . We can define a function *Df* as its derivative $\dfrac{d(f(x))}{dx}$ (use F6 2: Dot in [Y=] to plot the derivative). Note that this may not be defined on the whole domain. We can see the discontinuity in the derived function's graph, but must check the limits on the [HOME] screen.	
Right limit is: F3 [3] *df(x)* [,] *x* [,] 2 [,] 1 [)] [ENTER] Left limit is: F3 [3] *df(x)* [,] *x* [,] 2 [,] [(-)]1 [)] [ENTER]	
Since the limits are not the same the function is not differentiable at x=2 and *df*(2) is undefined.	

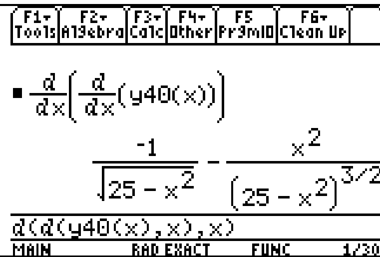
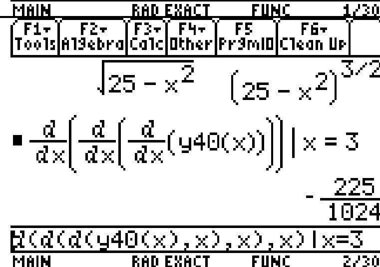
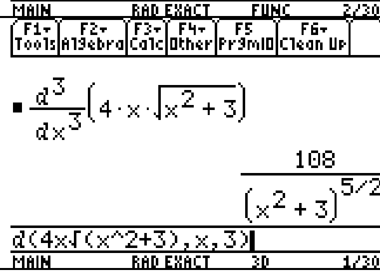
Example 2.2

Instructions	Screen Shot
Example. Find the derivative of $f(x) = x^n$ Define the function $f(x) = x^n$. When we define the value of power, n = 1, 2, 3, 4, 10 the functions are changed to the actual functions, x, x^2, x^3, x^4, x^{10}.	

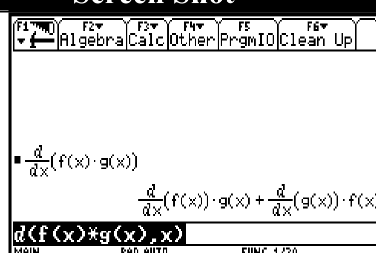
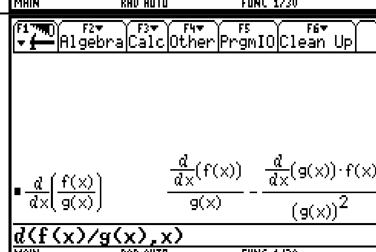
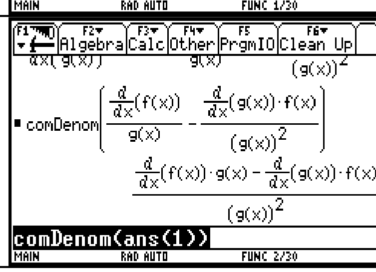
If we define the slope function *r* as the average rate of change,	
	
then we can see that the derivative of the functions are 1, $2x$, $3x^2$, $4x^3$, $10x^9$.	
Using the rate of function *r,* we can get that the general derivative of x^n is nx^{n-1}	
Thus $\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = nx^{n-1}$	

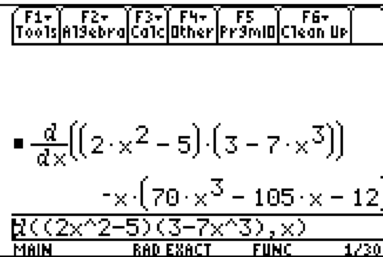
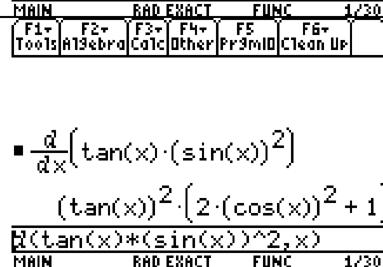
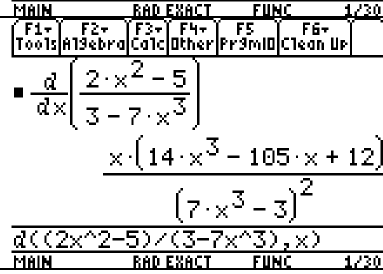
2.2.1 Second and higher derivatives

Instructions	Screen Shot
These can be accomplished by using repeated applications of the CAS function *d*. Here functions $y4(x)$ and $y40(x)$ from the [Y=] list have been used. Note the inclusion of the variable each time and the option of finding the value of a derivative at a specific value of x.	

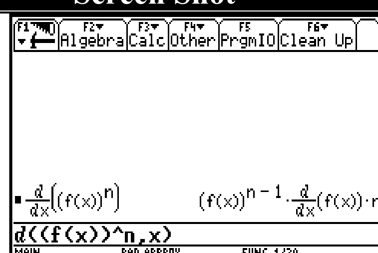
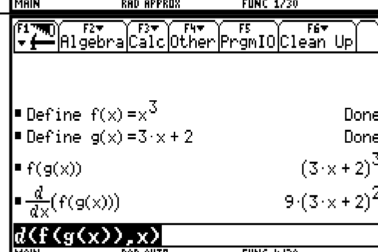
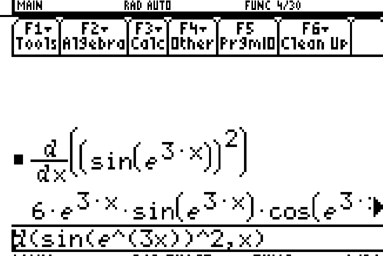
	
	
Alternatively we can specify the *n*th derivative with F3 1: *d*(differentiate function, *x*, *n*)	

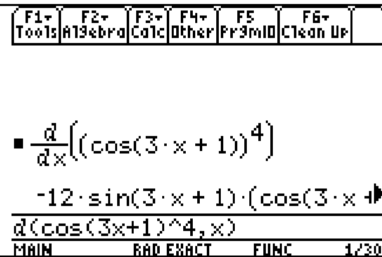
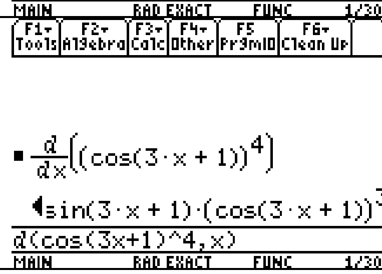
2.3 Differentiation rules

Instructions	Screen Shot
The TI-89 can act on functions that are unknown, to give the differentiation formulas.	
	
The common denominator function F2 6: comDenom(has been used to simplify an answer by combining two terms over a common denominator.	

Otherwise the TI-89 can be used to check differentiation of these functions by direct entry, as here.	
	
	

2.4 The Chain Rule

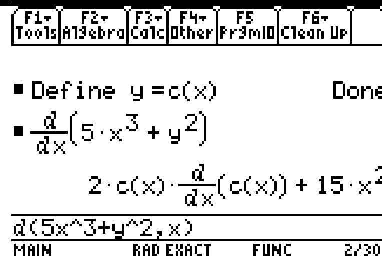
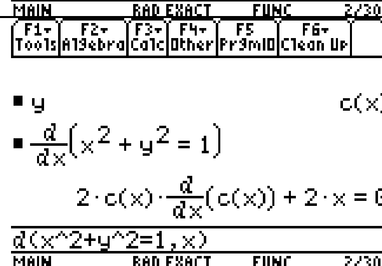
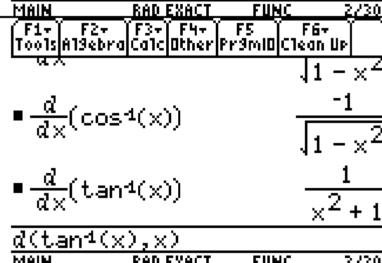
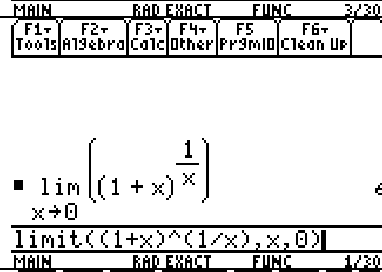
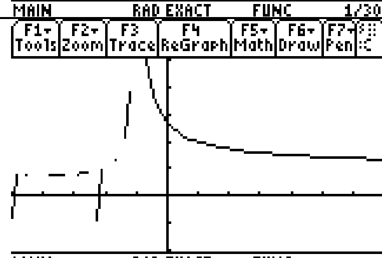
Instructions	Screen Shot
The rule for this can be seen similarly for *f(x)* raised to the power of *n*.	
Again functions are entered directly, although we can define them separately.	
	

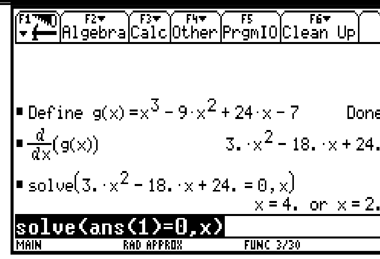
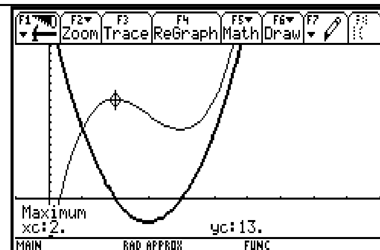
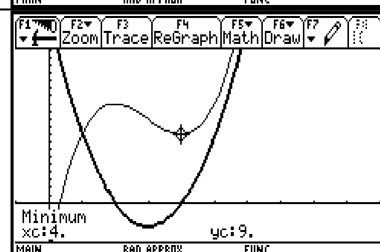
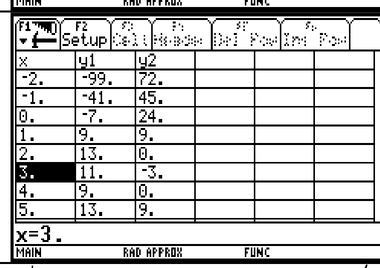
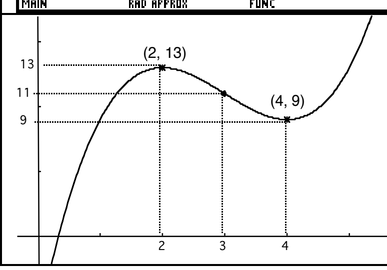
F1 Tools F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
■ d/dx((cos(3·x + 1))^4)
-12·sin(3·x + 1)·(cos(3·x +
d(cos(3x+1)^4,x)
MAIN RAD EXACT FUNC 1/30

F1 Tools F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
■ d/dx((cos(3·x + 1))^4)
sin(3·x + 1)·(cos(3·x + 1))^3
d(cos(3x+1)^4,x)
MAIN RAD EXACT FUNC 1/30

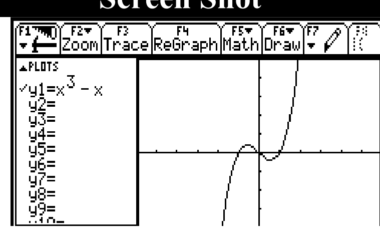
2.5 Implicit differentiation

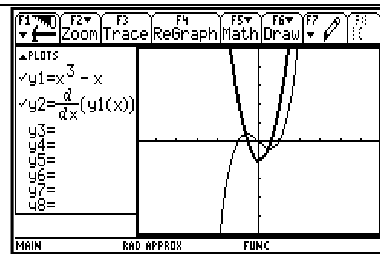
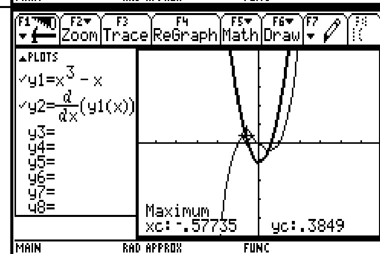
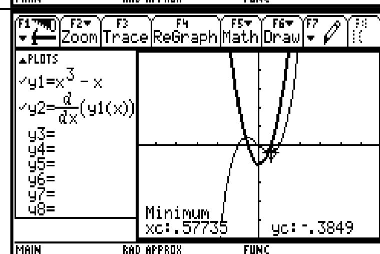
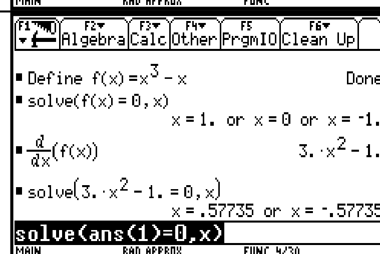
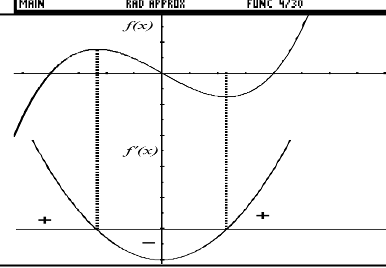
Instructions	Screen Shots
This can be accomplished by defining *y* to be some function of *x*, here *c*.	
	
Inverse trig functions are entered directly.	
The TI-89 confirms the value of the limit giving *e*.	
Note the asymptote at $x = -1$.	

3.1 Maximum and minimum values and
3.2 Derivatives and the shapes of curves

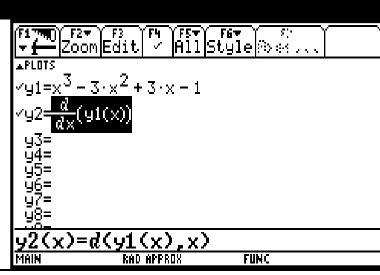
Instructions	Screen Shots
Relative Extrema: Find all relative extrema of the function $g(x)=x^3-9x^2+24x-7$ and confirm your result by sketching the graph. The TI-89 method combines use of the differentiation command, the solve command for $\frac{dy}{dx}=0$,…	
graphs of the function and its derivative to relate the algebraic solution to the pictures,…	
	
and a table of values to get coordinates of points, check limits, etc.	
	

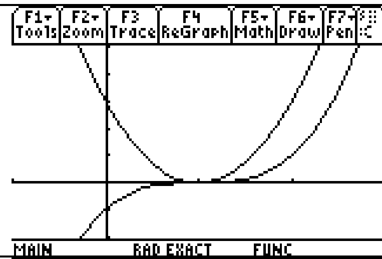
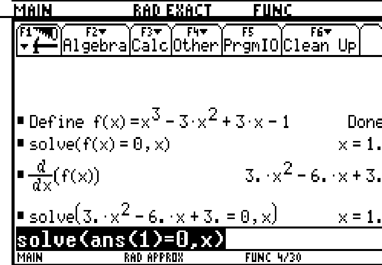
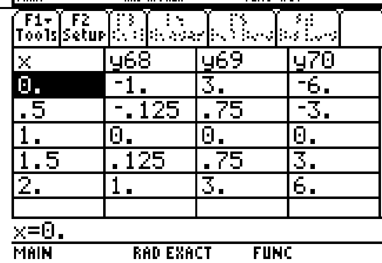
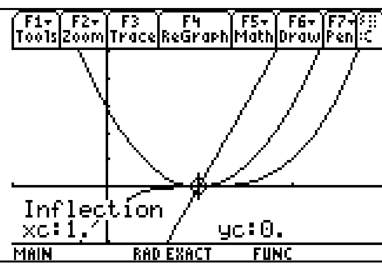
Example 3.1

Instructions	Screen Shot
Find the relative maximum and minimum values of the function $f(x) = x^3 - x$. First we can get an idea of the solutions by sketching the graphs of the function and its derivative	

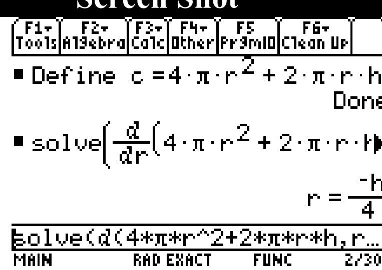
Note the use of $y2(x) = \frac{d(y1(x))}{dx}$ to sketch the graph of the derivative.	
	
	
Answers obtained from using F5 Maths 3: Minimum or 4: Maximum from the graph screen can be checked algebraically for accuracy in the [HOME] screen.	
	

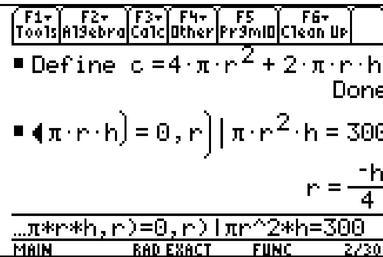
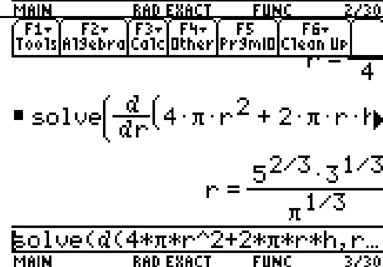
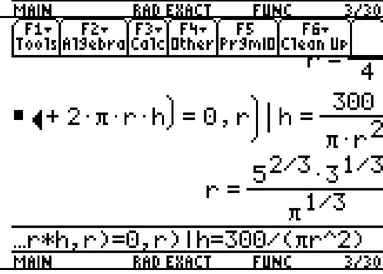
Example 3.2

The derivative can be zero without there being a relative maximum or relative minimum. Example. $f(x) = x^3 - 3x^2 + 3x - 1$	

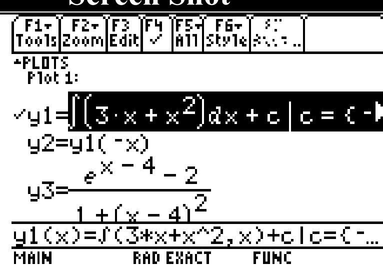
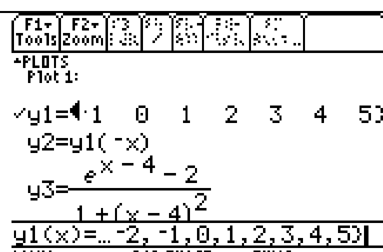
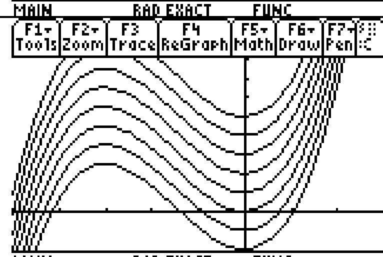
	
	
The second derivative test can be used to confirm that we have a point of inflection at x=1. Put $y3(x)$ equal to the second derivative and view the table of values, around x=1. We see that $\frac{d^2y}{dx^2}$ changes sign from negative to positive through x=1 (and is zero at x=1, check on the [HOME] screen and note the Intermediate Value Theorem) and so we have a point of inflection ($y68=f$, $y69=f'$ and $y70=f''$ here).	
We can check this on the graph screen by using F5 Maths 8: Inflection.	

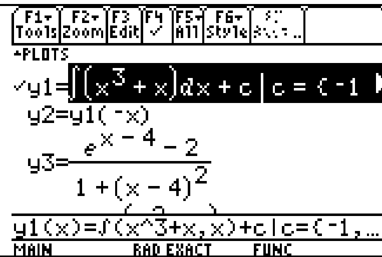
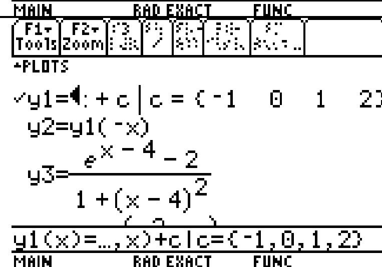
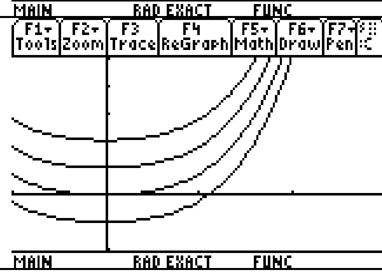
3.3 Optimisation problems

Instructions	Screen Shot
Often in these questions we have to find the optimum value of a function of two or more variables by first substituting for one of the variables a function previously formed. This can be done in a relatively easy way on the TI-89. Taking example 2 on the manual in section 3.3, we have to minimise the cost, $C = 2(2\pi r^2) + 2\pi rh$ subject to $\pi r^2 h = 300$. Note that the form of the condition (using |) means that the answer comes out well or does not come	

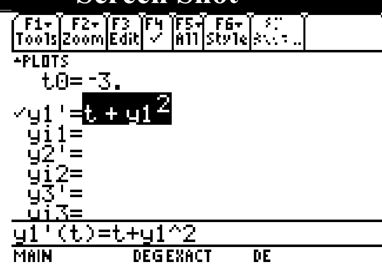
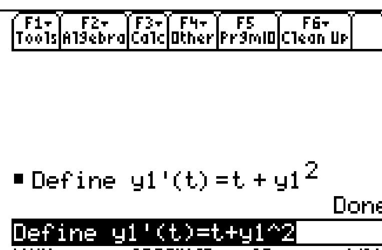
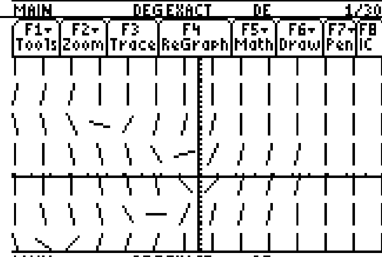
	
	
	

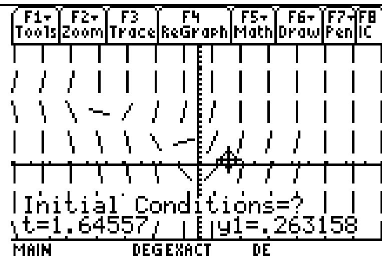
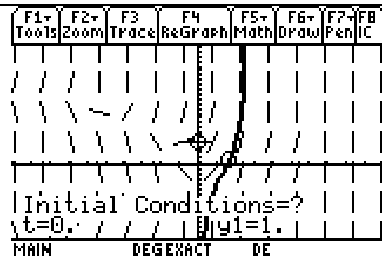
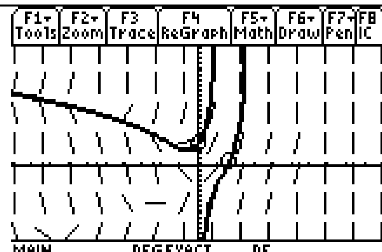
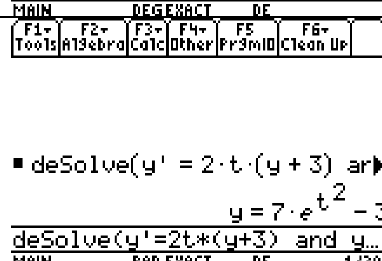
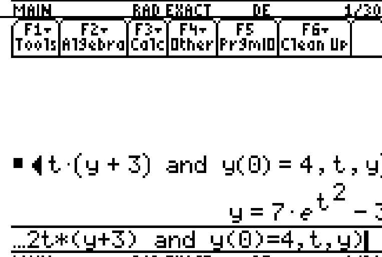
3.4 Antidifferentiation

Instructions	Screen Shot
Use the symbol ∫ found at [2nd] [7] for the antiderivative. We can enter on the [Y=] screen the function $y1(x)$ = [2nd] [7] function, x) + c when c={list of values separated by commas}. This will give us a number of antiderivatives of the function.	
$y1$ is then the function $F(x)$.	
Graphing the function will show what these functions look like and the relationship between them.	

	
	
Since they only differ by a constant the graphs are all translations of each other parallel to the *y*-axis.	

Displacement, velocity and acceleration

Instructions	Screen Shot
Set the graph drawing mode to differential equations using MODE Graph 6: DIFF EQUATIONS. In the Y= mode the DEs are then set up ready for you to enter. The TI-89 uses *t* not *x*. The variable *t* is given a key of its own on the TI-89, like *x, y,* and *z* namely ‹.	
To draw a direction field using the TI-89.	
	

Select the [Y=] screen and enter the differential equation using *t* (and *Y*1—or *Y*n—if needed). There is no use of *x*. Use [◆] [Window] (F2) to set the window dimensions to an appropriate *t* and *Y* size. Choose 'GRAPH' and it will put in the direction field.	
Selecting F8 IC enables a particular antiderivative solution to be drawn: IC stands for Initial Conditions, meaning a point (or points) known to be on the graph of the antiderivative required. Enter the co-ordinates or move the cursor to a chosen point and press ENTER.	
The solution curves for the antiderivative through the given point(s) is drawn.	
To solve a DE algebraically we use the command F3 C: deSolve(Use [2nd] = for the y'. Example 189 use F3 C: deSolve(*y*[2nd] = = 2*t*(*y*+3) and y(0)=4, *t*, *y*)	
	

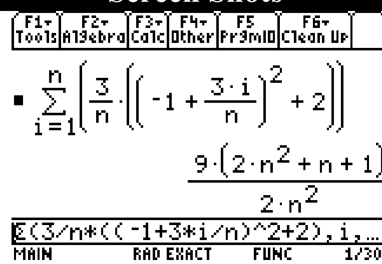
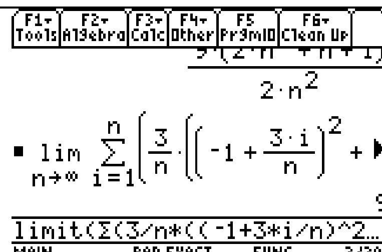
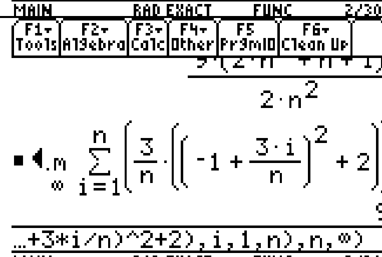
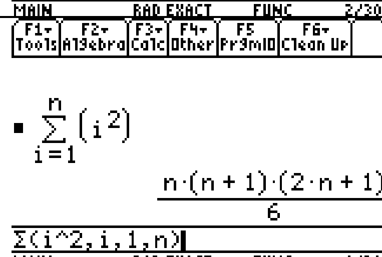
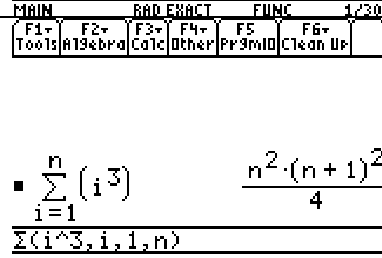
4. Integration

4.1 The area problem

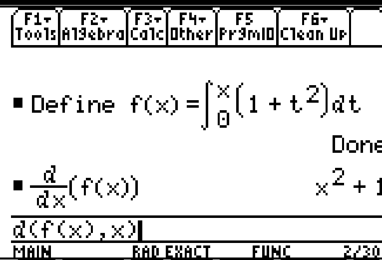
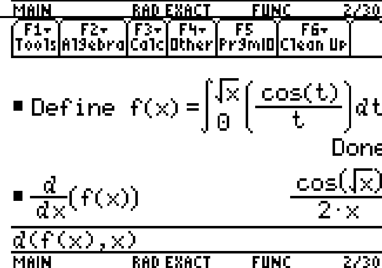
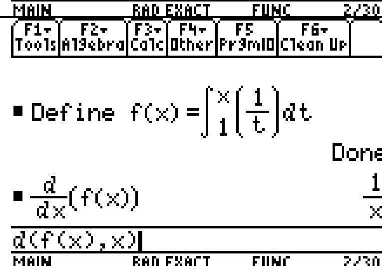
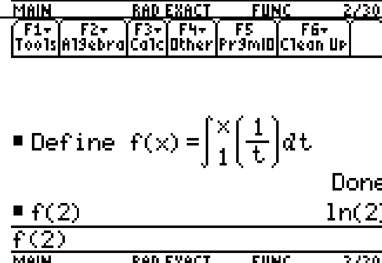
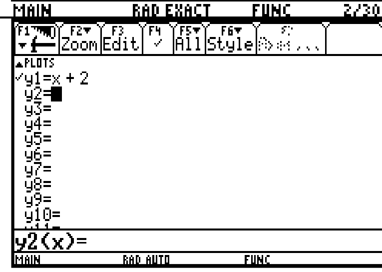
To find the area under the curve $f(x) = x^2 + 2$, from $x = -1$ to $x = 2$. using rightsum we have $x_0 = -1$, $x_1 = -1 + 3/n$, $x_2 = -1 + 2.3/n, \ldots$ $x_i = -1 + 3i/n, \ldots$ $x_n = -1 + 3n/n = -1+3=2$.

So the area can be obtained by taking the limit (if it exists) of the Riemann sum as $n \to\infty$.

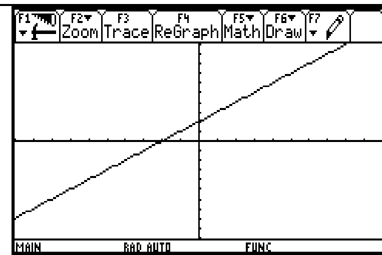
Area = $\underset{n\to\infty}{Lim}\left(\sum_{i=1}^{n}\frac{3}{n}\left(f(x_i)\right)\right) = \underset{n\to\infty}{Lim}\left(\sum_{i=1}^{n}\frac{3}{n}\left(\left(-1+\frac{3i}{n}\right)^2+2\right)\right)$.

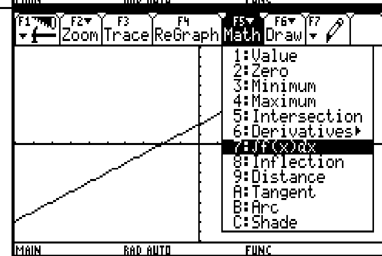
Instructions	Screen Shots
On the TI-89 this is entered as: F3 3: limit(F3 4: Σ(sum expression), *x*, *n*, 1, ∞), or enter the sum first and then take the limit. The ,*x* tells the calculator to sum with respect to *x*, and the *n*, 1, ∞) is part of the limit (from *n*=1 to ∞). Don't forget to make sure that *n,* and *i* do not have values in them (use F4 4: Delvar if they do).	
	
	
The summation function F3 4: Σ(sum will also give the general summation results in **Theorem 4.2.1**.	
	

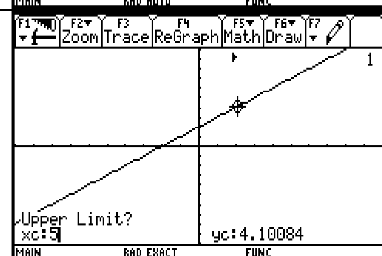
4.4 Fundamental Theorem of the Calculus

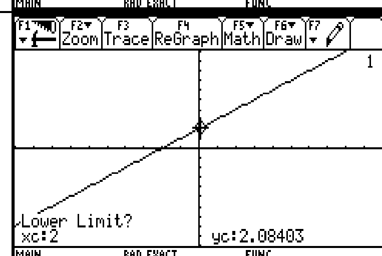
Instructions	Screen Shots
The derivatives of the integral functions can be found on the TI-89 by defining the function first using F4 1: Define, and then finding its derivative with respect to *x* (or these two steps can be done together).	
	
	
Using CAS with the theorem can help us with antiderivatives for functions such as $\frac{1}{x}$.	
Areas under curves are best calculated by evaluating the correct definite integral. This can be done numerically on the graph screen, or on the [HOME] screen. For example to find the definite integral $\int_2^5 (x+2)dx$ we can use the graph of *f(x)* on the TI-89 to see the area represented by the integral and numeric integration to calculate it. ◆ [Y=] *x* + 2 ENTER ◆ [GRAPH] F5 7 2 ENTER 5 ENTER	

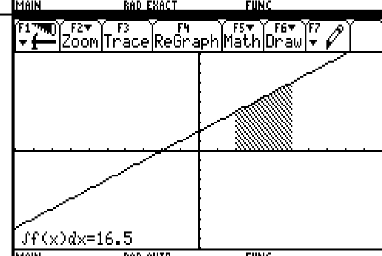
Note : Only the ***x*** value of the lower and upper limit needs to be typed in. Ignore the *y*-value. This should appear on your screen.



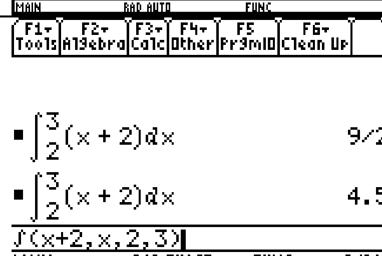


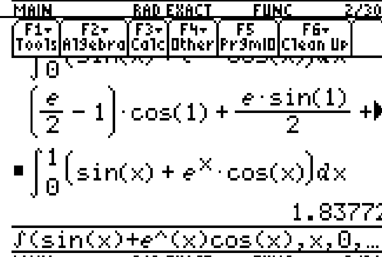






On the [HOME] screen we just enter the function into F3 2:∫ integrate, and the lower and upper limits. Note the use of [♦] [ENTER] again (approximate mode) to get the decimal answer.

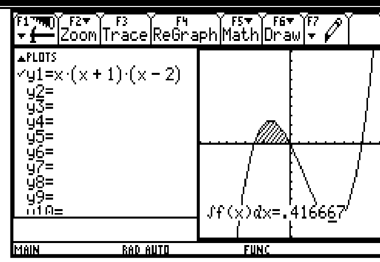
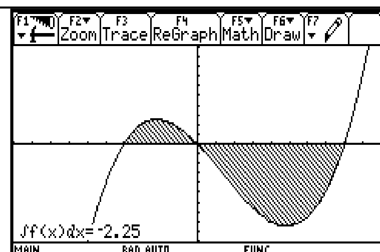
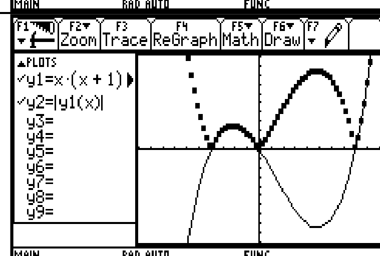
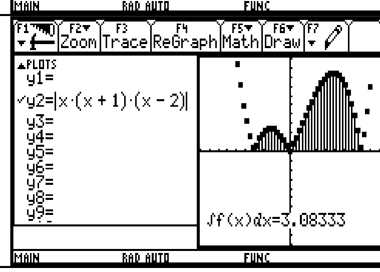




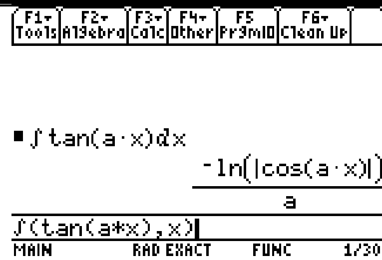
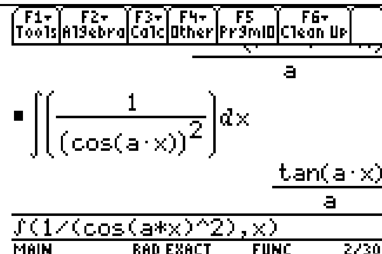
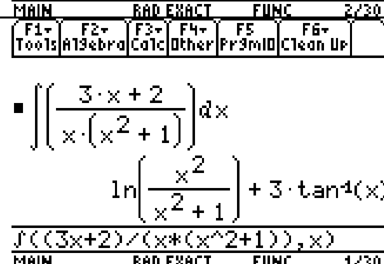
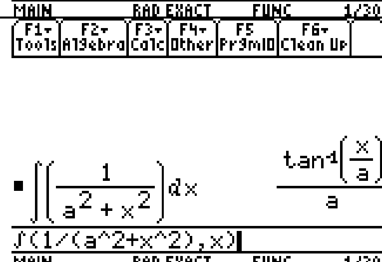
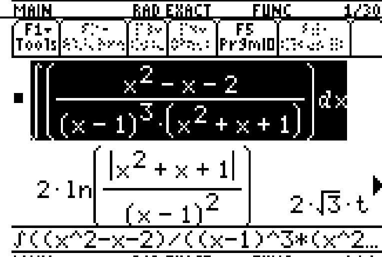
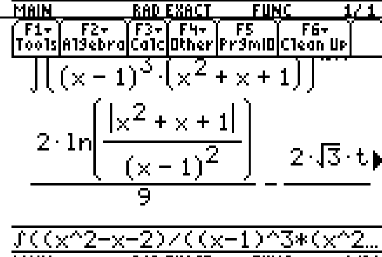
Area = $\int_a^b \{f(x) - g(x)\}dx$, where $x=a$ and $x=b$ are the x-values of the two points of intersection (if they exist).

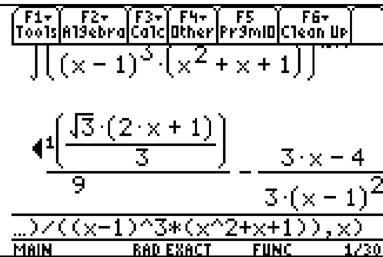
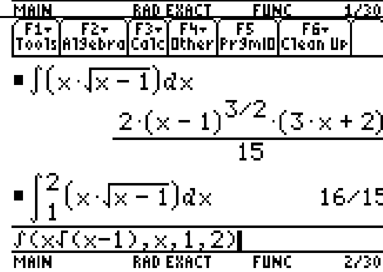
We can also use the formula $\int_a^b |f(x)|\,dx$ to find the area between the graph of f and the x-axis, and then we do not have to worry about where the function intersects the axis or the signs of the integrals. This works well on the TI-89 since we have the function abs.

For example calculate the area between $f(x)=x(x+1)(x-2)$ and the x-axis from $x=-1$ to $x=2$. It is always good to look at the graph of the function to see what is going on.

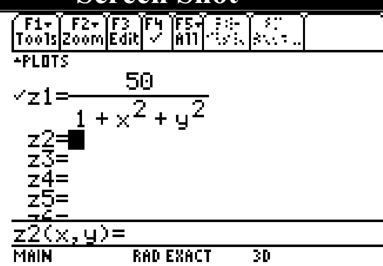
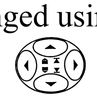
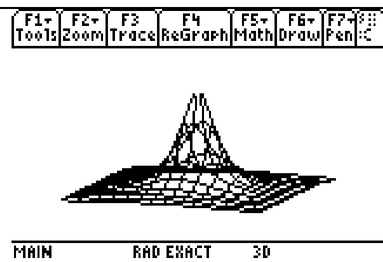
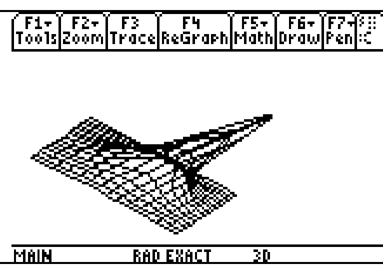
Instructions	Screen Shot
Define $y1 = x(x + 1)(x - 2)$ and draw the graph. Entering $y2$ as abs($y1(x)$) (which gives a reflection of $y1$ in the x-axis) enables the area to be found without finding the intersections with the axis. Note that the area is NOT equal to $\int_{-1}^{2} x(x+1)(x-2)dx$ (compare screens 2 and 4)	
	
	
	

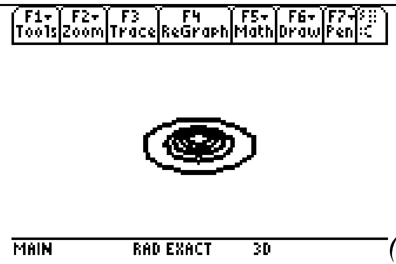
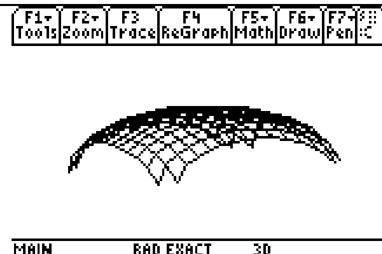
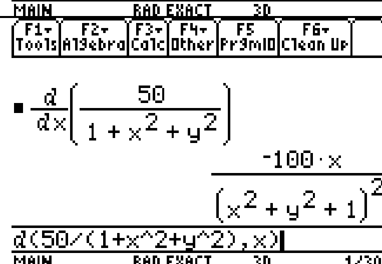
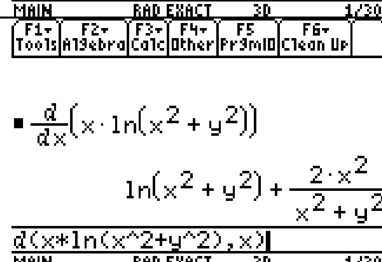
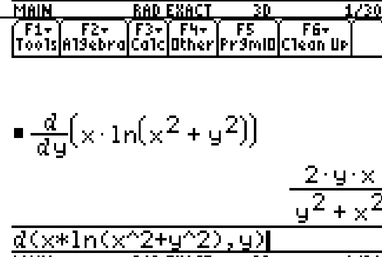
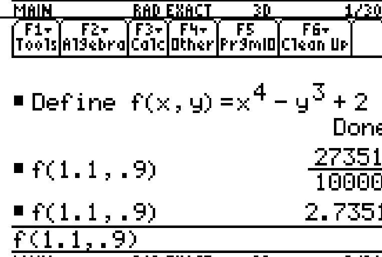
5. Integration techniques

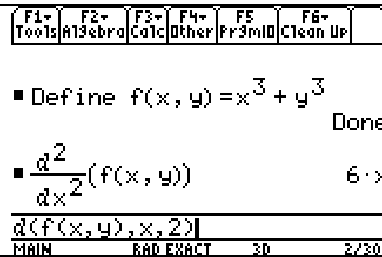
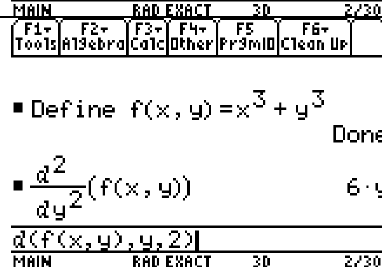
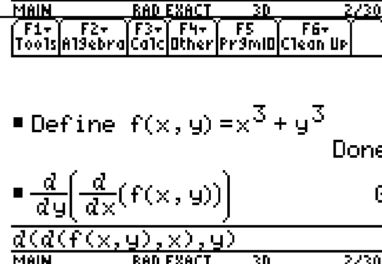
<table>
<tr><th>Instructions</th><th>Screen Shot</th></tr>
<tr><td>Specific techniques for integration are not required when using the TI-89 since it will integrate all integrable functions, using the ∫ function. However, we can verify some of the formulas for general results, as well as more specific functions.</td><td>
</td></tr>
<tr><td>Note how functions such as $\sec^2(ax)$ are entered, and the need for ( ) around the whole of a numerator and/or a denominator in $\frac{1}{a^2+x^2}$ and $\frac{3x+2}{x(x^2+1)}$.</td><td>
</td></tr>
<tr><td>Integrating a rational function</td><td>
</td></tr>
<tr><td>An inverse trig function.</td><td>
</td></tr>
<tr><td>Integrating a rational function</td><td>
</td></tr>
<tr><td></td><td>
</td></tr>
</table>

	
Indefinite and definite integrals.	

6 Functions of Two Variables

Instructions	Screen Shot
Set the graph drawing mode to 3D using MODE Graph 5: 3D. In the ◆ Y= mode the DEs are then set up ready for you to enter *z*1= etc. The TI-89 uses *y* and *x* for these functions.	
Use ◆ [GRAPH] to draw the graph (this may take a few seconds) You may need to resize the window using ◆ [WINDOW] where you can set all three variables. The viewing angle can also be changed using the *eye* variables or by using the  keys.	
Pressing [ENTER] will rotate the graph dynamically.	

We can draw contours too. Select the ◆ Y= mode and press ◆ | Change Style 1: WIRE FRAME to 3: CONTOUR LEVELS and press ENTER. Use ◆ [GRAPH] and then F6 Draw 7: Draw Contour command in the graph mode to enter the x and y values (here each 0). This can also be rotated discretely or dynamically.	
Here is the graph of $y = \sqrt{16 - x^2 - y^2}$.	
For partial derivatives, the TI-89 assumes letters to be constants unless told they are variables, so will do these as shown using F3 1: *d*(differentiate Here with respect to x	
Here differentiate with respect to x…	
…and here with respect to y Remember that these are the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ not what the CAS notation implies.	
We can use the F4 1: Define to define a function in two variables and hence find the value of the function.	

<table>
<tr><td>We can get f_{xx} by differentiating twice with respect to x…</td><td>
</td></tr>
<tr><td>…and f_{yy} by differentiating twice with respect to y</td><td>
</td></tr>
<tr><td>For f_{yx} and f_{xy} we differentiate twice, once for each variable.</td><td>
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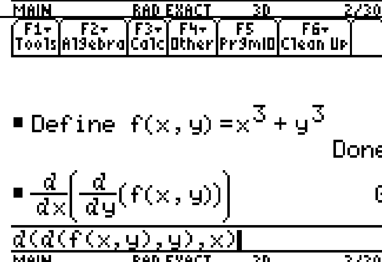
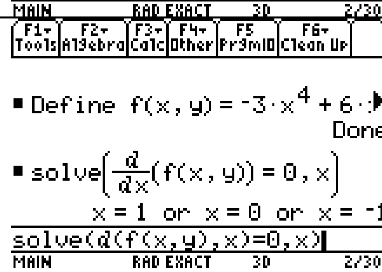
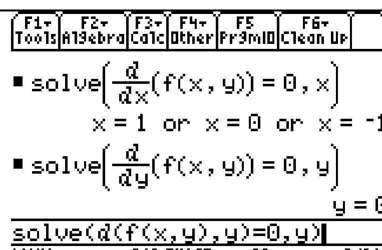
<tr><td></td><td>
</td></tr>
<tr><td>Example 207</td><td>
</td></tr>
<tr><td>Then use the Hessian obtained as above to test each point.</td><td>
</td></tr>
</table>

7 Linear Systems

7.1 Gaussian Elimination

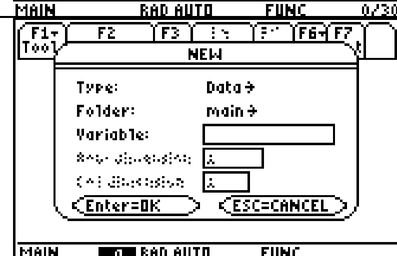
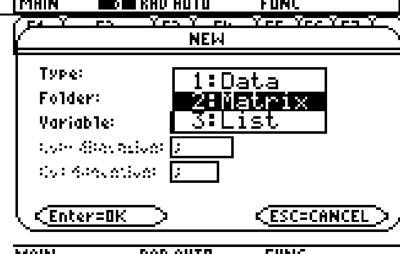
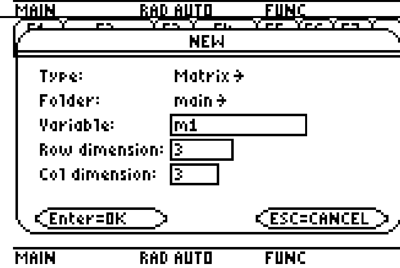
Matrix notation

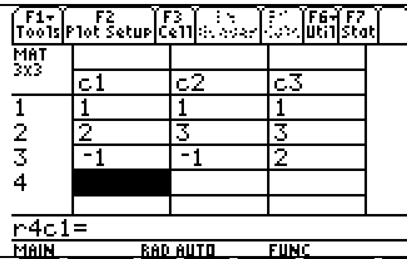
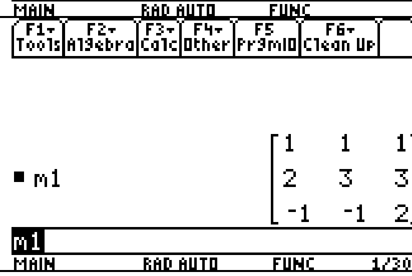
When there are 3 equations – in x, y, and z – we start by eliminating the first variable (x) in the last 2 equations and then eliminate the second variable (y) in the last equation. This leaves us with a set of equations in ***echelon form***. Once the equations are in *echelon form*, they can be solved by **back substitution**.

This can be done using the row operations on the TI–89, or using functions which give echelon form and reduced echelon form. First, we need to know how to enter a matrix into the Data/matrix Editor or into the Home screen.

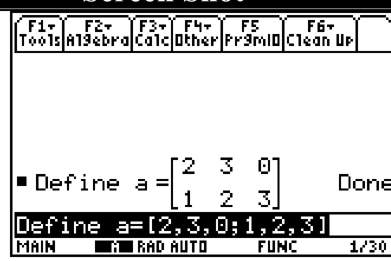
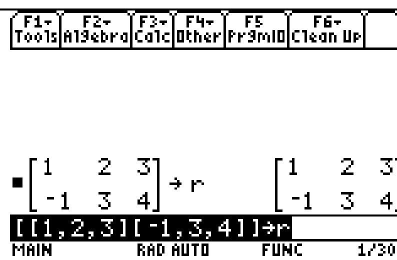
References: **TI-89 Guidebook** 229-233

Entering a matrix into the Data/matrix editor:

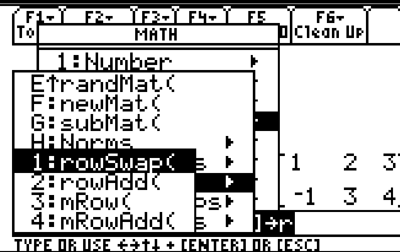
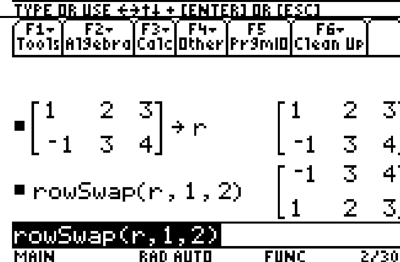
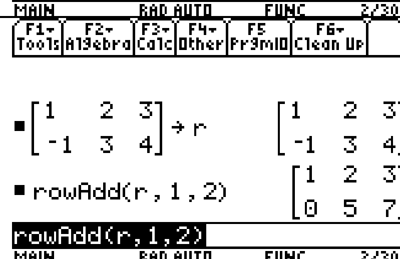
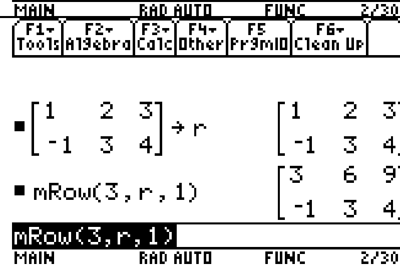
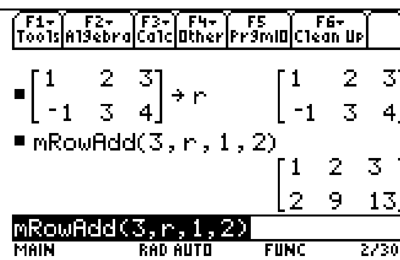
Instructions	Screen Shot
Press [APPS] 6, open the Data/matrix editor and then select 3. New	
For *Type*, select Matrix, as following.	
Press B and select 2: Matrix.	
Press D D and enter the variable name M1. (Some names are reserved, if you try to use a reserved name you will get an ERROR message). Enter the row and column dimensions of the matrix.	

Type in the first three rows and columns of the matrix. You will need to use the arrow keys to move around. Press ENTER to register each entry. You can use fractions and operations when you enter values.	
Press the " key and enter M1 return. You should now see the matrix in this standard form.	

Entering a matrix into the Home Screen

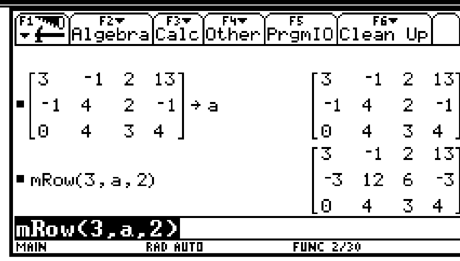
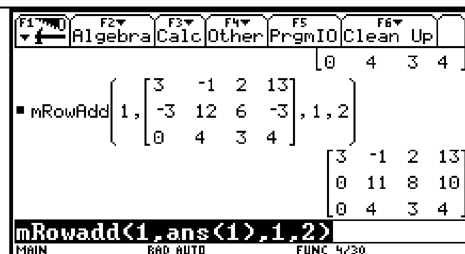
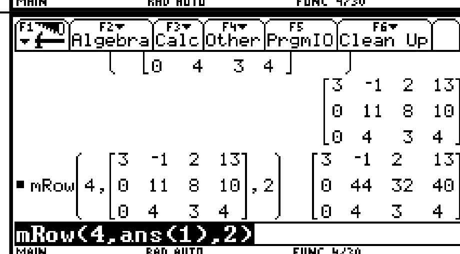
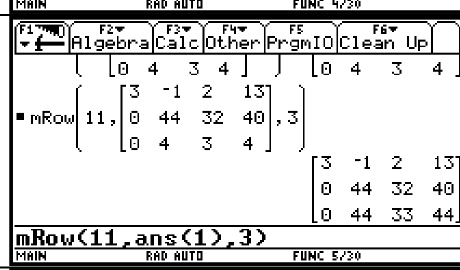
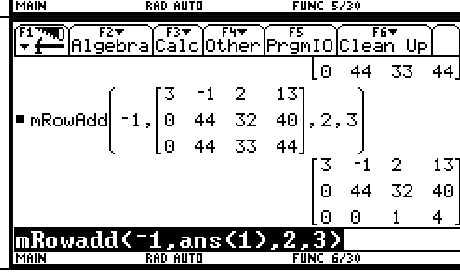
Instructions	Screen Shot
Method 1: From the Home screen, enter a matrix by using **Define**(which can be accessed by F4 1 or could be typed in). Use the square bracket [ ] to enclose the matrix. We enter the matrix by typing the first row and then the second and so on. Use commas to separate entries and semicolons to separate rows.	
Method 2: To enter a matrix into the Home screen, use one set of brackets around the entire matrix and one set of brackets around each row. Use commas to separate the entries in a row. Then press **STO→**, type a name for the matrix, and press **ENTER.** Example: [[1,2,3][-1,3,4]] **STO→** r **ENTER**	

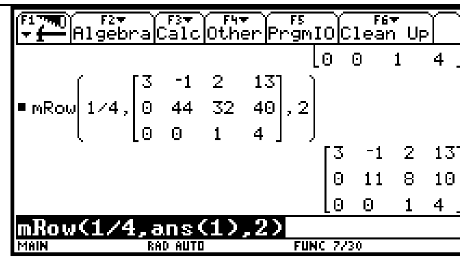
7.2 Matrix Row Operations

Instructions	Screen Shot
To swap two rows in one matrix, use 2nd [MATH] 4:Matrix J:Row ops 1:rowSwap(.	
Example: We can change rows 1 and 2 of matrix r with the command *rowSwap(r,1,2)*.	
To add the entries of one to those of another row, use 2nd [MATH] 4:Matrix J:Row ops 2:rowadd(. **Example:** Add the entries of row 1 to those of row 2 and store them into row 2 with the command *rowAdd(r,1,2)*.	
To multiply the entries of one row by a value, use 2nd [MATH] 4:Matrix J:Row ops 3:mRow(. **Example**: Multiply the entries of row 1 by 3 and store them into row 1 with the command *mRow(3,r,1)*.	
To multiply the entries of one row by a value and add the products to another row, use 2nd [MATH] 4:Matrix J:Row ops 4:mRowAdd(. **Example:** Multiply the elements of row 1 by 3, add the products to row 2, and store them into row 2 with the command *mRowAdd(3,r,1,2)*.	

Example 7.2.1: Solve the following system**:**

$$\begin{aligned} 3x - y + 2z &= 13 \\ -x + 4y + 2z &= -1 \\ 4y + 3z &= 4 \end{aligned}$$

Instructions	Screen Shot
The augmented matrix is: $\begin{bmatrix} 3 & -1 & 2 & 13 \\ -1 & 4 & 2 & -1 \\ 0 & 4 & 3 & 4 \end{bmatrix}$ Use the following instructions to row reduce this matrix. [3, –2, 2, 13 ; –1, 4, 2, –1 ; 0, 4, 3, 4] [STO▸] a [ENTER] [2nd] [MATH] 4:Matrix J:Row ops 3:mRow (3, a, 2) [ENTER] multiply the entries of row 2 by 3 and store them into row 2.	
4:mRowAdd (1, ans(1), 1, 2) [ENTER] multiply the elements of row 1 by 1, add the products to row 2 and store them into row 2.	
Multiply the elements of row 2 by 4.	
Multiply row 3 by 11.	
Multiply row 2 by –1, add the products to row 3, and store them in row 3.	

Here is the echelon form. $3x - y + 2z = 13$ $11y + 8z = -1$ $z = 4$ By back substitution, $z = 4$, $11y + 8(4) = -1$, so $y = -2$ $3x - (-2) + 2(4) = 13$, so $x = 1$	
On the TI-89 this is also obtained by: [3, –2, 2, 13 ; –1, 4, 2, –1 ; 0, 4, 3, 4] STO▸ a ENTER 2nd [MATH] 4:Matrix 3:ref (a) ENTER