Introduction to Using the TI-89 Calculator

Note: If this is the first time that you have used the TI-89 computer algebra system (CAS) calculator then you should first work through the Introduction to Using the TI-89. Some of the information found there in terms of key presses, menus, etc. will be assumed in what follows. While much of the information provided here describes how to carry out many mathematical procedures on the TI-89, using technology is not simply about finding answers, and we would encourage you NOT to see this calculator as primarily a quick way to get answers. Research by the authors, and others, shows that those who do this tend to come to rely on the calculator to the detriment of their mathematical understanding. Instead learn to see the TI-89 as a problem-solving, and investigative tool that will help you to understand concepts by providing different ways of looking at problems, thus helping you reflect on the underlying mathematics.

1. Saying 'Hello' to your graphics calculator



Basic Facilities of the TI-89

Function Keys	Cursor Pad
[F1] through [F8] function keys let you select	The cursor is controlled by the large blue
toolbar menus.	circle on the top right hand side of the
	calculator. This allows access to any part of
F4 Y- F7 WINDOW F4 GRAPH Tbl5H TABLE F1 F2 F3 F4 F5	an expression.

Application Short Keys	Calculator 1	Keypad
Used with the \checkmark key to let you select commonly used applications: [Y=] [WINDOW] [GRAPH] [TblSet] [TABLE] F6 Y- F7 WINDOW F8 GRAPH TblSet TABLE F1 F2 F3 F4 F5	Performs a variety of mathematical and scientific operations.	$\begin{array}{c} U_{1} & \Psi^{\ast} \\ \Psi^{\ast} &$

[2nd] • • • and j modify the action of other keys:

Modifier	Description
2nd	Accesses the second function of the next key you
(Second)	press
•	Activates "shortcut" keys that selects
(Diamond)	applications and certain menu items directly
	from the keyboard.
t	Types an uppercase character for the next letter
(Shift)	key you press.
j	Used to type alphabetic letters, including a space
	character. On the keyboard, these are printed in
	the same colour as the j key.
	[2nd] j used to type alphabetic letters.

	Key	Description
	APPS	Displays a menu that lists all the applications available
		on the TI-89.
	ESC	Cancels any menu or dialogue box.
	ENTER	Evaluates an expression, executes an instruction, selects
(alpha) (APPS)		a menu item, etc.
	MODE	Displays a list of the TI-89's current mode settings,
(HOME) (MODE) (CATALOG)		which determine how numbers and graphs are
		interpreted, calculated, and displayed.
	CLEAR	Clears (erases) the entry line.
	[CATALOG]	Press b or c to move the d indicator to the function or
		instruction. (You can move quickly down the list by
		typing the first letter of the item you need.)
		Press ENTER. Your selection is pasted on the home
		screen.

Application	Lets you:
[Home]	Enter expressions and instructions, and performs calculations
[Y=]	Define, edit, and select functions or equations for graphing
[Window]	Set window dimensions for viewing a graph
[Graph]	Display graph
[Table]	Display a table of variable values that correspond to an entered
	function

Press:	To display
F1, F2, etc.	A toolbar menu– Drops down from the toolbar at the top of most application screens. Lets you select operations useful for that application
2nd [CHAR]	CHAR menu– Lets you select from categories of special characters (Greek, math, etc.)
2nd [MATH]	MATH menu– Lets you select from categories of mathematical operations

Press	To perform
2nd [F6]	Clean Up to start a new problem:
Clear a–z	Clears (deletes) all single-character variable names in the current folder. If any of the variables have already been assigned a value, your calculation may produce misleading results.

Problem?	Try this!
If you make a typing error	If you make a typing error use 🖛 to undo one
	character at a time
	If necessary, also press M to clear the complete
	line.
If you want to clear the home screen completely	Press F1 n

Mode Settings

Instructions	Screen Shot
Press MODE, which shows the modes and their current settings	MODE Page 1 Page 2 Page 3 Graph Graph Gurrent Folder Main 7 Display Disits FLDAT 6 3 Current Folder Bisplay Disits FLDAT 6 3 Complex Format REL 2 Vector Format RECTANGULAR 3 Pretty Print Discense Main RAD AUTO
If you press F2 then 'Split Screen' specifies how the parts are arranged: FULL (no split screen), TOP-BOTTOM, or LEFT-RIGHT	MODE F1 F2 Pa3e 1Pa3e 2[Pa3e 3] * Split Screen Split 1 App Home + Split 2 App Graph + Number of Graphs Station + Station + <t< td=""></t<>

(a) Entering a Negative Number

Instructions	Examples
Use for subtraction and use \sum for negation.	To enter the number –7, press (-) 7.
To enter a negative number, press Σ followed by	$9 \times (-) 7 = -63,$
the number.	$9 \times - 7 = $ displays an error message
	To calculate $-3 - 4$, press (-) $3 - 4$ ENTER

(b) Implied Multiplication

If you enter:	The TI-89 interprets it as:
2 <i>a</i>	2^*a
ху	Single variable named <i>xy</i> ; TI-89 does not read as x^*y

(c) Substitution

Instructions	Examples
Using [2nd] []key	eg) (-) $x^{2+2} x=3$ -7 This calculates the value of $-x^{2} + 2$ when $x = 3$
Using 'STORE' key: STO►	eg) Find $f(2)$ if $f(x) = -x^3 + 2$ $f(x) = -x^3 + 2 \rightarrow f(x)$ f(2) = -6

(d) Rational Function Entry

Instructions	Example
$\frac{f(x)}{g(x)} = \frac{(f(x))}{(g(x))} = $ (numerator) \div (denominator)	$\frac{x+1}{2x-1} \rightarrow (x+1) \div (2x-1)$

(e) **Operators**

addition: + subtraction: - multiplication: × division: ÷ Exponent: ^ (f) Elementary Functions

Exponential: $e^{(x)}$	Trigonometric:
natural logarithm: $ln(x)$	$\sin(x), \cos(x), \tan(x), \sin^{-1}(x), \cos^{-1}(x), \tan^{-1}(x)$
square root: √	
absolute value: abs(x)	If you want $\underline{sec}(x)$ then put $1/\cos(x)$, $\cscc(x)$ is
	$1/\sin(x)$.
	Note: The trigonometric functions in TI-89
	angles are available in both degrees and radians.
	If you want degrees (180°) or radians (π) change
	using the MODE key previously discussed.

(g) Constants

To find:	Work
<i>i</i> : imaginary number	with 2nd key
π : Pi	with 2nd key
∞ : infinity	with 🔹 key

(h) Recalling the last answer

Instructions	Exam	ple
[2nd] [ANS]	ans(1)	Contains the last answer
	ans(2)	Contains the next-to-last answer

(i) Cutting, Copying and Pasting

Press:	To:
$\mathbf{t} \odot \text{ or } \mathbf{t} \odot$	highlight the expression.
• 5, • 6 and • 7	cut, copy and paste.
[2nd] [ENTRY]	replace the contents of the entry line with any previous entry.

(j) When differentiating with respect to x

To find:	Туре:
Limit $\lim_{x \to a} f(x)$:	$\lim(f(x), x, a)$
Indefinite Integral $\int f(x)dx$:	$\int (f(x), x, c)$
Definite integral $\int_{a}^{b} f(x) dx$:	$\int (f(x), x, a, b)$
Area between $f(x)$ and $g(x)$ on the interval $[a, b]$:	$\int_{a}^{b} \left f(x) - g(x) \right dx$
Differentiation $\frac{d}{dx}f(x)$:	d(f(x), x)

2. [Y=] and [Table]

(a) The [Y=] menu

Instructions	Screen Shot
Press • [Y=] to see the following:	F1 700 F2× F3 F3 F5× F5 F6× F1 y1= y2= y3= y4= y4= y5= y3= y3= y3= y3= y3= y3= y3= y3= y3= y4= y4= y5= y4= y5= y4= y5= y4= y5= y4= y4= y4= y4= </td

If there are any functions to the right of any of these eight equal signs, place the cursor on them (using the arrow keys) and press CLEAR

Place the cursor just to the right of $y_1 = and$ follow the sequence below.

Press	See	Explanation
$2x \cdot 3$	y1(x) = 2x + 3	You have entered
		$y_1 = 2x + 3$
[HOME]		This returns you to a blank
		Home Screen.
$y_1(x)$	y1(x)	This pastes y1 on the Home
	2x + 3	Screen.
y1(4) ∏	y1(4)	This finds the value of y1
	11	when $x = 4$.

(b) Table

Instructions	Screen Shot	
Press \bullet [TABLE] to see the table of values for $2x + 3$, as shown below:	Fill Fill <th< td=""><td></td></th<>	
Press • [TblSet], try change the settings and see the effect in [TABLE].	F1 F2 F2<	
	Figure Figure	
	Image: Setup [c]] # 1 # 1 # 1 # 1 # 1 # 1 # 1 # 1 # 1	
	F2 F2<	

Instru	ctions	Screen Shot
By changing [TblSet] from [1. AUTO] to	Setup Stipes Del Porting Port
[2.ASK], complet	e the table below:	tblStart: -2.
Remember: y1 is stil	l set to $2x + 3$	0 atbl: 1.
		1 Graph <-> Table: OFF→ 2 Independent: 1:80TO
		4 (Enter=SAVE)
		$5.$ 13. x = $^{-2}$.
		MAIN RAD AUTO FUNC
Х	y1	Setup Cell Header Del Row Ins Row
11	?	× 91 11. 25.
-3	?	-3357
-5	?	1. 5
	·	3. 9
		5. 13.

3. Graphing

(a) Displaying Window Variable in the Window Editor

Instructions Press (IWINDOW) or APPS 3 to display the Window Editor.	Screen Shot Tools200m xmin=-10. xmax=10. xscl=1. ymin=-10. yscl=1. xres=2
	xmin xscl yscl ymin

Variables	Description
<i>x</i> min, <i>x</i> max, <i>y</i> min, <i>y</i> max	Boundaries of the viewing window.
xscl, yscl	These <i>x</i> and <i>y</i> scales set the distance between tick marks on the
	x and y axes (see above right)
xres	Sets pixel resolution (1 through 10) for function graphs. The
	default is 2.

(b) Overview of the Math Menu

Press F5 from the Graph screen	F1+ F2+ F3 F4 F5+ F6+ F7+ ⁵ :: ToolsZoomTraceRegraph Math DrawPen:: 1: Value 2: Zero 3: Minimum
	4:Maximum 5:Intersection 6:Derivatives ► 7:Jf(x)dx 8↓Inflection
	E MAIN RAD AUTO FUNC

Math Tool	Description
Value	Evaluates a selected $y(x)$ function at a specified x value
Zero, Minimum,	Finds a zero (x-intercept), minimum, or maximum point within an
Maximum	interval.
Intersection	Finds the intersection of two functions.
Derivatives	Finds the derivative (slope) at a point.
$\int f(x)dx$	Finds the approximate numerical integral over an interval.
∆·Tangent	Draws a tangent line at a point and displays its equation

(b) Finding the Maximum & Minimum Values of a Function from its Graph

Instructions	Screen Shot
1. Display the Y=Editor .	F1 7m0 F2 ▼ ↓ F3 F4 ↓ F5 ↓ ↓ PLDTS y1= y1= ↓ ↓ ↓ ↓ y2= y3= ↓ ↓ ↓ ↓ y3= y3= ↓ ↓ ↓ y5= y6= ↓ ↓ ↓ y6= ↓ ↓ ↓ y7= ↓ ↓ ↓ y10= ↓ ↓ ↓ ↓ ↓ ↓ ↓
2. Enter the function	$\begin{array}{c} \begin{array}{c} r_{1} = r_{1} \\ r_{2} \\ r_{3} \\ r_{4} \\ r_{4} \\ r_{5} \\ r_$
 Enter graph mode (F3). Open the Math Menu F5, and select 4: Maximum. 	Fix Fix </td
4. Set the lower bound.	Fiz
5. Set the upper bound.	F17 m0 F2 F3 F4 Zoom F10 m2 F8 F7 v Image: Image
6. Find the maximum point on the graph between the lower and upper bounds.	$\begin{array}{c} \begin{array}{c} \text{Figure}{} & Fi$
7. Transfer the result to the Home screen, and then display the Home screen.[Home]	

(c) Overview of the Zoom Menu

Instructions	Screen Shot
Press F2 from <i>y</i> =Editor, window Editor, or Graph screen	F1+ F2+ F3 F4 F5+ F6+ F7+ F3+ 1001s 200m TraceRegraph Math Draw Pen S2+ 1: 200m 200m TraceRegraph Math Draw Pen S2+ 1: 200m 200m TraceRegraph Math Draw Pen S2+ 1: 200m Coom TraceRegraph Math Draw Pen S2+ Draw Pen S2+ Draw Draw Pen S2+ Draw Draw

Zoom tool	Description
1:ZoomBox	Lets you draw a box and zoom in on that box.
2:ZoomIn 3:ZoomOut	Lets you select a point and zoom in or out by an amount defined by
	SetFactors.
4:ZoomDec	Sets Δx and Δy to 0.1, and centres the origin.
6:ZoomStd	Sets Window variables to their default values.
	xmin=-10, xmax= 10, xscl=1, ymin=-10, ymax= 10, yscl= 1, xres= 2

Note: To get out of the graphing mode use 2 K. This will not work while the **BUSY** icon is flashing in the bottom right hand corner. Adjust your graph by selecting **F2** and choosing **2:ZoomIn**, **3:ZoomOut**, or **A:ZoomFit**

Example: Graph $y = x^2$ by following these instructions.

Instructions	Screen Shot
• [Y=] x ^ 2 [ENTER]	F1+ F2+ F3+ F5+ F6+ ::> Toots/200m/Edit Att Style::>< > *PLDTS 92= 92= 93= 94= 93= 95= 94= 95= 94= 95= 94= 95= 94= 95= 94= 95= 94= 95= 94= 92= Main RAD AUTO FUNC
● [GRAPH]	F1+ F2+ F3 F4 F5+ F6+ F7+ F3::: ToolsZoom Trace Re3raph Math Draw Pen ::: MAIN RAD AUTO FUNC

To draw a new graph go to [y=] and change the formula in the y1 position using the cursor to move up to it to delete it. This effectively clears the previous graph as well. Alternatively, using y2 will add the new graph to $y = x^2$.

[HOME] returns you to the Home screen.

4. The Algebra Menu

Menu Item	Description F2 MENU First	
1: solve	Solves an expression for a specified variable. This returns solutions only,	
	regardless of the Complex Format mode setting (For complex solutions, select	
	A:Complex from the algebra menu).	
2: factor	Factorises an expression with respect to all its variables or with respect to only a	
	specified variable.	
3: expand	Expands an expression with respect to all its variables or with respect to only a	
-	specified variable.	
4: zeros	Determines the values of a specified variable that make an expression equal to	
	zero.	
5: approx	Evaluates an expression using floating-point arithmetic, where possible.	
6: comDenom	Calculates a common denominator for all terms in an expression and transforms	
	the expression into a reduced ratio of a numerator and denominator.	
7: propFrac	Returns an expression as a proper fraction expression.	

Using the TI-89 in Mathematics

Topic 0 Preliminaries 0.2 Inequalities and the absolute value

Instructions	Screen Shot
Inequalities	F1+ F2+ F3+ F4+ F5 Toole919ebra(cole90bar/8+9mill(1ean.up
We can directly solve these, for example	
$3x - 2 \ge 7x + 10$	
F2 $3x - 2 \bullet [>] 7x + 10 , x) ENTER$	■ solve(3·x - 2≥7·x + 10, x)
	×≤-3
	MAIN RAD AUTO FUNC 1/30
We can also transform an inequality into the	F1+ F2+ F3+ F4+ F5 ToolsA19ebralCalcl0therPr9mi0Clean UP
form $x \ge$ or $x \le$ by performing the same	■ 3·× - 2≥7·× + 10
operation on both sides.	$3 \cdot x = 2 \ge 7 \cdot x + 10$ = $(3 \cdot y = 2 \ge 7 \cdot y + 10) = 7 \cdot y$
For example we can solve the inequality	-(3*× - 2 ≥ 1 × + 10) - 1 × × -4 · × - 2 ≥ 10
$3x - 2 \ge 7x + 10$ [ENTER]	■(-4·×-2≥10)+2
by adding $-7x$ to both sides of the equation,	$-4 \cdot \times \geq 12$
then adding 2	MAIN RAD AUTO FUNC 3/30
2 nd [ANS] = 7x [ENTER]	
2nd ANS + 2 ENTER	
	F1+ F2+ F3+ F4+ F5
and dividing by -4 gives the answer.	$= (3 \cdot x - 2 \ge 7 \cdot x + 10) - 7 \cdot x$
	$-4 \cdot \times -2 \ge 10$
[2nd] [ANS] [÷] [(-)] 4 [ENTER]	$=(-4 \cdot \times - 2 \ge 10) + 2$ -4 · ∨ ≥ 12
Note that the CAS reverses the inequality when	4·×≥12
dividing by the –ve quantity.	-4 x 2 3
	MAIN RADIAUTO FUNC 4/30
The absolute value function is found in the	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mlOClean Up
[MATH] menu (press 2nd 5), select 1: Number,	
select 2: abs((or Press D and ENTER)) and press	I -3.56 89/25
ENTER).	■[-3.56] 3.56
(This function also gives the modulus of a	- La + F. (1)
complex number.). To switch from exact to	-[a + 5 + 5i] $(a + 25)$
approximate mode we press • ENTER	MAIN RAD EXACT FUNC
Inequalities with absolute values can be solved	F1+ F2+ F3+ F4+ F5 ToolsAlgebraCalcotherPrgml0Clean Up
when they are broken down into single	
inequalities,	
	■solve(-5 <x+2,x) x="">-7</x+2,x)>
	• solve(x + $2 < 5$, x) x < 3
	SOIVE(X+2(5,X) MAIN RAD EXACT FUNC

or sometimes by squaring both sides of the inequality (note the unusual notation for this).	F1+ F2+ F3+ F4+ F5 Too1sA19ebraCa1clOther Pr3miD C1ean Up
F2 1: Solve((2nd 5 1: Number 2: $abs(x-5) < (2nd 5 1: Number 2: abs(x+3))^2, x)$	■ solve $((x - 5 < x + 3)^2, x)$ x > 1 <u>solve((abs(x-5)(abs(x+3))</u> Main Rep.Fract. Find 1/30

Note: The use of the • key to switch between exact and approximate modes (the TI-89 tries to use fractions in exact mode).

0.3 Domain, range and graph of a function

Instructions	Screen Shot
We can use the $[Y=]$ menu obtained by	F1+ F2+ [3] 51; 51; 51; 51; 51; 51; Tools Zoom(3) 31; 7 \$85; 85; 75; 85; 85; 75; 75; 85; 75; 75; 75; 75; 75; 75; 75; 75; 75; 7
the cursor just to the right of $v1$ = and	y1=x·cos(x) y2=y1(-x)
enter the function required. Note that we	$u_3 = \frac{e^{x-4}-2}{e^{x-4}-2}$
can use previously defined functions in	$1 + (x - 4)^2$
later ones.	
To enter a split domain function we use	F1+ F2+ F3+ F4+ F5 Too1sA19ebraCa1cluther Pr9ml0C1ean Up
the when () function and nest them if	
piecewise function. This has been done	[{1,×,<0,×<,
by defining a function g and using $y1=g$.	• Define $g(x) = \begin{cases} x, else \\ x - 3, else \end{cases}$
	Done Define g(x)=when(x(4,when
We can use g a number of times this way	MAIN RAD EXACT FUNC 1/30
	roominiseer de archader (* Sonale rean av
	(()
	$ \begin{cases} 1, \times < 0 \\ 1, \times < 1 \\ 2, \times $
	Done
	n(x<4,when(x<0,1,x),x-3) MAIN RAD EXACT FUNC 1/20 MAIN RAD EXACT FUNC 1/20 MAIN RAD EXACT FUNC 1/20
Note that these graphs look better when	ToolsZoom Trace ReGraph Math Draw Pen ::
[Y=] screen under F6 Style 2. Dot	
We can test the value of the function g at	MAIN RAD EXACT FUNC F1+ F2+ F3+ F4+ F5 ToolsA13ebra(Callather)Pr3m(D)Clean Up
the points $x=0$, and $x=4$ on the [HOME]	
screen, as shown.	
	(x − 3,eise ■g(0) 0
	■ g(4) <u>1</u>

Or we could use a table of values	F1+ F2 C3 F4 C C3 C3 C
of we could use a more of values.	
	5 1.
	0. 0.
	.5 .5
	y1(x)=0.
	MAIN BAD EXACT FUNC

Note: Use • [TblSet] to zoom in on the table values.

0.4 Trigonometric functions

Instructions	Screen Shot
The graphs of the functions $f(x)=x\cos(x)$ and $f(x)=x^2\sin^2(x)$ (entered as $\sin(x)^2$) are shown on the TI-89. We can verify that one is an odd function and the other even, by checking $f(a)$ against $f(-a)$ on the [HOME] screen.	Tools zoom Trace Regraph Hath Draw Pen :
It's an odd function.	 y1(a) a · cos(a) y1(-a) -a · cos(a) y1(-a) -a · cos(a) y1(-a) -a · cos(a)
Graph of the function $f(x) = x^2 \sin^2(x)$	F1+) F2+) F3 F4 F5+) F6+) F7+) F5+) Tools Zoom Trace ReGraph High High High MAIN BAD EXACT FUNC
It's an even function.	$\begin{array}{c} F_{1} \cdot F_{2} & F_{3} \cdot F_{4} & F_{5} \\ \hline Tools[A13ebro]Colc[Other]Pr3ml0]Clean Up \\ \hline \\ \texttt{y1(a)} & a^{2} \cdot (sin(a))^{2} \\ \hline \\ \texttt{y1(-a)} & a^{2} \cdot (sin(a))^{2} \\ \hline \\ \hline \\ \texttt{y1(-a)} & \text{AD EXACT} & FUNC & 2/30 \\ \hline \end{array}$

0.5 Translations and compositions of functions

Instructions	Screen Shot
We can check the effect of a	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebra[Calc Other Pr9ml0 Clean Up
transformation by looking at multiple	
graphs of a function, using the command	
to set values of a variable (which can be	<pre>Define f(x) = x Done</pre>
read as 'when').	■Graph 5·f(x+k) k=(-2 🕨
Enter F4 1. Define i $F(r) = 2nd 5 1$.	Done

Number 2: abs(x)	
	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl3ebraCalcOtherPr9mIOClean Up
	■ Define f(x)= x Done
	• $\langle k \rangle k = \langle -2 -1 0 1 2 \rangle$ Done
The graph function is at F4 Other 2: Graph	TooTsZoomTraceReGraphMathDrawFen-C
Composite functions can be obtained	F1+F2+F2+F3+F4+F5 ToolsA13ebraCa1COtherPr3mlOC1ean Up
the notation $f(g(x))$.	<pre>Define h(x) = x + sin(x) Done</pre>
	• $h(f(x))$ sin($ x $) + $ x $
	■ f(h(x)) sin(x) + x
	MAIN RAD EXACT FUNC 4/30

0.6 One-to-one and inverse functions

Instructions	Screen Shot
Graphs of functions which are inverses, such as <i>exp</i> and <i>ln</i> will not look like reflections in $y=x$ on the TI-89 unless the same scale can be used on each axis.	F1+ F2+ F3 F4 F5+ F6+ F7+81 ToolsZoomTraceReGraphMathDrawPenic
This can be done using F2 Zoom 5: Zoom Sqr (as shown in these graphs).	MAIN BAD EXACT FUNC F1+ F2+ F3 F4 Too1s/200m/Trace/ReGraph/Math/Draw/Penic Y
Note there is also a function in graph mode F6 Draw 3; DrawInv to draw an inverse function's graph	F1+ F2+ F3 F4 ToolsZoom/Trace/ReGraph/Math/Draw/Penic Y Y MBIN BAD AUTO FUNC



Note: We can not use $f(x)^{-1}$ for inverse functions. This gives the reciprocal of the function.

Topic 1 Limits

1.1 Limits of a function

Instructions	Screen Shot
Use F3 3: Limit(to find limits. The order	F1+ F2+ F3+ F4+ F5 F6+ ToolsAlgebraCalcOtherPrgmlDClean Up
is Limit(function, variable, value	
approached).	■ lim a(x) 2
	x+2
	■ lim g(x) undef
	×→4 limit(g(x),x,4) MAIN RAD EXACT FUNC 2/30
We can also find one-sided limits by	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCalcOtherPr3mIDClean UP
writing 1 or –1 before closing the bracket	<pre>lim g(x) undef</pre>
for right and left limits respectively (NB	$\times \neq 4$
do NOT enter +1, only 1). If the limit	×+4 ⁻ 4
does not exist we are given the answer	lim g(x) 1
undef(ined). Checking the right and left	$\frac{x \neq 4^*}{\text{limit}(q(x), x, 4, 1)}$
limits may help us see why this is so.	MAIN RAD EXACT FUNC 4/30
Checking a table of values can also be	Tools Setup Collinger in Standard Stand
useful.	x y1 3-8 3-8
	3.9 3.9
	4. 1. 4.1
	4.2 1.2
	×=4.2
While not proving them, we can verify	MAIN RAD EXACT FUNC (F1+\ F2+ \F3+\F4+\ F5 \ F6+ \)
limit laws for some examples. You	Tools A19ebra Ca1c Other Pr9mIO Clean UP
should check them with some functions	(h(x))
of your own. Note the use of the	lim g(x)
calculator's approximate mode here	$=\frac{x+2}{\lim h(x)}$.687451
calculator s'approximate mode nere.	x+2
	limit(g(x),x,2)/limit(h(x Main Rad Exact Func 3/30

Example 1.1

Instructions	Screen Shot
Example $\lim_{x\to 0} \sin \frac{1}{x}$	F177m0) F2▼ ▼∰— A1gebra Ca1c Other PrgmIO Clean Up
Some limits do not exist. We can build an	
understanding of some reasons for this.	• Define $f(x) = sin\left(\frac{1}{x}\right)$ Done
	■ lim f(x) undef x+0
	1imit(f(x),x,0) MAIN 600 AUTO FUNC 2/30
We can plot the graph and zoom in on	fi Tano F2 F3 F3 F4 F5 F5 F6 F7 F6 F1 F1 Zoom Trace ReGraph Math Draw 🗸 🖉 🗄
x = 0.	9
	Au
	×

Or from the table we can see that no matter how much we zoom in on $x = 0$ values do not tend towards the same number (left and right limits do not exist).	F1 F2 F2 F3 <
	F2 F2
	F1 F2 F2 F2 Setup (201) (201) (201) (201) (201) F2 F2 x y1

Example 1.2

Instructions	Screen Shot
Example: Find $\lim_{x\to 0} \frac{\sin x}{x}$ This is an important limit, but one that cannot be found by putting $x = 0$, since the function is undefined for $x = 0$. Enter F3 (3) $f(x) = SIN(x)$; $x \to x \to 0$) ENTER	F1700 F2 F3 F4 F5 F6 • Define $f(x) = \frac{\sin(x)}{x}$ Done • f(.2) .993347 • f(.1) .998334 • f(.05) .999583 • f(.01) .999983
If we change the value of x, taking steps closer to 0 then the value of $f(x)$ gets closer to 1. F3 (3) $f(x) = SIN x$ F3 (3) $f(x)$, x, $) \div x$, x, 0 (0) ENTER) ENTER	F1700 F2× F3× F4× F5× F6× F6× <td< td=""></td<>
Looking at the graph can help with what the limit might be.	F1700 F2+ F3 F4 F5+ F7 € € Y 1 Y 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1

XX7 4 1 1	
We can use a table	👻 🚰 Setup Calt Baseda - Dal Pow Ins Pow
	x y1
	B. undef
	4 97355
	.6 .94107
	.8 .8967
	184147
	1.2 .7767
	1.4 .70387
	👻 🚰 Setup Calt Haskes Del Postre Inc Post
	x y1
	8. undef
	.2 .99335
	.3 .98507
	.4 .97355
	.5 .95885
	.6 .94107
	.7 .92031
	ELTERNY EL Y EL
	👻 🚰 Setup (si) (Hisbidge (Deli Pow (Ins. (Pow
	x y1
	0.1 Laggage
	.02 .99993
	.03 .99985
	.04 .99973
	.05 .99958
	07 99919
	x=0
	MAIN RAD AUTO FUNC
We find that:	
we mu mat.	✓ == Setup[Cell[Header [Del Pow]Ine Pow]
$\sin x$	
$\lim - 1$	
$x \rightarrow 0$ χ	.002 1.
	.003 1.
	.007 .99999
	v=0
	MAIN RAD AUTO FUNC

1.3 Continuity

Instructions	Screen Shot
The function g used in 0.3 is	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcather Pr9mlaClean Up
discontinuous at $x=0$ and $x=4$. The limits	
at <i>x</i> =4 were calculated in Example 1.1.	<i>(</i> (), (),
	{ 1,×<0,×<4
	• $fine g(x) = [x - 3, else]$
	Done n(x(4.when(x(0.1.x).x=3)
	MAIN BAD EXACT FUNC 1/30
	Tools Zoom Trace Regraph Math Draw Pen :C
	MAIN BAD EXACT FUNC

To be continuous at $x=4$ we would need the right limit to equal the left.	F1+ F2+ F3+ F4+ F5 Tools A19ebra Calc Other Pr9ml0 Clean Up ■ lim g(x) undef x → 4
	■ lim g(x) 4 x→4 ⁻
	■ lim g(x) 1 _x→4*
	limit(g(x),x,4,1) MAIN RADEXACT FUNC 4/30

1.3.2 The intermediate value theorem

Instructions	Saraan Shat
This is very useful for showing that there	Tóo1s Zóom Trace ReGraph Math Draw Pén :C
is a root of $f(x)=0$ between two domain	
values. If $f(a) < 0$ and $f(b) > 0$ (or vice versa)	
then there is a zero of f between a and b .	
Using the table of values in a graph we can	
then zoom in on the root.	
	MAIN BAD EXACT FUNC
For the function below we see from the	F1+ F2 F3 F3 F5 F5 F5 F5
table that $f(4) < 0$ and $f(5) > 0$, so we zoom	X JA2
in to find the root using the Intermediate	38161
Value Theorem	535914
	6. 1.0778
	7. 1.8086
	x=3
This table shows it is between 4.6 and 4.8	
	4.44381
	4.61308
	535914
	5.2 .54103
	x=5.2
	MAIN BAD EXACT FUNC
This table shows it is between 4.65 and	Tools Setup Coloradar (Colorados Cons
4.7. This can be continued to the required	X 93 4 55 - 2048
accuracy.	4.61308
	4.650594
	4.75 .00923
	X=4.55 Main Bad Exact Func
We could of course get the TI-89 to find	F1+ F2+ F3 F4 F5+ F6+ F7+811 Too1sZoomTraceReGraphMathDrawPen(<
the root directly from the [Graph] or	
[Home] screens, but we need to	
understand that this theorem is one basis	
for finding it. For the graph use F5 Math	
2. Zero enter the lower and upper bounds	Zero I 💛
(4 and 5 from the theorem) and we get	xc:4.69315 yc:6.755e-14
1.60315 for the root of $f(r) = 0$ or the zero	
f	
01	

In the [Home] screen we use F2 1: Solve	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCa1cOtherPr9miDClean UP
(and enter $f(x)=0, x$). We need	
approximate mode (holding down •)	<pre>solve(f(x) = 0, x)</pre>
when pressing <u>(LWLII</u>) to get the decimal	$x = \ln(2) + 4$
answer.	<pre>solve(f(x) = 0, x)</pre>
	× = 4.69315
	solve(f(x)=0,x)
	MOIN DOD EVOLT FILME 2/20

1.4 Limits involving infinity

Instructions	Screen Shot
Limits involving infinity are entered as	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIOClean Up
before but using [∞] key (● [CATALOG])as if it is the value approached.	$= \lim_{\substack{x \to \infty}} \left(\frac{1-2 \cdot x}{3 \cdot x + 5} \right) - 2/3$ $= \lim_{\substack{x \to -\infty}} \left(\frac{1-2 \cdot x}{3 \cdot x + 5} \right) - 2/3$ $\frac{\lim_{\substack{x \to -\infty}} \left(\frac{1-2 \cdot x}{3 \cdot x + 5} \right) - 2/3}{\lim_{\substack{x \to -\infty}} \left(\frac{1-2 \cdot x}{3 \cdot x + 5} \right) - 2/3}$ $\frac{\lim_{\substack{x \to -\infty}} \left(\frac{1-2 \cdot x}{3 \cdot x + 5} \right) - 2/3}{\lim_{\substack{x \to -\infty}} \left(\frac{1-2 \cdot x}{3 \cdot x + 5} \right) - 2/3}$

1.4.3 Asymptotes

Instructions	Screen Shot
Use the limits to find the horizontal	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCa1cOtherPr9mIOClean Up
asymptotes. For sloping asymptotes we can use the TI-89 to divide the numerator of a function by its denominator, using F2, 3: Expand (the function F2, 7: propFrac(will give the same result here). The asymptote here is $y=-x/3 + 10/9$ since as $x \rightarrow \infty$ the remainder of the expansion approaches 0.	• expand $\left(\frac{3 \cdot x - x^2}{3 \cdot x + 1}\right)$ $\frac{-10}{9 \cdot (3 \cdot x + 1)} - \frac{x}{3} + 10/9$ <u>expand((3x-x^2)/(3x+1))</u> MAIN RAD EXACT FUNC 1/30
The answer can be checked by drawing both graphs.	F1+ F2+ F3 F4 F5+ F6+ F7+ F3 Tools/Zoom/Trace/ReGraph/Math/Draw/Penic

2.1 Tangents and rate of change

Instructions	Screen Shot
Consider the function $y=x^2$. We can define a rate of change function r to be the gradient of a chord of length h . That is: $r(h) = \frac{f(x+h) - f(x)}{h}$ (NB there is a built-in numeric derivative function at F3 A: nDeriv which could be used but looks rather different). We can then use this function r at a point, for example, $x = 2$. Whenever we change h taking steps of h closer to 0 then the value of r is getting closer to 4.	$ \begin{array}{c} f_{1} \underbrace{\texttt{TM}}_{\textbf{r}} \left[f_{2} \underbrace{\texttt{F}_{2}}_{\textbf{r}} \underbrace{\texttt{F}_{3}}_{\textbf{r}} \underbrace{\texttt{F}_{4}}_{\textbf{r}} \underbrace{\texttt{F}_{5}}_{\textbf{r}} \underbrace{\texttt{F}_{6}}_{\textbf{r}} \underbrace{\texttt{Up}}_{\textbf{r}} \right] \\ \bullet \text{ Define } f(x) = x^2 & \text{ Done } \\ \bullet \text{ Define } r(h) = \underbrace{\frac{\texttt{f}(x+h) - f(x)}{h}}_{\textbf{h}} & \text{ Done } \\ \bullet \text{ Define } x = 2 & \text{ Done } \\ \bullet r(.1) & 4.1 \\ \bullet r(.01) & 4.01 \\ \bullet r(.001) & 4.001 \\ \hline \textbf{R(0,001)} & \\ \hline \textbf{R(0,01)} & \\ \hline R$
We can confirm this by asking for the limit of <i>r</i> as <i>h</i> approaches 0.	F17000 F2v F3v F4v F5 F6v ■ Define x = 2 Done ■ r(.1) 4.1 ■ r(.01) 4.01 ■ r(.061) 4.001 ■ lim r(h) 4 ■ d/dx(f(x)) 4 ■ d/dx(f(x)) 4

Note:
$$f'(x) = \lim_{h \to 0} r(h) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
 (where the limits exist). Thus here the rate of change at $x = 2$: $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = f'(2) = 4$.

2.2 The derivative as a function

Instructions	Screen Shot
To differentiate on the TI-89 we use the	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCa1cOtherPr9mIOClean Up
F3 Calc, 1: d(differentiate command,	
which is also found at 2nd 8. The format	
is d(function, variable to differentiate with respect to)	$= \frac{d}{d\times} \left(3 \cdot \times^3 - 7 \cdot \times^2 + 6 \right)$
	9·× ² - 14·×
	$\frac{d(3\times^{3}-7\times^{2}+6,\times)}{\text{Main}}$
In the second example we can use the	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCalcOtherPr9mIOClean Up
function Trig collect, found in F2, 9:	
Trig, 2: tCollect to simplify the answer.	$= \frac{d}{dx} \left((\sin(x))^2 - \cos(x) \right)$
Use 2nd (-) for ANS, the previous answer.	$2 \cdot \sin(x) \cdot \cos(x) + \sin(x)$
	<pre>tCollect(2 sin(x) cos(x) +)</pre>
	$\frac{\sin(2 \cdot x) + \sin(x)}{\text{tCollect}(ans(1))}$

Example 2.1

Instructions	Screen Shot
Find out whether the	
function $f(x) = \begin{cases} x^2 & \text{for } x < 2\\ 6 - x & \text{for } x \ge 2 \end{cases}$ is	F1+ F2+ F3 Too1sZoomTraceReGraphMathDrawPen=C
differentiable at $x=2$.	
Define the piecewise functions by using	
the following instructions. $F_4 = f(x) = when (1 \times 0 \text{ and } [x]) = x$	<u>\</u> Z
$\begin{array}{c} r \neq \prod f(x) = \text{when } (f(x) \neq 0) \\ 2 \prod 6 \prod r \prod \text{ENTER} \end{array}$	
Then we graph the function f We can	MAIN RAD EXACT FUNC
define a function <i>Df</i> as its derivative	
$\frac{d(f(x))}{dx}$ (use F6 2: Dot in [Y=] to plot	
the derivative). Note that this may not be	
defined on the whole domain.	
We can see the discontinuity in the	
derived function's graph, but must check	
the limits on the [HOME] screen.	
Right limit is: F3 $\exists df(x) , x , 2 , 1$	F1+ F2+ F3+ F4+ F5 Too1sA13ebra[Ca1c Other Pr3ml0]C1ean Up
Left limit is: F3 $\exists df(x) , x , 2 , (-)1$	
) ENTER	■ lim df(x) -1 x+2*
	lim df(x) 4
	$\frac{x \neq 2^{-1}}{\text{limit}(df(x), x, 2, -1)}$
Since the limits are not the same the	MAIN BAD EXACT FUNC 2/30 (F1+) F2+ (F3+) F5+ F6+)
function is not differentiable at $r=2$ and	Too1s A19ebra Ca1c Other Pr9m10 Clean Up
df(2) is undefined.	<pre>lim df(x) -1 x→2*</pre>
	■ lim df(x) 4 ×+2 ⁻
	■ df(2) undef
	df (2) Main Bad Exact Func 3/30

Example 2.2

Instructions	Screen Shot
Example. Find the derivative of $f(x) = x^n$ Define the function $f(x) = x^n$. When we define the value of power, $n = 1, 2, 3, 4$, 10 the functions are changed to the actual functions, x, x^2, x^3, x^4, x^{10} .	$\begin{array}{c} \begin{array}{c} F_{1*} & F_{2*} \\ \hline Tools & F_{2*} & F_{3*} & F_{4*} & F_{5} \\ \hline Tools & F_{2*} & F_{3*} & F_{4*} & F_{5} \\ \hline \hline Tools & F_{3*} & F_{3*} & F_{4*} & F_{5} \\ \hline \hline \\ \bullet & Define & f(x) & \\ \bullet & Done \\ \bullet & f(x) & \\ \hline & & & \\ \hline \\ \hline & & & \\ \hline \\ \hline \\ \hline \\$

If we define the slope function <i>r</i> as the average rate of change	F1+ F2+ F3+ F4+ F5 ToolsAl9ebraCalcletherPr3ml0Clean UP
average rate of change,	• Define $r(x, h) = \frac{f(x + h) - f}{h}$
	Bone
	$(1 \ 2 \cdot x + h \ 3 \cdot x^2 + 3 \cdot h \cdot x)$
	r(x,h) MAIN RAD AUTO FUNC 5/40
	F1+ 5: 1:3* 1:** F5 Tools 8: 8: 8: 8: 8: 8: 8: 8:-
	■r(x,h)
	$\P + h^2 + 4 \cdot x^3 + 6 \cdot h \cdot x^2 + 4 \cdot I^2$
	h+0
	$\frac{(1 2 \cdot x 3 \cdot x^2 4 \cdot x^3 10)}{d(f(x), x)}$
	MAIN RAD AUTO FUNC 1/6
then we can see that the derivative of the functions are $1 - 2x - 2x^2 - 4x^3 - 10x^9$	ToolsAnJeen F3+ F4+ F5 ToolsAnJeenaCalclotherPr9miD(Clean Up = 110 F(X, T)
Tunctions are $1, 2x, 5x, 4x, 10x$.	h+0
	$(1 2 \times 3 \times 4 \times 10)$
	$\frac{1}{4x}(\tau(x))$
	CT 2/4 3/4 4/4 10/ C(T(x),x) Main Rep auto FUNC 2/40
Using the rate of function <i>r</i> , we can	f17mm) F2▼ ∫F3▼ ∫F4▼ ↓F5 ▼ ← AlgebraCalc OtherPrgmIO Clean Up
get that the general derivative of x^n is	■ Define f(x)=x ⁿ Done
nx^{n-1}	■Define r(h)= f(x+n)-f(x) Done
	• expand(r(h)) $\frac{(x+h)^n}{h} - \frac{x^n}{h}$
	■ lim r(h) n·x ^{n = 1} h≠0
	limit(r(h),h,0) MAIN RAD AUTO FUNC 4/30
Thus $\frac{dy}{dt} = f'(x) = \lim \frac{(x+h)^n - x^n}{dt} = nx^{n-1}$	fi]7700 F2▼ \F3▼ \F4▼ \F5 ▼
$dx \qquad h \rightarrow 0 \qquad h$	• expand(r(h)) $\frac{(x+h)^n}{h} - \frac{x^n}{h}$
	• lim r(h) n x ⁿ - 1
	$\bullet \frac{d}{dx}(f(x)) \qquad n \cdot x^{n-1}$

2.2.1 Second and higher derivatives

Instructions	Screen Shot
These can be accomplished by using repeated applications of the CAS function <i>d</i> . Here functions $y4(x)$ and $y40(x)$ from the [Y=] list have been used. Note the inclusion of the variable each time and the option of finding the value of a derivative at a specific value of <i>x</i> .	$= y4(x) \qquad 4 \cdot x \cdot (x^{2} + 3)$ $= \frac{d}{dx} \left[\frac{d}{dx} (y4(x)) \right] \qquad 24 \cdot x$ $\frac{d}{d(d(y4(x), x), x)}$
derivative at a specific value of x.	MAIN RAD EXACT FUNC 2/30

	F1+ F2+ F3+ F4+ F5 Too1sA13ebra[Ca1c Other Pr3ml0 Clean Up
	$= \frac{d}{d\times} \left(\frac{d}{d\times} (940(\times)) \right)$
	$\frac{-1}{\sqrt{25-x^2}} - \frac{x^2}{(25-x^2)^{3/2}}$
	$\frac{d(d(y40(x), x), x)}{Main \qquad \text{Rad Exact Func } 1/30}$
	F1+ F2+ F3+ F4+ F5 F6+ ToolsAlgebra(CalcOther)PrgmlD(Clean Up)
	$(25 - x^2)^{-1}$
	$= \frac{\alpha}{d\times} \left[\frac{\alpha}{d\times} \left(\frac{\alpha}{d\times} (940(\times)) \right) \right] \times = 3$
	- <u>225</u> 1024
	2(d(d(y40(x),x),x),x) x=3 MainBAD EXACTFUNC2/30
Alternatively we can specify the <i>n</i> th	(F1+) F2+ [F3+]F4+]F5 F6+ Too1sA19ebra[Ca1C Other Pr9ml0[C1ean Up
derivative with F3 1: d (differentiate function, x , n)	$= \frac{a^3}{a^{\times 3}} \left(4 \cdot \times \cdot \sqrt{x^2 + 3} \right)$
	5/2
	(x ² + 3) ^{3/2}
	<u>d(4×√(×^2+3),×,3)</u> Main rad Exact 30 1/30

2.3 Differentiation rules

Instructions	Screen Shot
The TI-89 can act on functions that are unknown, to give the differentiation	(f1770) ↓ ↓ Algebra Calc Other PrgmIO Clean Up
formulas.	
	$\bullet \frac{d}{dx} (f(x) \cdot g(x))$
	$\frac{d}{dx}(f(x)) \cdot g(x) + \frac{d}{dx}(g(x)) \cdot f(x)$
	RC 1 (X) + 9 (X) , X) MAIN RAD RUTO FUNC 1/30
	(1770) ↓ ↓ Algebra Calc Other PrgwIO Clean Up
	den a de comercia
	$= \frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) \qquad \frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) - \frac{d}{dx} \left(\frac{g(x)}{g(x)} \right)^{-1} \left(\frac{d}{dx} \left(\frac{g(x)}{g(x)} \right)^{-1} \right)^{-1} $
	d(f(x)/g(x),x) Main BAD AUTO FUNC 1/20
The common denominator function F2 6:	(f17m0) + ← (F1gebra Calc)Other Prgm10(Clean Up) (XX 4X) = 4(X) = 4(X)
comDenom(has been used to simplify an	$\begin{bmatrix} \frac{d}{dx}(f(x)) & \frac{d}{dx}(g(x)) \cdot f(x) \end{bmatrix}$
common denominator.	• comDenom $\left[\frac{\alpha \times \cdots \times \alpha}{g(x)} - \frac{\alpha \times \cdots \times \alpha}{(g(x))^2}\right]$
	$\frac{d}{dx}(f(x)) \cdot g(x) - \frac{d}{dx}(g(x)) \cdot f(x)$
	$(g(x))^2$
	MAIN RAD AUTO FUNC 2/30

Otherwise the TI-89 can be used to check differentiation of these functions by	F1+ F2+ F3+ F4+ F5 Too1sA19ebraCa1clotherPr9ml0Clean Up
direct entry as here	
uncer entry, as here.	
	$= \frac{\alpha}{d \times} \left[\left(2 \cdot \times^2 - 5 \right) \cdot \left(3 - 7 \cdot \times^3 \right) \right]$
	-×·(70·× ³ - 105·× - 12)
	$\frac{2((2\times^2-5)(3-7\times^3), x)}{4}$ MAIN RAD EXACT FUNC 1/30
	F1+ F2+ F3+ F4+ F5 Too1s A19ebra Ca1c Dther Pr3mi0 C1ean Up
	$= \frac{d}{dx} \left(\tan(x) \cdot (\sin(x))^2 \right)$
	$(\tan(x))^2 \cdot \left(2 \cdot (\cos(x))^2 + 1\right)$
	N(tan(x)*(sin(x))^2,x)
	F1+ F2+ F3+ F4+ F5 ToolsAl3ebraCalcOtherPr3mIDClean Up
	$= \frac{d}{d\times} \left(\frac{2 \cdot x^2 - 5}{3 - 7 \cdot x^3} \right)$
	$\times (14 \cdot \times^3 - 105 \cdot \times + 12)$
	$(7 \cdot \times^3 - 3)^2$
	d((2×^2-5)/(3-7×^3),×) MAIN RAD EXACT FUNC 1/30

2.4 The Chain Rule

Instructions	Screen Shot
The rule for this can be seen similarly for $f(x)$ raised to the power of n .	(f1770) F2▼ ↓ F3▼ Algebra Calc Other PrgmIOClean Up
	$ \begin{array}{c} \bullet \frac{d}{dx} \Big[(f(x))^n \Big] & (f(x))^{n-1} \cdot \frac{d}{dx} (f(x)) \cdot n \\ \hline \frac{d\zeta(f(x))^n, x)}{d\zeta(f(x))^n, x} & \\ \hline \end{array} $
Again functions are entered directly, although we can define them separately.	F17700 F2▼ F17700 F2▼ F17700 F12▼ F17700 F12 F17700 F12 F177
	■ Define f(x) = x ³ Done ■ Define g(x) = 3 · x + 2 Done ■ f(a(x)) (3 · x + 2) ³
	$= \frac{d}{d_{X}} (f(g(x))) \qquad 9 \cdot (3 \cdot x + 2)^{2}$ $\frac{d \langle f(g(x)), x \rangle}{M \text{HIN}} \qquad \text{EAD AUTO} \qquad \text{FUNC 4/30}$
	F1+ F2+ F3+ F4+ F5 Too1s A19ebra Ca1c Other Pr9mi0 C1ean Up
	$= \frac{d}{dx} \left(\left(\sin(e^{3 \cdot x}) \right)^2 \right)$
	6·e ^{3·×} ·sin(e ^{3·×})·cos(e ^{3·}) 2(sin(e^(3×))^2,×) MAIN BAD EXACT FUNC 1/30

F1+ F2+ F3+ F4+ F5 F6+ Too1sA19ebraCa1cOtherPr9mIOCTean UP
$= \frac{d}{d \times} \left(\left(\cos(3 \cdot \times + 1) \right)^4 \right)$
$\frac{-12 \cdot \sin(3 \cdot x + 1) \cdot (\cos(3 \cdot x + 1))}{\pi^2 (\cos(3 \cdot x + 1))}$
a(COS(3x+1)'4, x) Main Rad Exact Finc 1/30 Finc 1/20
Tóðis Atjébra Caic Other Primil Cléan Up
$= \frac{d}{d\times} \left(\left(\cos(3 \cdot \times + 1) \right)^4 \right)$
$\frac{4\sin(3\cdot x+1)\cdot(\cos(3\cdot x+1))^3}{d(\cos(3\cdot x+1))^{4\cdot x}}$
MAIN RAD EXACT FUNC 1/30

2.5 Implicit differentiation

Instructions	Screen Shots
This can be accomplished by defining y	[F1+] F2+ [F3+] F4+] F5
to be some function of x here a	Tools H19ebra Ca1C Other Pr9MID Clean UP
to be some function of x , here c .	■Define y=c(x) Done
	$= \frac{d}{d\times} (5 \cdot \times^3 + y^2)$
	$2 \cdot c(x) \cdot \frac{d}{dx} (c(x)) + 15 \cdot x^2$
	$\frac{d(5\times^{3}+y^{2},x)}{d(5\times^{3}+y^{2},x)}$
	THIN BAD EXACT FUNC 2730 F1+ F2+ F3+ F4+ F5 F6+ Too1s A19ebra Ca1c Other Pr9m10 Clean Up
	■y c(x)
	$= \frac{d}{d\times} \left(\times^2 + y^2 = 1 \right)$
	$\frac{2 \cdot c(x) \cdot \frac{d}{dx}(c(x)) + 2 \cdot x = 0}{1 + 2 \cdot x = 0}$
	<u>d(x^2+y^2=1,x)</u> MAIN RAD EXACT FUNC 2/30
Inverse trig functions are entered directly.	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebra[Calc Other Pr9m10 Clean Up
	41 - x - 1
	$\frac{u}{d\times}(\cos^4(\times)) \qquad \frac{1}{\sqrt{1-x^2}}$
	$= \frac{d}{dx}(\tan^4(x)) \qquad \frac{1}{x^2 + 1}$
	$\frac{\alpha}{d(\tan^4(x),x)}$
The TL 80 confirms the value of the limit	MAIN RAD EXACT FUNC 3/30 (F1+) F2+ (F3+) F4+ (F5) F6+ (
giving <i>e</i> .	Too1s A19ebra Ca1c Other Pr3mlO C1ean UP
	(1)
	$= \lim_{x \to 0} \left[(1+x)^{\times} \right] e$
	limit((1+x)^(1/x),x,0)
	MAIN RAD EXACT FUNC 1/30 (F1, F2, F3, F3, F4, F5, F6, F6, F7, F7, F6, F7, F7, F7, F7, F7, F7, F7, F7, F7, F7
Note the asymptote at $r=-1$	
	MAIN BAD EXACT FUNC

3.1 Maximum and minimum values and

3.2 Derivatives and the shapes of curves

Instructions	Saraan Shata
Relative Extrema: Find all relative	(f1770) → f2→ F2→ F3→ F4→ F5 → f→ Algebra Calc Other PrgmIO Clean Up
$g(x)=x^3-9x^2+24x-7$ and confirm your result by sketching the graph The TI-89	• Define $g(x) = x^3 - 9 \cdot x^2 + 24 \cdot x - 7$ Done • $\frac{g}{2}(-x \cdot x)$ = 7 $x^2 + 24 \cdot x - 7$
method combines use of the differentiation command, the solve	$= \frac{1}{dx}(9(x)) \qquad $
command for $\frac{dy}{dx} = 0$,	SOLVE(ans(1)=0,x) Main Rad Reprox func 3/30
graphs of the function and its derivative to relate the algebraic solution to the pictures,	Ham Bad AFFED: FUNC
	Image: State of the state o
and a table of values to get coordinates of points, check limits, etc.	17 12 <

Example 3.1

Instructions	Screen Shot
Find the relative maximum and	(F17700) F2▼ F3 ▼
minimum values of the function	
$f(x) = x^3 - x.$	
First we can get an idea of the solutions	
by sketching the graphs of the function	
and its derivative	<u>99=</u> //



Example 3.2

The derivative can be zero without there	F17700 F2▼ F3 F4 F5▼ F6▼ 50 50 50 50 50 50 50 50 50 50 50 50 50	
being a relative maximum or relative	▲PLOTS ✓41=× ³ - 3·× ² + 3·× - 1	
minimum.	√y2 <u>d</u> (y1(x))	
Example. $f(x) = x^3 - 3x^2 + 3x - 1$	y3= y <u>4</u> =	
	95= 96= 92=	
	9/= 	
	<u>y2(x)=a(y1(x),x)</u> Main rad approx func	



3.3 Optimisation problems

Instructions

Often in these questions we have to find the optimum value of a function of two or more variables by first substituting for one of the variables a function previously formed. This can be done in a relatively easy way on the TI-89. Taking example 2 on the manual in section 3.3, we have to minimise the cost, $C = 2(2\pi r^2) + 2\pi rh$ subject to $\pi r^2 h = 300$. Note that the form of the condition (using |) means that the answer comes out well or does not come

Screen Shot

$$\begin{bmatrix}
F_{1} + F_{2} + F_{3} + F_{3$$

[F1+] F2+ [F3+] F5 [F6+]]
Too1s A19ebra Ca1c Other Pr9mI0 Clean UP
■Define c=4·π·r ² +2·π·r·h
Done
$\bullet \left(\pi \cdot \mathbf{r} \cdot \mathbf{h} \right) = 0, \mathbf{r} \left(\pi \cdot \mathbf{r}^2 \cdot \mathbf{h} = 300 \right)$
$r = \frac{-h}{4}$
π*r*h, r)=0, r) πr^2*h=300 MAIN RAD EXACT FUNC 2/30
F1+ F2+ F3+ F4+ F5 Too1s A19ebra Ca1c Other Pr3mi0 C1ean Up T = 4
• solve $\left(\frac{d}{dr}\left(4\cdot\pi\cdot r^2+2\cdot\pi\cdot r\cdot r\right)\right)$
$r = \frac{5^{2/3} \cdot 3^{1/3}}{\pi^{1/3}}$
501ve(d(4*π*r^2+2*π*r*h,r MAIN RAD EXACT FUNC 3/30
ToolsAlgebra[Calc Other Prgmi0]Clean UP ToolsAlgebra[Calc Other Prgmi0]Clean UP 4
$= \left(+ 2 \cdot \pi \cdot \mathbf{r} \cdot \mathbf{h} \right) = 0, \mathbf{r} \right) \left \mathbf{h} = \frac{300}{\pi \cdot \mathbf{r}^2} \right $
$r = \frac{5^{2/3} \cdot 3^{1/3}}{\pi^{1/3}}$
r*h,r)=0,r) h=300/(πr^2) MAIN BAD EXACT FUNC 3/30

3.4 Antidifferentiation

Instructions	Screen Shot
Use the symbol \int found at 2nd $\boxed{7}$ for the antiderivative. We can enter on the [Y=] screen the function $y1(x) = 2nd$ $\boxed{7}$ function, x) + c when c ={list of values separated by commas}. This will give us a number of antiderivatives of the function.	$\begin{array}{c} \begin{array}{c} F_{1+} F_{2+} F_{3} F_{4} F_{5} F_{6+} F_{5+} F_{5+} \\ \hline Tools Zoom[Edit] & [All Style St$
y1 is then the function $F(x)$.	$\begin{array}{c c} F_{4}^{F_{4}} \left[F_{2}^{F_{4}} \right] & \begin{array}{c} F_{4}^{F_{4}} \left[F_{4}^{F_{4}} \left[F_{4}^{F_{4}} \right] & \begin{array}{c} F_{4}^{F_{4}} \left[F_{4}^{F_{4}} \left[F_{4}^{F_{4}} \right] & \end{array} & \begin{array}{c} F_{4}^{F_{4}} \left[F_{4}^{F_{4}} \left[F_{4}^{F_{4}} \right] & \end{array} & \end{array} & \end{array} & \end{array} & \end{array} & \end{array} \\ \end{array} \right] \end{array}$
Graphing the function will show what these functions look like and the relationship between them.	F4- F2- F3 F4 F5- F6- F7-51 TootsZoomTraceReGraphMathDrawPenI-C



Displacement, velocity and acceleration

Instructions	Screen Shot
Set the graph drawing mode to	F1+ F2+F3 F4 F5+ F6+ S ToolsZoomEdit / AllStyleskaal
differential equations using MODE	+PLOTS t0=-3.
Graph 6: DIFF EQUATIONS. In the Y=	√y1'= <mark>t + y1²</mark>
mode the DEs are then set up ready for	
you to enter. The 11-89 uses t not x .	914-
The variable t is given a key of its own on	<u>y1'(t)=t+y1^2</u>
the 11-89, like x , y , and z namely $\langle .$	MAIN DEGEXACT DE
To draw a direction field using the TL_{89}	[F1+]F2+ [F3+].F4+]. F5]F6+]
To draw a direction field using the 11-69.	Tools Algebra calc Other PrgmIO Clean Up
	■Define y1'(t)=t+y1 ²
	Done Define_u1'(t)=t+u1^2
	MAIN DEGEXACT DE 1/30
	TootsZoomTraceReGraphMathDrawPénIC
	///////////////////////////////////////
	<u>}}}<!--2/16/11/11</u--></u>
	1

Select the $[Y=]$ screen and enter the differential equation using <i>t</i> (and <i>Y</i> 1—or <i>Y</i> n—if needed). There is no use of <i>x</i> . Use \bullet [Window] (F2) to set the window dimensions to an appropriate <i>t</i> and <i>Y</i> size. Choose 'GRAPH' and it will put in the direction field.	F1+ F2+ F3 F4 F5+ F6+ F7+ F8 Tools Tools <td< th=""></td<>
Selecting F8 IC enables a particular antiderivative solution to be drawn: IC stands for Initial Conditions, meaning a point (or points) known to be on the graph of the antiderivative required. Enter the co-ordinates or move the cursor to a chosen point and press ENTER.	$\begin{array}{c c c c c c c c c c c c c c c c c c c $
The solution curves for the antiderivative through the given point(s) is drawn.	F1: F2: F3: F3: F3: F5: F6: F7: F8: ToolsZoom TraceReGraph F1: F6: F7: F8: I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I I MAIN DEGESACT DE I
To solve a DE algebraically we use the command F3 C: deSolve(Use 2nd = for the y'. Example 189 use F3 C: deSolve(y 2nd = = $2t(y+3)$ and y(0)=4, t, y)	Fire F2+ F3+ F4+ F5 Tools Algebra Calc atter Promin Clean up • deSolve(y' = 2 · t · (y + 3) ar) $y = 7 \cdot e^{t^2} - 3$ deSolve(y'=2t*(y+3) and y) Main Bab EXACT FF 1/20
	■ $t \cdot (y + 3)$ and $y(0) = 4, t, y$ $y = 7 \cdot e^{t^2} - 3$ $\frac{2t \cdot (y + 3)}{2}$ and $y(0) = 4, t, y$

4. Integration

4.1 The area problem

To find the area under the curve $f(x) = x^2 + 2$, from x = -1 to x = 2. using rightsum we have $x_0 = -1$, $x_1 = -1 + 3/n$, $x_2 = -1 + 2 \cdot 3/n$, ... $x_i = -1 + 3i/n$, ... $x_n = -1 + 3n/n = -1 + 3=2$.

So the area can be obtained by taking the limit (if it exists) of the Riemann sum as $n \rightarrow \infty$.

Area =
$$\lim_{n \to \infty} \left(\sum_{i=1}^{n} \frac{3}{n} (f(x_i)) \right) = \lim_{n \to \infty} \left(\sum_{i=1}^{n} \frac{3}{n} \left(\left(-1 + \frac{3i}{n} \right)^2 + 2 \right) \right).$$

Instructions	Screen Shots
On the TI-89 this is entered as:	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCa1cOtherPr9mIDClean Up
F3 3: limit(F3 4: Σ (sum expression), <i>x</i> ,	$\frac{n}{2}\left[x\left((-x_{ij})^2\right)\right]$
$n, 1, \infty$), or enter the sum first and then	$\sum_{i=1}^{n} \left[\frac{3}{n} \cdot \left[\left[-1 + \frac{3}{n} \right] + 2 \right] \right]$
take the limit. The , x tells the calculator	$9 \cdot (2 \cdot n^2 + n + 1)$
to sum with respect to <i>x</i> , and the <i>n</i> , 1, ∞)	2·n ²
is part of the limit (from $n=1$ to ∞). Don't	$\sum(3/n*((-1+3*i/n)^{2}+2), i,)$
forget to make sure that n , and i do not	
have values in them (use F4 4: Delvar ii they do)	
they do).	
	F1+ F2+ F3+ F4+ F5 Too1sA19ebra[Ca1c Other Pr9m10 C1ean Up
	$\frac{5(2\pi)^{-1}(1+1)}{2\cdot n^2}$
	$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{3}{n} \cdot \left(\left(-1 + \frac{3 \cdot i}{n} \right)^2 + \right) \right)$
	$\frac{1}{1} \frac{1}{1} \frac{1}$
	First First <th< td=""></th<>
	2·n ²
	$\bullet \P_{m} \sum_{i=1}^{n} \left[\frac{3}{n} \cdot \left[\left(-1 + \frac{3 \cdot i}{n} \right)^2 + 2 \right] \right]$
	9 +3*i/n)^2+2),i,1,n),n,∞) MANN RAD EXACT FUNC 2/30
The summation function F3 4: Σ (sum	F1+ F2+ F3+ F4+ F5 Too1sA19ebraCa1c@therPr9mIDClean Up
will also give the general summation	n /
results in Theorem 4.2.1.	$= \sum_{i=1}^{n} [i^2]$
	$n \cdot (n+1) \cdot (2 \cdot n+1)$
	$\frac{6}{\Sigma(i^2, i, 1, n)}$
	MAIN RAD EXACT FUNC 1/30
	Too1s A13ebra Ca1c Other Pr9mlO C1ean Up
	• $\sum_{i=1}^{n} (i^3)$ $\frac{n^2 \cdot (n+1)^2}{4}$
	$\frac{1-1}{\Sigma(1^{3}, 1, 1, n)}$

4.4 Fundamental Theorem of the Calculus

Instructions	Screen Shots
The derivatives of the integral functions can be found on the TI-89 by defining the function first using F4 1: Define and then	$\begin{bmatrix} F_{1+} & F_{2+} & F_{3+} & F_{4+} & F_{5} \\ \hline Toots a T sebra Calc D ther Fr Smill Clean Up \\ \end{bmatrix}$ $\square Define f(x) = \begin{bmatrix} \times (1 + t^2) dt \end{bmatrix}$
finding its derivative with respect to r (or	Benne ((x)-jo(1 to juto
these two steps can be done together)	Done 2.4
	$ \frac{\mathbf{a}_{\mathbf{x}}(\mathbf{r}(\mathbf{x}))}{\mathbf{a}_{\mathbf{x}}(\mathbf{r}(\mathbf{x}))} \times \mathbf{r} + 1 $
	<u>d(f(x),x)</u> MAIN BAD EXACT FUNC 2/30
	F1+ F2+ F3+ F4+ F5 Too1sA13ebra[Ca1c Other Pr3ml0 Clean Up
	• Define $f(x) = \int_{0}^{\sqrt{x}} \left(\frac{\cos(t)}{t}\right) dt$ Done
	$= \frac{d}{d\times}(f(x)) \qquad \frac{\cos(\sqrt{x})}{2 \cdot x}$
	d(f(x),x) MAIN RAD EXACT FUNC 2/30
	F1+ F2+ F3+ F4+ F5 Too1sA19ebraCatcluther Pr9mIDC1ean Up
	• Define $f(x) = \int_{1}^{x} \left(\frac{1}{t}\right) dt$
	$= \frac{d}{dx}(f(x)) \qquad \qquad \frac{1}{x}$
Using CAS with the theorem can help us with antiderivatives for functions such as	FINN BUD BALL FUNC 2/30 F1+ F2+ F3+ F4+ F5 F6+ Too1s A19ebra Ca1c Other Pr9m10 Clean Up
$\frac{1}{x}$.	• Define $f(x) = \int_{1}^{x} \left(\frac{1}{t}\right) dt$
	Done
	$\frac{\bullet f(2)}{f(2)} = \frac{\ln(2)}{\ln(2)}$
	MAIN RAD EXACT FUNC 2/30
Areas under curves are best calculated by evaluating the correct definite integral. This can be done numerically on the graph screen, or on the [HOME] screen. For example to find the definite integral	P1790 P2* P3 P4 P5* P6* P4* × ✓ ✓ Allistyle > > × ✓ Ø > > × ✓ Ø > > × ✓ Ø > > × ✓ Ø > > × ✓ Ø > > × ✓ Ø > > y3= y4= y5= y6= y7= y9= Ø > >
$\int_{2}^{3} (x+2) dx$ we can use the graph of $f(x)$	<u>919=</u> <u>92(x)=</u> MAIN BED BUTD FUNC
on the TI-89 to see the area represented	
by the integral and numeric integration to	
calculate it.	
• $[Y=] x + 2$ ENTER • [GRAPH] F5 7 2 ENTER 5 ENTER	



Area = $\int_{a}^{b} \{f(x) - g(x)\} dx$, where x=a and x=b are the x-values of the two points of intersection (if they exist).

We can also use the formula $\int_{a}^{b} |f(x)| dx$ to find the area between the graph of *f* and the *x*-axis, and then we do not have to worry about where the function intersects the axis or the signs of the integrals. This works well on the TI-89 since we have the function abs.

For example calculate the area between f(x)=x(x+1)(x-2) and the x-axis from x=-1 to x=2. It is always good to look at the graph of the function to see what is going on.



5. Integration techniques

Instructions	Screen Shot
Specific techniques for integration are not	F1+ F2+ F3+ F4+ F5 ToolsA19ebra[Calc OtherPr9mi0 Clean UP
required when using the TI-89 since it	
will integrate all integrable functions,	
using the J function. However, we can	■∫tan(a·x)dx
verify some of the formulas for general	
functions	∫(tan(a*x),x)
runonons.	
Note how functions such as $\sec^2(ax)$ are	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebra[Calc Other Pr9ml0[Clean Up
entered, and the need for () around the	
whole of a numerator and/or a	$\left[\left(\frac{1}{2}\right)d\times\right]$
denominator in $\frac{1}{x^2 + 2}$ and $\frac{3x + 2}{x^2 + 2}$.	$\int \left[\left(\cos(\mathbf{a} \cdot \mathbf{x}) \right)^2 \right] $
$a^2 + x^2$ $x(x^2 + 1)$	
	$\frac{\int (1/(\cos(a*x)^2), x)}{\text{Main}_{\text{BAD}} \text{ Exact}_{\text{FUNC}} \frac{2/30}{2}}$
Integrating a rational function	(F1+) F2+ (F3+) F4+ F5 F6+ Too1sA13ebraCa1c Other Pr3ml0 C1ean Up
	$\int \left(\frac{3 \cdot x + 2}{2} \right)_{d \times d}$
	$\left[\int \left[\frac{1}{x \cdot (x^2 + 1)} \right]^{\alpha \times \alpha} \right]$
	$\ln\left(\frac{x^2}{2}\right) + 3 \cdot \tan^4(x)$
	$\frac{(x^2+1)}{(((3y+2))((y_1(y_2(y_1(y_2(y_1(y_1(y_1(y_1(y_1(y_1(y_1(y_1(y_1(y_1$
	MAIN RAD EXACT FUNC 1/20
An inverse trig function.	ToolsA19ebraCalcatePr9mIDClean UP
	$t_{an4}(\times)$
	$\left[\left(\frac{1}{a^2 + a^2} \right) dx - \frac{dan(a)}{a} \right]$
	J(a ⁻ + x ⁻) J(1/(a ² +x ²),x)
Integrating a rational function	MAIN BAD EXACT FUNC 1/30
integrating a rational failetion	$\frac{100 \text{ spectral process process}}{\sqrt{2 - y - 2}}$
	$\left\ \frac{1}{\left(x-1\right)^3 \cdot \left(x^2+x+1\right)} \right\ ^{dx}$
	$(x^2 + x + 1)$
	$2 \cdot \ln \frac{1}{(x-1)^2} = 2 \cdot \sqrt{3} \cdot t^{1/2}$
	$\frac{\int ((x^2 - x - 2)/((x - 1))^3 * (x^2 - x - 2)}{Main}$
	F1+ F2+ F3+ F4+ F5 Too1s A19ebra Ca1C Other Pr9milD C1ean Up
	$\int \left[(x-1)^3 \cdot \left[x^2 + x + 1 \right] \right]^{1/2}$
	$2 \cdot 10 \frac{ x^2 + x + 1 }{2}$
	$\frac{-1}{(x-1)^2} - \frac{2\sqrt{3}}{1}$
	Y
	<u>J((x^2-x-2)/((x-1)^3*(x^2</u> Main rad exact func 1/30

$\frac{\left[F_{1}^{F_{1}} \right] F_{2}^{F_{2}} + \left[F_{3}^{F_{3}} \right] F_{4}^{F_{4}} + \left[F_{3}^{F_{3}} \right] \left[F_{6}^{F_{4}} \right] U_{2}^{F_{3}} \\ \int \left[\left(\times -1 \right)^{3} \left(\times^{2} + \times +1 \right) \right] $
$4\frac{4\left(\frac{\sqrt{3}\cdot(2\cdot x+1)}{3}\right)}{9} - \frac{3\cdot x-4}{3\cdot(x-1)^2}$
F1+ F2+ F3+ F4+ F5 Too1sfA19ebraCa1cl0ther/Pr9milClean Up
$= \int (\times \cdot \sqrt{x-1}) dx$
$\frac{2\cdot(\times-1)^{3/2}\cdot(3\cdot\times+2)}{15}$
$\frac{\int (x-1), x, 1, 2}{\text{MAIN}}$

Functions of Two Variables

Instructions	Screen Shot
Set the graph drawing mode to 3D using	F1+ F2+ F3 F4 F5+ E3 F4 F5+ ToolsZoomEdit / A11 (1/5) (2001)
MODE Graph 5: 3D. In the \checkmark Y= mode	-PLOTS 50
the DEs are then set up ready for you to	$\sqrt{21} = \frac{1}{1 + x^2 + y^2}$
enter $z1$ = etc. The TI-89 uses y and x for	z2=
these functions.	z4= z5=
	MAIN RAD EXACT 3D
Use \bullet [GRAPH] to draw the graph (this	F1+ F2+ F3 F4 F5+ F6+ F7+5:: Too1sZoomTraceReGraphMathDrawPen:C
may take a few seconds)	
	/ħ
You may need to resize the window using	AT 35.
• [WINDOW] where you can set all	
three variables. The viewing angle can	
also be changed using the eye variables or	MAIN RAD EXACT 3D
by using the 🗢 keys.	
Pressing [ENTER] will rotate the graph	F1+ F2+ F3 F4 F5+ F6+ F7+5:: Too1sZoomTraceReGraphMathDrawPen:C
dynamically.	
	\$ ~ *
	i i i i i i i i i i i i i i i i i i i
	MAIN RAD EXACT 3D

We can draw contours too.	F1+ F2+ F3 F4 F5+ F6+ F7+8:: Too1s/Zoom/Trace/ReGraph/Math/Draw/Pen::C
Select the Y = mode and press	
CONTOUR LEVELS and press FNTFR	
Use \bullet [GRAPH] and then E6 Draw 7	
Draw Contour command in the graph	
mode to enter the x and y values (here	
each 0). This can also be rotated	
Here is the graph of $y = \sqrt{16 - r^2 - y^2}$	F1+ F2+ F3 F4 F5+ F6+ F7+8:) Toots/Zoom/Trace/ReGraph/Math/Draw/Pen::C
There is the graph of $y = \sqrt{10 - x} - y$.	
For partial derivatives, the TI-89 assumes	F1+F2+F2+F4+F4+F5+F5+F6+ Too1s A13ebra Ca1c Other Pr3m10 C1ean Up
letters to be constants unless told they are	_ a (50)
F3 1: $d($ differentiate	$\frac{d \times \left[1 + x^2 + y^2\right]}{1 + x^2 + y^2}$
Here with respect to x	$\frac{-1000 \times}{(\sqrt{2} + \sqrt{2} + 1)^2}$
	<u>(x + y + 1)</u> <u>d(50/(1+x^2+y^2),x)</u>
	Tools A13ebra[Ca1c Other Pr3ml0 Clean Up
Here differentiate with respect to <i>x</i>	
	$= \frac{\alpha}{d \times} [\times \ln(\times^2 + y^2)]$
	$\ln(x^2 + y^2) + \frac{2 \cdot x^2}{x^2 + x^2}$
	$x^{-} + y^{-}$ <u>d(x*ln(x^2+y^2),x)</u>
	First First <t< td=""></t<>
\dots and here with respect to y	
Remember that these are the partial	$= \frac{d}{dy} \left(\times \ln(\times^2 + y^2) \right)$
derivatives $f_x(x,y)$ and $f_y(x,y)$ not what the	<u>2·y·x</u> 2 · 2
CAS notation implies.	$\frac{g^2 + x^2}{d(x*\ln(x^2+y^2),y)}$
We can use the F4 1: Define to define a	MIN BRD EXACT 20 1720 F1+ F2+ F3+ F4+ F5 F6+ Too1s A13ebra Catc Dther Pr3mid Clean Up
function in two variables and hence find	• Define $f(x, y) = x^4 - y^3 + 2$
the value of the function.	Done
	• $f(1.1, .9) = \frac{27331}{10000}$
	f(1.1,.9) 2.7351 f(1.1,.9)
	MAIN RAD EXACT 3D 3/30

We can get f_{xx} by differentiating twice	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCalclotherPr9ml0Clean Up
with respect to x	
	Define f(x,y) = x ³ + y ³
	Done
	$= \frac{d^2}{2} (f(x, y)) \qquad 6 \cdot x$
	dx ²
	<u>((+(x,y),x,2)</u> MAINRAD EXACT3D 2/30
and f_{yy} by differentiating twice with	F1+ F2+ F3+ F4+ F5 F6+ Too1sA13ebraCa1clotherPr3ml0Clean Up
respect to y	
	■Define f(x,y)=x ³ +y ³
	Done
	$= \frac{d^2}{c} (f(x, y)) \qquad 6 \cdot y$
	dy ²
	<u>d(f(x,y),y,2)</u> MAIN RADEXACT 30 2/30
For f_{vx} and f_{xv} we differentiate twice, once	F1+ F2+ F3+ F4+ F5 F6+ Too1sA13ebraCa1cOther Pr3mlOClean Up
for each variable.	
	$= \operatorname{Define}_{\mathcal{L}} \mathcal{L}(x_1, y_1) = x_1^3 + y_2^3$
	Done
	$-\frac{d}{d}\left(\frac{d}{d}\left(c(x_{1},y_{2})\right)\right) = 0$
	<u>a(a(f(x,y),x),y)</u> Main rad exact 30 2/30
	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl3ebraCalcather Pr3ml0Clean Up
	Done
	$= \frac{d}{d} \left(\frac{d}{d} \left(f(x, y) \right) \right) = 0$
	<u>a(a(f(x,y),y),x)</u> Main Rad Exact 30 2/30
Example 207	F1+ F2+ F3+ F4+ F5 F6+ Too1sA13ebraCa1clather Pr3ml0Clean Up
	• Define $f(x, y) = -3 \cdot x^4 + 6 \cdot$
	Done
	$= \text{solve}\left[\frac{\alpha}{d\times}(f(x, y)) = 0, x\right]$
	$\times = 1$ or $\times = 0$ or $\times = -1$
	SOTUE(@(f(X, y), X)=0, X) MAIN RAD EXACT 3D 2/30
Then use the Hessian obtained as above	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCa1cOtherPr3miDClean Up
to test each point.	$= \operatorname{solve}\left(\frac{d}{d}(f(x,y)) = 0, y\right)$
	x = 1 on y = 0 on y = -1
	x = 1 or $x = 0$ or $x = 1$
	• solve $\left[\frac{1}{dy}(f(x,y)) = 0, y\right]$
	y = 0
	SOIVE(@(†(X, y), y)=0, y) Main Pan Evert on 2/20

7 Linear Systems

7.1 Gaussian Elimination Matrix notation

When there are 3 equations - in x, y, and z - we start by eliminating the first variable (x) in the last 2 equations and then eliminate the second variable (y) in the last equation. This leaves us with a set of equations in *echelon form*. Once the equations are in *echelon form*, they can be solved by **back substitution**.

This can be done using the row operations on the TI–89, or using functions which give echelon form and reduced echelon form. First, we need to know how to enter a matrix into the Data/matrix Editor or into the Home screen.

References: TI-89 Guidebook 229-233

Entering a matrix into the Data/matrix editor:

Instructions	Screen Shot
Press APPS 6, open the Data/matrix editor and then select 3. New	F1- APPLICATIONS T>012 1: Home 2: Y= Editor 3: Window Editor 4: Graph 5: Table 1: Current 1: Current am Editor 2: Open am Editor 5: New
For <i>Type</i> , select Matrix, as following.	F1 F2 F3 F4 F6 F7 Tool NEW F1 F6 F7 F7 F6 F7 Tool NEW F1 F6 F7 F7 F6 F7 Tool NEW Type: Data + Folder: main + Variable: S6
Press B and select 2: Matrix.	Image: Provide and Provid
Press D D and enter the variable name M1. (Some names are reserved, if you try to use a reserved name you will get an ERROR message). Enter the row and column dimensions of the matrix.	NEW Type: Matrix + Folder: main + Variable: m1 Row dimension: 3 Col dimension: 3 Enter=0K ESC=CANCEL

Type in the first three rows and columns of the matrix. You will need to use the arrow keys to move around. Press ENTER to register each entry. You can use fractions and operations when you enter values.	$\begin{array}{c c c c c c c c c c c c c c c c c c c $
Press the "key and enter M1 return. You should now see the matrix in this standard form.	F1+ F2+ F3+ F3+ F5 Tools Algebra Calcather Pright Clean Up M1 MAIN RAD AUTO FUNC 1/30

Entering a matrix into the Home Screen

Instructions	Screen Shot
Method 1: From the Home screen, enter a matrix by using Define (which can be accessed by F4 1 or could be typed in). Use the square bracket [] to enclose the matrix. We enter the matrix by typing the first row and then the second and so on. Use commas to separate entries and	Fit F2+ F3+ F3+ F5 F6+ ToolsA13ebra[Calc]ather[Pr3ml0]Clean UP Define $a = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ Done Define $a=[2,3,0;1,2,3]$ MAIN A RAD AUTO FUNC 1/30
semicolons to separate rows. Method 2:	F1+ F2+ F3+ F4+ F5 Toolsh19ebralCa1clDtherPr3mlDCtean UP
To enter a matrix into the Home screen, use one set of brackets around the entire matrix and one set of brackets around each row. Use commas to separate the entries in a row. Then press STO \rightarrow , type a name for the matrix, and press ENTER . Example: [[1,2,3][-1,3,4]] STO \rightarrow r ENTER	$ \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 4 \end{bmatrix} + r \\ \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 4 \end{bmatrix} $ $ \begin{bmatrix} (1,2,3)[-1,3,4]] + r \\ MAIN \\ RAD AUTO \\ FUNC \\ 1/30 \end{bmatrix} $

7.2 Matrix Row Operations

Instructions	Screen Shot
To swap two rows in one matrix, use 2nd	F1+1 F2+ 1F3+1 F4+1 F5 F6+ To MATH DC1ean Up
[MATH] 4:Matrix J:Row ops 1:rowSwap(.	1:Number EfrandMat(F:newMat(G:subMat(H:Norms 1:nowSwap(5:rowRdd(>:nRow(>:nRow(>:nRow(>:nRowAdd(>:nRowAdd(
Example : We can change rows 1 and 2 of	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCalcOtherPr9mIOClean UP
matrix r with the command $rowSwap(r, 1, 2)$.	
	$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 7 & 4 \end{bmatrix} \rightarrow r \begin{bmatrix} 1 & 2 & 3 \\ -1 & 7 & 4 \end{bmatrix}$
	■rowSwap(r,1,2) [1 2 3]
	rowSwap(r,1,2) Main Radiauto Func 2/30
To add the entries of one to those of another	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCa1clOtherPr3mlOClean Up
row, use 2nd [MATH] 4:Matrix J:Row ops	
2:rowadd(.	$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 7 & 4 \end{bmatrix} \neq r \begin{bmatrix} 1 & 2 & 3 \\ -4 & 7 & 4 \end{bmatrix}$
Example: Add the entries of row 1 to those of row 2 and store them into row 2 with the	
command rowAdd(r 1 2)	•rowHdd(r,1,2) [0 5 7]
communa / o what (1,1,2).	rowAdd(r,1,2) MainRAD_AUTOFUNC2/30
To multiply the entries of one row by a value, use 2nd [MATH] 4:Matrix J:Row ops	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebra[Calc Other Pr9ml0 Clean Up
3:mRow(.	$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
Example : Multiply the entries of row 1 by 3	
and store them into row 1 with the command $\frac{1}{2}$	• mRow(3,r,1) -1 3 4
mKow(3,r,1).	MRow(3, r, 1) MAIN RAD AUTO FUNC 2/30
To multiply the entries of one row by a value	F1+ F2+ F3+ F4+ F5 F6+
and add the products to another row, use 2nd	
[MATH] 4:Matrix J:Row ops 4:mRowAdd(.	-1 3 4 $+r$ -1 3 4
Example: Multiply the elements of row 1 by 3,	■ mRowAdd(3,r,1,2)
add the products to row 2, and store them into	
row 2 with the command $mRowAdd(3,r,1,2)$.	$\frac{[2 7 13]}{\text{mRowAdd}(3, r, 1, 2)}$
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Example 7.2.1: Solve the following system:

$$3x - y + 2z = 13$$
$$-x + 4y + 2z = -1$$
$$4y + 3z = 4$$

Instructions	Screen Shot
The augmented matrix is:	F1770 F27 F37 F47 F5 F67
$\begin{bmatrix} 3 & -1 & 2 & 13 \end{bmatrix}$	
	3 -1 2 13 -1 4 2 -1 + a -1 4 2 -1
	■ mRow(3,a,2) -3 12 6 -3
Use the following instructions to row	
reduce this matrix.	MAIN RAD AUTO FUNC 2/30
[3, -2, 2, 13; -1, 4, 2, -1; 0, 4, 3, 4]	
STON a ENTER	
[2nd] [MATH] 4:Matrix J:Row ops	
3:mRow (3, a, 2) ENIER	
store them into row 2	
store them into row 2.	
4:mRowAdd (1, ans(1), 1, 2) [ENTER]	F17700 . F27 . F37 . F47 . F5
	UICLEAN UP
multiply the elements of row 1 by 1, add	$\begin{bmatrix} 3 & -1 & 2 & 13 \\ -3 & 12 & 6 & -3 & 1 & 2 \end{bmatrix}$
the products to row 2 and store them into	
row 2.	3 -1 2 13 0 11 8 10
	MAIN RAD AUTO FUNC 4/30
Multiply the elements of row 2 by 4.	F1770 F27 Algebra Calc Other PrgmIO Clean Up
	■ MRow 4,0 11 8 10,2 0 44 32 40
Multiply row 3 by 11.	F17700 F2+ → ← Algebra Calc Other PrgMIO Clean Up
	■ mRow 11, 0 44 32 40, 3
	0 44 32 40
	[0 44 33 44] mRow(11,ans(1),3)
Multiply row 2 by -1 add the products to	
row 3 and store them in row 3	→ <u>+</u> → Hlgebra Calc Other PrgmIO Clean Up 0 44 33 44
	[3 -1 2 13] ■ "Poured -1 0 44 32 49 2 7
	0 44 33 44
	3 -1 2 13 0 44 32 40
	MAIN BAD AUTO FUNC 6/30

Here is the echelon form.	F17780 F2▼ F3▼ F4▼ F5 F6▼ ▼ ∰ Algebra Calc Other PrgmIO Clean Up
3x - y + 2z = 13	0 0 1 4
11y + 8z = -1	■ mRow 1/4, 0 44 32 40, 2
z = 4	
By back substitution,	0 11 8 10
z = 4,	
11y + 8(4) = -1, so $y = -2$	MAIN RAD AUTO FUNC 7/30
3x - (-2) + 2(4) = 13, so $x = 1$	
On the TI-89 this is also obtained by:	F17700 F2▼ → ← Algebra Calc Other PrgmIO Clean Up
[3, -2, 2, 13; -1, 4, 2, -1; 0, 4, 3, 4]	[3 -1 2 13] [3 -1 2 13]
STO► a ENTER	$\begin{bmatrix} -1 & 4 & 2 & -1 \\ 0 & 4 & 3 & 4 \end{bmatrix} \rightarrow a = \begin{bmatrix} -1 & 4 & 2 & -1 \\ 0 & 4 & 3 & 4 \end{bmatrix}$
STO► a ENTER 2nd [MATH] 4:Matrix 3:ref(a) ENTER	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
STO► a ENTER 2nd [MATH] 4:Matrix 3:ref(a) ENTER	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$