Mathematics Colloquium 2002
Abstracts of the Talks

Stone walls and Mathematics: Searching for Connections

Willy V. Alangui
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One of the tasks of ethnomathematics is to search for cultural practices that can be identified as "a structured body of knowledge and therefore legitimised as mathematics" (D'Ambrosio, quoted in Barton, 1995). The challenge is in showing how a body of knowledge may lead us to mathematics while being true to the cultural context in which such knowledge is rooted.

A research on the practice of stone walling in two indigenous communities in northern Philippines was conducted during the first half of this year. This talk will present initial data from the research and then consider the following question within the context of the challenge posed above: what connections can we find between the practice of stone walling and mathematics?

Uno's Invariant Conjecture for General Linear and Unitary Groups

Jianbei An
Department of Mathematics
The University of Auckland

Recently, Isaacs and Navarro proposed a new conjecture which is a significant generalization of the Alperin-McKay conjecture, and Uno raised an alternating sum version of the Isaacs-Navarro conjecture which is a refinement of the Dade conjecture.

In this talk, I will states some of the conjectures and report on the current works for general linear and unitary groups.

Undergraduate ESOL Students: What Difference does language make to their mathematics learning?

Bill Barton
Department of Mathematics
The University of Auckland

The dynamics of learning mathematics in English when it is not your first language are not fully understood. What English-language competencies are required to learn mathematics at an undergraduate level? Is a general English-language competency sufficient to study mathematics, or is a specific competency required, such as English for academic purposes or a knowledge of mathematical discourse? Do L1 and L2 students learn mathematics in different ways? As a first step towards investigating the detail of such questions, a test has been developed that assesses mathematical understanding in five modes. This is being given to first year undergraduate mathematics students at Auckland University. We will report on the first batch a data from 50 students.

Cell cycle modelling of human melanoma tumors

Britta Basse
University of Canterbury

A population of cells differentiated by DNA content, can be divided into four phases: G1 phase, S phase (DNA replication), G2 phase and M phase (cell division). We have devised a system of partial differential equations to model the cell cycle. The model has four compartments corresponding to each phase with transition probabilities between phases.

We have applied our model to a human melanoma cell line, where taxol, an anticancer agent used in the treatment of ovarian cancer, was added to this cell line to halt cell division. We have validated our model using flow cytometry, a technique that can measure the DNA content of a particular cell.

Mathematics Enrichment
How can we attract more young people to our subject?

Toni Beardon
Faculty of Education
University of Cambridge

The speaker founded a group of enrichment projects involving the Mathematics and Education Faculties at the University of Cambridge and the Royal Institution of Great Britain. One of these projects, the NRICH Online Mathematics Club, has 130,000 members, the majority children, in 130 countries and the website had 5 million hits in October 2002 proving that young people are attracted to mathematics. This talk will explore ways of using communication technology to support the learning and teaching of school mathematics; the value of interactions between keen university and school students; the development of mathematical thinking and communication skills; making the learning of mathematics exciting; making closed questions on the core curriculum more open to encourage students to think for themselves; encouraging students to want to explore extensions of topics they learn in school; and inspiring students to want to do mathematics outside the normal limits of school work and preparation for examinations.

Radial basis functions: Embedding theorems and fast algorithms

Rick Beatson
Department of Mathematics and Statistics
University of Canterbury

Radial basis functions is a recent branch of Approximation Theory. The theory and applications of this scattered data fitting tool are both beautiful. The theory being abstract function theory and harmonic analysis, and the applications being built upon complicated algorithms derived from the function theory. In this talk I will give a general introduction to the basic theory and some recent applications. The emphasis will be on accessibility and broad concepts at the expense of the full gore of technical detail. The talk will be heavily laced with animations and videos.

Studying the cardio-respiratory system

Alona Ben-Tal

The cardio-respiratory system is responsible for maintaining desired levels of oxygen and carbon dioxide in the blood. This process involves complex interactions between two oscillators, the heart and the lung. The system is nonlinear and contains different time scales. My efforts to study the cardio-respiratory system will be described. The main focus will be on gas exchange in the lungs.

Toroidal triangulations are geometric

C. Paul Bonnington
Department of Mathematics
University of Auckland

Steinitz's Theorem states that a graph is the 1-skeleton of a convex polyhedron if and only if it is 3-connected and planar. The polyhedron is called a geometric realisation of the embedded graph. Its faces are bounded by convex polygons whose points are co-planar. When does a graph embedded on the torus have a geometric realisation in 3-space? In this talk we prove that every toroidal triangulation has a geometric realisation. The proof involves an interesting detour into 4-space. (This is joint work with Dan Archdeacon, University of Vermont and Jo Ellis-Monaghan, Saint Michael's College, Vermont).

Taking the right steps

John C. Butcher
The University of Auckland

It is a characteristic question in the computer solution of an evolutionary problem as to how the stepsize should be chosen. For most interesting problems, it doesn't make sense to use a constant stepsize to advance the approximate solution from the initial point to the time value where the solution is required. For sections of the trajectory where the errors are accumulating rapidly, it is necessary to take small steps but this is balanced by the larger steps that can be taken in sections where there is a slower rate of error growth. A quantitative analysis of these questions can be carried out using calculus of variations arguments, combined with elements from control theory. The theory extends to numerical schemes for which variable order strategies are available.

Multiterminal resistors and monotonicity

Bruce Calvert
University of Auckland

We show that a network H may be constructed from multiterminal resistors with resistance functions which are Lipschitz and strongly monotone, and that H is again a multiterminal resistor of this type. The techniques include new notation, nodal analysis and loop space analysis. One conclusion is that an infinite network of multiterminal resistors can serve as an approximation for all large finite subnetworks. Another conclusion is that we may obtain the hierarchy of infinite networks of A. Zemanian from the set of multiterminal resistors.

Cheap computation of Hamiltonian problems in quadratic form

Robert Chan
Department of Mathematics
The University of Auckland
Tamaki Campus

Abstract: We consider Hamiltonian problems with Hamiltonian defined by a quadratic form, $H(y) = 1/2y^TSy$, where S is a symmetric matrix. It is well known that symmetric or symplectic methods preserve the Hamiltonian and are important for the long term integration of Hamiltonian problems. Explicit Runge-Kutta methods are cheap to implement but are known to be neither symmetric nor symplectic. In this talk we report on a study which involves modifying explicit Runge-Kutta methods by allowing coefficients of the method to depend on the constant stepsize used, and discuss the much improved accuracy obtained. As an example we apply the methods to a wave equation with boundary and initial conditions, and present some numerical results.

Some Problems with the Emphasis on Numeracy

Megan Clark
Centre for Mathematics Education
School of Mathematical and Computing Sciences
Victoria University

In New Zealand, along with other Western countries there has been a recent thrust toward numeracy programs in the primary schools and consequent shift in the emphasis given to parts of the curriculum. I will argue that this may lead to some undesrable outcomes including a lack of skills needed for some technical occupations and some disadvantaging of particular groups in society.

Sums of squares and sums of triangular numbers

Shaun Cooper
Institute of Information and Mathematical Sciences
Massey University - Albany

Let $r_k(n)$ and $t_k(n)$ denote the number of representations of n as a sum of k squares, and k triangular numbers, respectively. That is, $r_k(n)$ is the number of solutions in integers of the equation

$$x_1^2 + x_2^2 + \cdots + x_k^2 = n$$

and $t_k(n)$ is the number of solutions in *non-negative* integers of the equation

$$\frac{x_1(x_1+1)}{2} + \frac{x_2(x_2+1)}{2} + \cdots + \frac{x_k(x_k+1)}{2} = n.$$

For example, $r_2(50) = 12$, since $(x_1, x_2) = (\pm 1, \pm 7),\ (\pm 5, \pm 5),\ (\pm 7, \pm 1)$, and $t_2(6) = 3$, since $(x_1, x_2) = (0, 3),\ (2, 2),\ (3, 0)$. Geometrically, $r_k(n)$ and $2^k t_k(n)$ count the number of lattice points (points with integer coordinates) on k-dimensional spheres centred at $(0, 0, \cdots, 0)$ and $(-\frac{1}{2}, -\frac{1}{2}, \cdots, -\frac{1}{2})$, respectively.

The study of $r_k(n)$ and $t_k(n)$ has a long and interesting history, going back to Fermat and earlier. Significant results were discovered by Euler, Lagrange, Gauss, Jacobi, Eisenstein, Smith, Minkowski, Liouville, Ramanujan and many others.

In this talk we shall prove that for $1 \leq k \leq 7$, there is a constant c_k for which $r_k(8n + k) = c_k t_k(n)$, for all n. There is no similar result for $k \geq 8$.

This is joint work with P. Barrucand and M. Hirschhorn.

Regular Cayley Maps for Finite Groups

Marston Conder
University of Auckland

For the purposes of this talk, a *regular Cayley map* is a 2-cell embedding of a Cayley graph $C(G, S)$ of a group G (with respect to some generating set S) into an orientable surface, such that the group of all incidence- and orientation-preserving symmetries of the embedded graph is sharply transitive on the arcs of the graph. In particular, the group G acts regularly on vertices of the map, and is complementary to a cyclic group of symmetries fixing a given vertex. Such objects are closely related to finite quotients of $(2, p, q)$ triangle groups, and may also be studied in terms of so-called *skew-morphisms* of the group G.

These concepts will be explained, and illustrated in the special case where the group G is abelian. A number of questions and answers will be given, including some pathological examples, along with a recent spin-off concerning the structure of groups expressible as a product AB of two subgroups A and B where A is abelian and B is cyclic.

This is joint work with Robert Jajcay and Tom Tucker (on regular Cayley maps) and Marty Isaacs (on products of cyclic and abelian subgroups).

The minimob and approximation

Rod Downey
Department of Mathematics
Victoria University of Wellington

Some years ago, with Mike Fellows, the author began a program to investigate a type of complexity based on the asymptotic behaviour of the running times of algorithms when parameters are taken into account. The result is called "parameterized complexity."

The basic class in the first studies is $W[1]$, the class:

INPUT: A Turing Machine M
PARAMETER: k
QUESTION: Does M accept in $\leq k$ steps?

The idea is to replace this with a new class based on controlling the size of the problem as a parameter. E.g consider satisfiability for a formula of size $k \log n$. This allows potentially for better non-approximability results. It results in a new class we call the minimob.

Two-layered Fluid Flow in Pipelines

Bernard Ee
Institute of Information and Mathematical Sciences
Massey University - Albany

Multiphase pipelines frequently experience liquid slugging caused by a number of mechanisms, one of which is the topography of the pipeline which normally includes many uphill and downhill sections. Typically liquid outflow may rise and fall in a fairly well defined cyclic pattern, with peak flows up to ten times the average flow. This means that there must be transitions which separate different flow regimes within the pipeline. Some may occur at points where the pipeline slope changes. Others may move along the pipeline; as these transitions move, the discharge rates of the different phases will change at any given point, although usually the only measured effect is at the outfall.

In this talk, I will be focusing on determining the conditions under which transitions, both short and long, can exist in a pipeline and if so, whether are they of a permanent form or not. The equations and assumptions used in this analysis will be discussed, along with some preliminary results.

Phragmén-Lindelöf Theorems

P.C. Fenton
University of Otago

The results here have to do with the least harmonic majorants of subharmonic functions in sectorial domains of bounded angular extent, a domain D being *sectorial* if its boundary consists of two simple curves Γ_1 and Γ_2 joining 0 to ∞, which are non-intersecting unless they are identical, and D having *angular extent* at most 2η, where $0 < \eta \leq \pi$, if, for all positive r, the measure of $\{\arg z : z \in D_r\}$ is at most 2η. Here $D_r = D \cap \{z : |z| = r\}$. The proofs use a technique of Beurling in a new way.

Coalgebras, Bisimulation, and Comonads

Rob Goldblatt
Victoria University of Wellington

Coalgebras of functors T : **Set** $\to$ **Set** on the category of sets have been of recent interest in theoretical computer science, due to their role in representing various kinds of data structure such as lists, streams and trees, as well as "state-based" systems such as automata, labelled transition systems, and classes in object-oriented programming languages.

In many of these examples, the forgetful functor from the category of T-coalgebras to **Set** has a right adjoint providing a coalgebra cofreely generated over each set. The adjunction gives rise to a certain kind of structure $\mathbb{G}^T$ called a *comonad*. Comonads themselves generate new classes of coalgebras by a construction due to Eilenberg and Moore.

A class K of coalgebras is a *behavioural covariety* if it is closed under disjoint unions and images of bisimulation relations, the latter being relations of "observational indistinguishability" of states. Such a K may be thought of as the class of all coalgebras that satisfy some computationally significant property.

In this talk we explain that behavioural covarieties K are precisely the Eilenberg-Moore categories of coalgebras for certain comonads $\mathbb{G}^K$ naturally associated with $\mathbb{G}^T$. A categorical characterization can be given of the comonads that can be represented in the form $\mathbb{G}^K$ for behavioural K, involving a pullback condition on the components of a natural transformation.

Reference:

A Comonadic Account of Behavioural Covarieties: www.mcs.vuw.ac.nz/~rob

Cartan, de Rham and quantum gravity

Rod Gover
Department of Mathematics
The University of Auckland

A current focus in quantum gravity is the so-called AdS/CFT correspondence which is based on the asymptotically hyperbolic space 'Poincare metric' space. On the other hand the Cartan bundle and connection have a long and important history for conformal and similar geometries and are intimately connected to classical Lie theory. I will describe in elementary terms how a new construction directly links these and leads to a new way to calculate de Rham cohomology.

This is collaborative work with Thomas Branson (University of Iowa) based on recent work with Andreas Cap (Erwin Schrodinger Institute, Vienna), and Larry Peterson (University of North Dakota).

Unifying Randomness

Evan Griffiths
Victoria University of Wellington

There are several different notions of algorithmic randomness for real numbers, including Martin-Löf random, computably random, Schnorr random, and Kurtz random. Such notions are usually defined using either measure theoretic terms (random reals should not be in effective null sets), martingales (no betting strategy on the digits should succeed), or incompressibility (initial segments should be hard to compute). Indeed many of these randomness notions have at least two equivalent formulations, and the connection between Martin-Löf's null set definition and prefix-free complexity of strings is particularly celebrated.

The definitions of these notions and links among their characterisations will be discussed, along with their connections to Turing degrees and some other areas of mathematics, in an effort to approach a unified view of algorithmic randomness. (This includes recent work with Rod Downey and Geoff LaForte, as well as results of D.Hirschfeldt, A.Nies, S.Reid and others).

Constructing the Higman-Sims graph

Paul R. Hafner
The University of Auckland

The Higman-Sims graph is a strongly regular graph with parameters $(100, 22, 0, 6)$ whose automorphism group contains the sporadic simple group of order 44,352,000 as subgroup of index 2. We will give a construction of this graph, based on a geometric interpretation of N. Robertson's construction of the Hoffman-Singleton graph.

Having started 'from nothing', this construction can also be used to obtain the Steiner system $S(3, 6, 22)$, which is the point of departure of the original construction by Higman and Sims. Finally, the extended Steiner system $S(4, 7, 23)$ will also be constructed using the Higman-Sims graph.

MAPLE labs for advanced calculus and differential equations - the Canterbury experience

John Hannah
University of Canterbury

The computer package MATLAB has been an integral part of linear algebra courses at Canterbury for 15 years, but until recently there have been only sporadic attempts at incorporating similar packages into our calculus courses. In this talk I shall discuss what a "computer algebra system" such as MAPLE has to offer in an otherwise traditional course on advanced calculus and differential equations. Examples will be taken from a series of MAPLE worksheets intended for use in a laboratory-type environment. My main point is that MAPLE can be used to encourage higher level thinking than we would normally expect of students in such a course. Examples of such thinking include synthesizing different viewpoints of a problem, and experimenting with conjectures and refutations. Feedback from students will be used to highlight potential gains, and also possible pitfalls.

A rising bubble with a slowly adsorbing surfactant

John F.Harper
School of Mathematical and Computing Sciences
Victoria University of Wellington

A small gas bubble rising in a surfactant solution has long been known to have a stagnant spherical cap around its bottom stagnation point and a shear-free spherical cap on the rest of its surface. Theories for the distribution of surfactant around the bubble are well known if it is determined either by convective diffusion (with adsorption being so fast that the surface can be assumed in equilibrium with the bulk) or by slowness of adsorption and desorption (with diffusion being so fast that the bulk concentration can be assumed uniform). This paper appears to be the first attempt to take both mechanisms into account. There are singular Volterra integral equations for both the free cap and the stagnant cap, and they are solved numerically. Both caps give singular perturbation problems in the limit of infinitely fast adsorption.

Much of this work has been done since the author spoke on the same subject at the Manawatu-Wellington Applied Mathematics Conference in September 2002.

Word problems and 3-manifolds

John F.P.Hudson
Institute of Fundamental Sciences
Massey University

Conjecture There is a compact 3-manifold with an unsolvable word problem.

It is well known that there is a 4-manifold with an unsolvable word problem. But the 3-dimensional case seems improbable.

Waldhausen has shown the any irreducible sufficiently large 3-manifold has a solvable word problem. Moreover the standard approach to constructing group presentations with unsolvable word problems uses HNN extensions. But any 3-manifold whose fundamental group is an HNN extension must be sufficiently large.

How could an example be constructed?

Modelling the Growth and Structure of Passive Films on Metals

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Passive metals are protected from the environment by a thin (3-5 nm) oxide film that forms on their surface. These oxide films can reduce the corrosion rate of a metal to roughly 0.1 μm/year. This extraordinary kinetic stability of otherwise highly reactive metals is exploited in many modern technologies. However, our understanding of the chemistry and physics of these films is still rudimentary. Our team has focussed on the passive oxide film that forms on iron at anodic potentials in weakly alkaline solutions. Surface x-ray diffraction has been used to determine the atomic structure of this film, and quantum mechanical calculations have been used to relate the atomic structure to the electronic structure. This microscopic understanding of the film has been used to develop a macroscopic model of film growth. In turn, by comparing the macroscopic film growth model to experiment, we can gain a greater understanding of the microscopic features of the film that are important for passivity.

This work was supported by the New Economy Research Fund contracts CO8X9903 and CO8X0222.

Implementation of stiff General Linear Method

Junying (Shirley) Huang
Department of Mathematics
The University of Auckland

We discuss general linear methods for the numerical study of stiff differential equations, characterised by a property known as "inherent Runge-Kutta stability". This implies that the stability matrix has only one non-zero eigenvalue and this eigenvalue is a rational approximation to the exponential function, just as the stability function of an implicit Runge-Kutta method. In addition to theoretical properties of the method, such as stability and order, we will discuss implementation questions including starting methods, predictors for the stages, efficient iteration schemes, truncation error estimation and stepsize control. This is a joint work with John Butcher (The University of Auckland)

Perturbed Markov Chains

Jeffrey J. Hunter
Institute of Information and Mathematical Sciences
Massey University - Albany

The effects of perturbations of the transition probabilities, associated with finite irreducible discrete time Markov chains, on the stationary distribution of the chain are explored. A unifying treatment, based upon the application of generalized matrix inverses is presented. It is shown that the changes in the stationary probabilities are intimately connected with the behaviour of associated mean first passage times. Some interesting qualitative results regarding the nature of the relative and absolute changes to the stationary probabilities are obtained together with some improved bounds. In addition, a procedure for updating the mean first passage times to determine changes in the stationary distributions under successive perturbations is also presented.

Wiggly homoclinic bifurcation curves near saddle-node/Hopf instabilities

Vivien Kirk
Department of Mathematics
University of Auckland
and
Alan Champneys
Department of Engineering Mathematics
University of Bristol

A distinctive pattern of entwined, wiggly homoclinic bifurcation curves has recently been seen in a number of systems of differential equations containing simultaneous saddle-node and Hopf instabilities. This talk gives an explanation for this phenomenon, based on calculations involving perturbed normal form equations for the saddle-node/Hopf bifurcation.

Source term estimation Of atmospheric pollution from accidental releases of gas

Padmanathan Kathirgamanathan
Centre for Mathematical Modelling
Massey University

This paper describes the development of an inverse model, which may be used to estimate the source term parameters for a polluting gas released into the atmosphere from a point above the ground. The model uses measured pollution concentrations at a minimum of three observations sites on the ground as well as meteorological data such as wind speed and cloud cover. The inverse model is formulated as a least squares minimisation problem coupled with the solution of an advection-diffusion equation. The statistical basis of the least-squares technique allows quantification of the uncertainty of the calculated estimates, which in turn allows estimation of the uncertainty of the simulation model predictions.

The minimisation problem where the pollution is released instantaneously is well-posed and the source term is calculated with reasonable accuracy. However, that for a non-steady extended release source is ill-posed; consequently its solution is extremely sensitive to errors in the measurement data. Tikhonov regularisation, which stabilises the solution process, is used to overcome the ill-posedness of this problem. The optimal value of the regularisation parameter in the problem is estimated using both the linear and non-linear L-curve criterion, and a generalised cross-validation approach. The accuracy of the model is examined by using some real experimental data, as well as concentration data (generated by the forward model) to which normally distributed relative noise has been added.

Long-term coexistence for a competitive system of spatially varying gradient reaction-diffusion equations

Andrei Korobeinikov, John Norbury and Graeme C. Wake
Mathematical Institute, University of Oxford

Spatial distribution of interacting chemical or biological species is usually described by a system of reaction-diffusion equations. In this work we consider a system of two reaction-diffusion equations with spatially varying diffusion coefficients which are different for different species and with forcing terms which are the gradient of a spatially varying potential. Such a system describes two competing biological species. We are interested in the possibility of long-term coexistence of the species in a bounded domain. Such long-term coexistence may be associated either with a periodic in time solution (usually associated with a Hopf bifurcation), or with time-independent solutions. We prove that no periodic solution exists for the system. We also consider some steady-states (the time-independent solutions) and examine their stability and bifurcations.

Stabilization of “bumps” by noise.

Carlo R. Laing
Institute of Information and Mathematical Sciences
Massey University - Albany

Spatially localized regions of active neurons (“bumps”) have been proposed as a mechanism for working memory, the head direction system, and feature selectivity in the visual system. Stationary bumps are ordinarily stable, but including spike frequency adaptation in the neural dynamics causes a stationary bump to become unstable to a moving bump through a supercritical pitchfork bifurcation in bump speed. Adding spatiotemporal noise to the network supporting the bump can cause the average speed of the bump to decrease to almost zero, reversing the effect of the adaptation and “restabilizing” the bump. This restabilizing can be understood by examining the effects of noise on the normal form of the pitchfork bifurcation where the variable involved in the bifurcation is bump speed. This noise–induced stabilization is a novel example in which moderate amounts of noise have a beneficial effect on a system, specifically, stabilizing a spatiotemporal pattern. Determining which aspects of our model system (integral rather than diffusive coupling, a slow variable, traveling structures that appear through a pitchfork bifurcation in speed) are necessary for this type of behaviour remains an open problem.

Numerical computations to understand long-time qualitative behaviour of numerical solution of ODE

Pierre Leone
The University of Auckland

We would like to show how we can make use of an algorithm mainly due to J. Laskar (bureau des longitudes, Paris) to understand the long-time qualitative behaviour of the numerical solutions of ODEs. We present an application of the algorithm to the numerical solution of Hamiltonian systems integrated with symplectic integrators. The numerical results are totally in accordance with known theoretical results. We plan to use similar computations to understand the long-time behaviour of general linear methods.

Rankings on Multisets of Cardinality Three

Mark Li-Chen Liu
Department of Mathematics
The University of Auckland

Multisets are collections of objects which may contain several identical copies of the same object. To define a multiset M on a finite set $A = \{a_1, a_2, \ldots, a_n\}$ one has to specify how many times each element of A occur in M, i.e. their multiplicities. In other words, an n-element multiset M on A is a pair $M = (A, \mu)$. where μ: $A \to \mathbb{N}$ is a nonnegative integer-valued function such that $\mu(a_1) + \mu(a_2) + \ldots + \mu(a_m) = n$. By $\mathcal{P}_n(A)$ we denote the set of all multisets on A consisting of exactly n elements.

In applications these multisets may correspond to n-element committees formed from members of m political parties or allocations of n indivisible identical goods to m customers, etc.

Any reflexive, complete, transitive and antisymmetric relation is called a ranking. In this talk we assume that the set A is equipped with a ranking $'' >''$ such that $a_1 > a_2 > \ldots > a_m$ and we are interested in the ways this ranking can be consistently extended to a ranking of $\mathcal{P}_n(A)$. The ranking on $\mathcal{P}_n(A)$ is called consistent [1] if it agree with the initial order on A and is preserved under the operation of multiset union.

Rankings on multisets have been used both in Economics [1] and in Computer Science [2] for proving program termination. A ranking $\succ$ on $\mathcal{P}_n(A)$ is said to be additive if there exist positive real numbers $w_1, \ldots, w_m$ such that for any two multisets $M = (A, \mu)$ and $N = (A, \nu)$

$$M \succeq N \Longleftrightarrow \sum_{i=1}^{n} \mu(a_i) w_i \geq \sum_{i=1}^{n} \nu(a_i) w_i$$

Here we classified all consistent linear orderings on $\mathcal{P}_3(A)$, both additive and nonadditive, when A contains four elements. The additive ones have a nice graphical representation. This is a joint talk with Arkadii Slinko.

References:

1. Sertel, M., Slinko, A.: "Ranking Committees, Words or Multisets." Report Series 482, Department of mathematics, The University of Auckland, 2002.

2. Derhowitz, N.: "Termination of Rewriting." In: *Proc. First Internat. Conf. on Rewriting Techniques and Applications,* Lecture Notes in Computer Science, Vol 202 (Springer, Berlin, 1985), 180-224.

Abel Function Transforms

Alastair McNaughton
The University of Auckland
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Associated with a suitable real-valued function f, there is an Abel function F such that Abel's functional equation $F(f(x)) = F(x) + 1$ is satisfied. These Abel functions can be used to determine general functional roots for the function f. In addition the Abel function may be used to define a transform of some other function g. The resulting graphs present some fresh insights on the relative behaviour of various types of functions for large values of x.

Slugging in oil-gas pipelines

Robert McKibbin
Centre for Mathematics in Industry
Institute of Information and Mathematical Sciences
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At MISG2002, this year's Mathematics in Industry Study Group workshop, one problem was brought by an Australian gas producer, SANTOS. The task was to find simple models to explain the phenomenon of slugging in two-phase pipelines carrying gas and condensate from the gas-field. At the outlet of the pipeline, the gas output is more or less steady, but the liquid comes out in slugs, with peak flows being up to 10 times the average. The slugs are often cyclic in nature.

Slugging is a worldwide problem in pipelines carrying both liquid and gas. Not only does it make flows at the outlet difficult to handle, but it can induce severe mechanical vibrations in the pipe. In pipes across undulating terrain, such as those operated by SANTOS, a major cause of slugging is the topography. Liquid tends to build up and sit at the lowest points of the pipeline until it is forced onwards through the rest of the pipe by the pressure of the gas caught behind. SANTOS asked the MISG group to develop a simple way of estimating peak liquid flow rates, slug sizes and the period between the peaks.

This talk gives a brief overview of the problem, some approaches used by the MISG group, and the results of some calculations made since the workshop.

Ants, bees and algorithms: the mathematics of social insects

Mary R. Myerscough
School of Mathematics and Statistics
University of Sydney, Australia

Social insects such as ants, bees and termites live in colonies whose size ranges from 10 or 20 individuals up to many hundreds of thousands, depending on the species. Each individual insect in these colonies is comparatively simple and has simple interactions with its nestmates and its environment. At a colony level, however, the collective effects of these individuals and their local interactions produces a complex entity, which is flexible, efficient and finely tuned to changing environmental conditions.

Mathematical modelling has become an important tool in scientific research on how local, individual reactions are integrated to produce global, colony-scale behaviour. Examples of a number of different models for social insect behaviour will be given, including classic ant-trail models. Ideas from ant trail models have been adapted to produce successful ant-based optimisation models.

Choosing correct spatial and temporal scales is important in social insect models. Examples will be presented of models at different scales. These may include models for thermoregulation, for the spread of pheromone on comb wax and for foraging in bee colonies.

A model for nest site choice that incorporates honeybee dance communication will be used to show how bees use, filter and control information flow in the colony. This model also shows how the mathematical structure of a model can give insight into the way that s small change in individual behaviour can significantly change the nature of a process at colony level.

Derivation of a class of ODE Solvers

Nicolette Moir
Department of Mathematics
The University of Auckland

Runge-Kutta methods have been used to numerically solve ordinary differential equations for over 100 years. Although they generally work well, there are some disadvantages associated with them. Many attempts have been made to overcome these disadvantages, while retaining the multi-stage structure of Runge-Kutta methods. I will present a class of methods, known as Almost Runge-Kutta methods, which have been developed along these lines. Central to this talk will be an explanation of the ideas leading to this new class of methods.

Building and Fitting a model of human vocabulary change

Geoff Nicholls
Department of Mathematics
The University of Auckland

In collaboration with Russell Gray of the Psychology Department of the University of AUckland, I have recently been involved in a study of European vocabulary data. Similarities between the vocabularies of the Indo-European languages reflect their origin in a common ancestral language. Pairs of languages which are relatively closer in vocabulary diverged from one another in relatively more recent times. We build and fit a model of the language and vocabulary change.The point process of word births lives on the ancestral tree of languages. Words surviving to the leaves of the tree are observed in the corresponding modern languages. The observation process determines an age-dependent thinning of the word-birth process. We integrate the missing data, including thinned words and unknown tree topology and thereby obtain an estimate of the time depth of the Indo-european language tree, and a model-dependent quantification of the uncertainty in that estimate.

Non-standard eigenvalue problems arising from the extension of the Graetz convection problem to a porous medium

D. A. Nield
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The classical Graetz convection problem is the problem of thermally developing forced convection in a fluid flowing in a channel between plane parallel plates. It involves the solution of an eigenvalue system of order two of the standard Sturm-Liouville type. This problem has now been extended to the case of a channel occupied by a porous medium, with flow modeled by the Brinkman equation and with further complicating factors. The case of local thermal non-equilibrium leads to an eigenvalue system of order four rather than two. The case where the effect of axial conduction is included leads to a second-order system in which the eigenfunctions are no longer orthogonal in the usual sense. In each case the problem has been solved.

Constructing the automorphism group of a p-group

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We have recently developed a new and effective algorithm to construct the automorphism group of a p-group. It works down a lower central series for the group. We outline the basic algorithm, illustrate its application, outline the potential problems, and briefly discuss the methods used to address these.

Fluid and Magnetofluid Turbulence

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University of Waikato

A review of Kolmogorov's theory of energy cascade in a turbulent fluid will be given, including the straightforward extension of these ideas to the case of magnetohydrodynamics (MHD). We will then present recent advances which show that the magnetofluid case is fundamentally different, especially when there is strong mean.

Integrating Technology into Undergraduate Mathematics Courses

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There is considerable variation between tertiary institutions nationally and internationally with respect to the acceptance and use of technology in the teaching and learning of mathematics. Some mathematicians favour at most only limited use of technology in their teaching, especially in early courses, believing that the use of calculators and/or computers can impair students' development of basic mathematical and cognitive skills. Others view technology merely as a useful manipulative tool to enable students to work with more complex and potentially more meaningful examples.

By contrast, there is a growing body of research demonstrating that technology can be of substantial assistance pedagogically, with particular benefits in developing students understanding of concepts. An increasing number of tertiary institutions now have sizeable technology components in their undergraduate mathematics courses, reflecting a recognition of the potential benefits indicated by the mathematics education research, as well as the increasing importance of technological tools to research mathematicians.

Many proponents of the use of technology argue that courses should be developed where technology is fully integrated into all aspects of the curriculum. This notion of technology integration however is itself not clearly defined, with some debate on what constitutes a technology-integrated curriculum.

This talk will briefly discuss the conflicting beliefs about the implementation of technology in tertiary mathematics education, and give an example from an undergraduate course here at The University of Auckland which illustrates how recent developments in the power of technology, such as CAS-calculators, requires a re-examination of course curricula. I will propose an initial description of a technology-integrated curriculum, and discuss some of the practical and theoretical implications of this model.

The Boundary Crossing Theorem

Gabriela Popa

The existence of the unique point at which a line segment crosses out of a located convex set in $\mathbf{R}^N$ is proved constructively, and the continuity of the resulting mapping is investigated. It is well–known that the full form of the intermediate value theorem cannot be proved constructively (although there are hypothesis that can be added to the usual ones, and that apply to most functions of interest in elementary analysis, in order to find the exact point at which the intermediate value is attained). A consequence of this is that we cannot be sure of finding the point where the segment joining a point inside a convex subset Y of a Banach space to one outside Y crosses the boundary ∂Y, even if Y is **located** in the sense that for each $x \in X$ the distance

$$\rho(x, Y) = \inf \{\|x - y\| : y \in Y\}$$

exists.

If Y is a closed, located and convex subset of $\mathbf{R}^N$, then it seems intuitively clear that there is a unique point $h(z)$ at which the line segment $[\xi, z]$ intersects ∂Y. Earlier work by Bridges, Richman, and Wang shows that the distance from that segment to ∂Y is 0; but this is not sufficient to establish constructively that the segment actually intersects ∂Y. We show that in this case the segment really does meet the boundary, and that the resulting mapping h, taking z outside Y to the unique point of $[\xi, z] \cap \partial Y$, is continuous.

Topological Concepts and Language

Ivan L Reilly
The University of Auckland

How does a mathematician's conceptual understanding depend, if at all, on first language? We shall discuss our Marsden-funded research project, and especially an initial analysis of some of the pilot data that we have collected.

Scale Invariant Means and the Discrete Digit Problem

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It is a well established fact that in some circumstances, lists of what appear to be random numbers, show a striking non-uniform distribution of digits. In many instances, these numbers arise relative to a system of units. In such cases there is an underlying assumption of scale invariance, by which is meant that the choice of units may well be arbitrary.

The general problem of scale invariance is probably best looked at from the point of view of certain means on suitable function spaces. This approach gives a simple explanation for thedistribution of first significant digits.

There are two cases to consider. The first, of importance e.g. in physics, is when the underlying function spases are defined on the reals. The theory then is relatively straightforward. But the second, where the underlying semigroup is the positive integers under multiplication, seems much more difficult. This is the case which will be discussed and some open questions addressed.

A threshold quantity for disease eradication – beyond R_0

Mick Roberts
AgResearch

We propose a new threshold quantity for the analysis of the epidemiology of infectious diseases. The quantity is similar in concept to the familiar basic reproduction ratio R_0, but it singles out particular host types instead of providing a criterion that is uniform for all host types. Using this methodology we are able to identify the long-term effects of disease control strategies for particular sub-groups of the population, to estimate the level of control necessary when targeting control effort at a subset of host types, and to identify host types that constitute a reservoir of infection. These insights cannot be obtained by using R_0 alone. The methodology will be illustrated with reference to tropical diseases in humans and tuberculosis in possums.

**Resonance optical switch:
calculation of the resonance eigenvalues**

Kieran Robert and Boris Pavlov
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The University of Auckland

We consider a mathematical model of an imaginable resonance optical switch with one linear input fiber and three linear output fibers (terminals) attached on the boundary of a planar circular lens. If the places of contact of the lens with the terminals are properly selected, the switch may reorient the beam of light incoming from the input fiber to different terminals, depending on distribution of values of the refractive coefficient manipulated by the electric field applied to the device. In actual publication we concentrate on the choice of geometric parameters of the switch and verification of conditions of it's temperature stability. Both problems are reduced to the classical problem of spectral analysis for wave equation with special coefficients on a circular domain. We base on the corresponding calculation for the Schrödinger equation presented in previous papers, but in actual case we meet particular difficulties appearing from the fact that the potential of the Schrödinger equation defines an additive perturbation of the Laplace equation, but the refractive index $\rho := 1 + \delta_\rho > 0$ in wave equation

$$u_{tt} = \frac{1}{\rho^2(x)} \triangle u$$

defines a multiplicative perturbation. To overcome relevant analytic complications we introduce a new perturbation techniques which also permits to consider more general problem on large perturbations of operators with discrete asymptotically rear spectrum. This techniques is based on a special block-matrix representation of operators.

On circuit simulation and Abstract Differential Algebraic Systems

Steffen Schulz
Institute for Applied Mathematics
Humboldt University Berlin

Today circuit simulation using Differential Algebraic Equations plays a vital role in modern circuit design. When using Partial Differential Equations to model semiconductor devices we have to consider coupled systems of PDEs and DAEs called Partial Differential Algebraic Equations. Considering a simple PDE it will be shown how these equations can be treated as Abstract Differential Algebraic Systems existing in real Hilbert spaces. We put special emphasis on how the tractability index for DAEs can be generalized for these systems.

Liquid bridges between three particles

P Rynhart and R McLachlan
Massey University

In this talk the liquid bridge profile formed between three equally sized (spherical) primary particles is analysed. Primary particles are equally separated, with sphere centres placed on the vertices of an equilateral triangle. Local physics requires the fluid surface to have constant mean curvature (CMC), along with a fluid contact angle α, present at the intersection between the primary particles and the fluid surface (or 3-phase contact line). Equations for the problem are derived using these properties, and available symmetry inherent within the problem is used during this stage. A second order non-linear elliptic partial differential equation is derived for the mean curvature, which is subject to non-linear boundary conditions. The problem is approximated using a mesh, using 5 and 9 point numerical stencils to evaluate the numerical derivatives. A robust non-linear equation solver was employed to solve the system down to arbitrary tolerance. With the fluid profile now known, numerical integration was used to calculate the (non-dimensional) liquid bridge volume, surface area and energy density. To confirm the accuracy of the solver, the boundary for the 3 particle case was modified to the 2 particle problem (as the profile of the 2 particle case is known). Agreement was found between the two solutions.

Geodesic Geometry of Black Holes

Gabriela Slezáková

A geodesic is a curve whose tangent is paralelly transported along it. Study of geodesics offers a way to understand geometry of black holes. I shall explain how geodesics are classified, describe a method of finding solutions of equations by which geodesics are governed, and illustrate orbits derived on the basis of these solutions.

Obstructions to Clustered Planarity in Directed Graphs

Jamie Sneddon
Department of Mathematics
The University of Auckland

The well known Kuratowski's Theorem gives a characterisation of planarity by a set of obstructions, or excluded minors. A directed graph is said to be clustered planar if it can be embedded in the plane so that all of the in-arcs at each vertex consecutive in the rotation. A set of eleven operations which preserve clustered planarity are presented, forming minors of directed graphs. These operations are shown to produce a smaller directed graph in an ordering of the directed graphs, represented by a 'hierarchy of measures'. The obstructions to clustered planarity are presented, thereby characterising the property.

How Large a Coalitional Should Be to Manipulate an Election?

Arkadii Slinko
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The well-known impossibility theorem of Gibbard and Satterthwaite [1] states that every non-dictatorial social choice function at certain profiles is manipulable by a single individual. Nevertheless, in a number of publications it was shown that for large societies individual manipulability is not an issue for all classical SCFs since they are asymptotically nonmanipulable, i.e. the proportion of manipulable profiles to the total number of profiles tends to zero as the number of voters tends to infinity.

It should be noted that although the asymptotic nonmanipulability was observed for both of the two major models for the distribution of voting preferences, i.e. the IC (Impartial Culture conjecture) and the IAC (Impartial Anonymous Culture conjecture), the speed of convergence of the probability to encounter a manipulable profile (situation for the IAC) is different, which are in the order of $O\left(1/n^{1/2}\right)$ and of $O\left(1/n\right)$, respectively.

In the literature the concept of coalitional manipulability existed [2] and was studied under several models but there was no attention to the size of the coalition. A profile or voting situation was considered coalitionally manipulable if there existed a coalition of any size which could manipulate the election. In this paper we introduce a new concept of k-manipulability. We call a profile (or a voting situation) k-manipulable if the exist a coalition of size k or less which is capable of manipulation.

In this paper we show that, for the IC conjecture, when the number of participating agents n tends to infinity, the probability that a random profile will be k-manipulable by a coalition of size $k = Cn^{\alpha}$, with $0 \leq \alpha < 1/2$, is being of order $O\left(1/n^{1/2-\alpha}\right)$. For the IAC conjecture, the probability that a random voting situation will be k-manipulable for a coalition of size $k = Cn^{\alpha}$, with $0 \leq \alpha < 1$, is being of order $O\left(1/n^{1-\alpha}\right)$.

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Some recent results concerning weak Asplund spaces

Sivajah Somasundaram
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A Banach space X is called a *weak Asplund* [*Gâteaux differentiability*] space if every continuous convex function defined on it is Gâteaux differentiable at the points of a residual [dense] subset. While it is easy to see that every weak Asplund space is a Gâteaux differentiability space, it is not so obvious whether every Gâteaux differentiability space is a weak Asplund space. In this talk we show that there are in fact some Gâteaux differentiability spaces that are **not** weak Asplund. Thus we answer a question raised by David Larman and Robert Phelps from 1979.

Star Clusters and the Astronomical N-body Problem

Winston L. Sweatman
Institute of Information and Mathematical Sciences
Massey University

A globular star cluster is an astronomical object. It consists of a ball of about a million stars which move under their mutual gravitational attraction. The stars are ancient: they are some of the oldest known and so globular star clusters have been sometimes described as astronomical fossils.

Studying the star cluster as a whole is itself a million-body problem. Additionally, there are dynamical processes within the cluster which are dominated by fewer body interactions requiring study of two-, three- and four-body problems.

The boundary feedback stabilisation of a string-mass system

Steve Taylor
Department of Mathematics
The University of Auckland

We study the boundary stabilisation of a vibrating string with an interior point mass, zero Dirichlet condition at the left end and velocity feedback at the right end. Assuming finite energy initially, we show that the energy to the right of the point mass decays like C/t while that of the point mass decays like $C/\sqrt{t}$. The energy to the left of the point mass approaches zero but at no specific rate. This system is of interest because it serves as a prototype of similar phenomena in other systems.

It is joint work with Walter Littman of the University of Minnesota.

Solving Elliptic Partial Differential Equations, Without Factorization

Garry J. Tee
University of Auckland

An elliptic pde over a region is usually solved by constructing a finite set of nodes over the region, in a pattern as regular as possible. At each node, a standard finite-difference (or finite-element) approximation to the pde is applied, giving a finite set of equations. If the pde (and boundary conditions) are linear, then the problem is approximated by a finite set of linear algebraic equations. In solving that finite set of linear equations, almost all of the computational cost is devoted to factorizing the matrix into a product of lower and upper triangular matrices. For M nodes over a 2D region,the factorization costs $\mathrm{O}(M^2)$ arithmetic operations, and the local truncation error of the solution is $\mathrm{O}(1/M)$.

An alternative approach consists of constructing a regular net of nodes, then constructing sparse upper and lower triangular matrices such that the product of those triangular matrices approximates at each node the pde (and boundary conditions). No factorization is required, for solving the approximating equations.

For M nodes over a 2D region, the total cost of this solution is only $\mathrm{O}(M)$ arithmetic operations, and the local truncation error of this solution is $\mathrm{O}(1/\sqrt{M})$.

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The logistic "zipper" transform

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From an information theory perspective, coarse grained encodings of discrete-time non-linear systems such as the logistic map, make these ideal as information sources which known entropy characteristics. Alternatively one may view the map as a transform which uniquely maps an initial value to string and if we expressing the initial value as a binary expansion, we have then simply a string transform. For the special case, (r=4) a closed form solution exists for the logistic map. We show that for this case the symbolic dynamics may be computed from any initial state by way of a reversable XOR (exclusive or function) "zipper" transform that operates on adjacent bit pairs in the string. By implication the initial string and its transform must contain identical information content. We explore the "zipper" transform in the context of a novel measure of entropy.

A Mathematical Study of Cytosolic Calcium Oscillations

Krasimira T Tsaneva-Atanasova
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Calcium plays a significant role in the control of cellular behaviour, as a signalling factor for different cellular processes. These calcium signals, expressed by cytosolic calcium oscillations and waves, result from a delicate balance between calcium release from the endoplasmic reticulum (ER) and reuptake by calcium ATPases. Although the greatest part of calcium release and reuptake is to and from the ER, calcium fluxes across the cell membrane play an important role in the control of such oscillations. We use a mathematical model to study the interactions between membrane and ER fluxes, with particular reference to pancreatic acinar cells. A dynamical analysis of the model gives a reasonable explanation for some experimental results.

The Runge-Kutta-Munthe-Kaas method for the rigid body problem

Priscilla Tse
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The University of Auckland

Because classical numerical methods do not preserve geometric properties, a new approach based on the Lie group methods was proposed by H. Munthe-Kaas. An introduction will be given to these "Runge-Kutta-Munthe-Kaas" (RKMK) method. Also, the application of the RKMK method for the rigid body problem in free rotation will be considered.

Inverse Problems for a Semi-Linear Wave Equation

David J. N. Wall & T. John Connolly
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Department of Mathematics and Statistics
University of Canterbury

Inverse source problems associated with a semi-linear transport or one-way wave equation in one spatial dimension will be considered. It is shown that these problems arise naturally in areas such as electromagnetism, nonlinear optics and population dynamics.

Further, it is shown an analytic solution to one of the inverse problems can be given and moreover, that this inverse problem of determination of a source function is ill-posed, and must be regularised. A novel regularisation scheme which combines least squares monotone approximation and mollification of the noisy data is used to provide this regularisation. Numerical solutions from the inverse problems are presented showing that the method is robust to noisy signals.

Newtons Coefficient of Restitution

Graham Weir
Industrial Research
Lower Hutt

The coefficient of restitution (CoR) is a measure of energy loss during particle impact. This paper will review the theory of the CoR for low velocity impacts between spherical particles, briefly outling the historical development os the subject, and presenting the approximate velocity dependence of the CoR. It is suggested that the primary scaling of velocity is with respect to the elastic wave speeds of the particles, and that the CoR is not a particle property, but depends on the nature of the impact.

The Taming of the Monster

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The University of Birmingham

We survey the Classification of Finite Simple Groups with particular emphasis on the 26 so-called 'sporadic' simple groups. The largest of these is the 'Monster,' with nearly 10^{54} elements, which cannot be written as matrices smaller than 196882×196882. I shall describe in general terms how we can perform practical calculations in such an enormous group, and what we can deduce about the structure of the group.

Nonlocal eigenvalue problems arising in cell-growth models

Graeme Wake,
Biomathematics Research Centre,
University of Canterbury

The study size (=DNA content) distribution of a cohort of cells which are growing and simultaneously undergoing frequent cell division satisfies a nonlocal (but singular) eigenvalue problem Wake et al (2000). The first eigenvalue of such problems is related to the time constant for the population growth. The nonlocal effect $\mathbf{z} \rightarrow \mathbf{g}(\mathbf{z})$ has a fixed point, which if attracting, has a well-developed theory, but in the case of a g being repelling little is known.

We formulate an eigenvalue problem for holomorphic equations of the functional argument and show that the corresponding spectrum is discrete by reformulating the problem in terms of a compact operator. Results from the Fredholm Alternative adapted for nonlocal problems are used to derive existence results for the non-homogeneous problem.

This work is strongly connected to the underpinning work on tumour cell growth of Dr Britta Basse (see elsewhere in this volume) and is joint with Dr Bruce van-Brunt and Jonathan Marshall (Massey University). (See van-Brunt et.al (2001) and in press)).

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Complementing cappable degrees in the d.c.e. degrees

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We will prove that for any cappable c.e. degree $\mathbf{c} > \mathbf{0}$, there is a d.c.e. degree $\mathbf{d}$ such that $\mathbf{d} \cap \mathbf{c} = \mathbf{0}$, $\mathbf{d} \cup \mathbf{c} = \mathbf{0}'$. $\mathbf{d}$ is called a complement of $\mathbf{c}$. As a corollary, we have: $\mathbf{c} > \mathbf{0}$ is cappable if and only if $\mathbf{c}$ is complemented in the d.c.e. degrees. This is a joint work with Andsheng Li and Rod Downey.

Practical general linear methods

William Wright
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In this talk, a brief summary of the traditional methods for the numerical solution of ordinary differential equations will be presented. The focus will be on the advantages and disadvantages of each method.

General linear methods were derived as a unifying framework for the study of a wide class of options, identifying the standard Runge-Kutta and linear multistep methods as special cases. However, identifying practical methods from this large class has proved difficult.

A class of diagonally implicit general linear methods with a property known as "inherent Runge-Kutta stability" will be discussed. These methods are proposed as practical alternatives for the solution of both stiff and non-stiff problems. Several experiments comparing these methods with well known traditional methods will also be included.

Crossing Numbers of Surface-knots

Tsukasa Yashiro
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We define distance between surface–knots and distance between classical knots. We show that the distance between two distinct twist–spins of two classical knots is bounded by the distance between these classical knots. In particular, the unknotting number of a classical knot is bounded below by the distance from a twist spin of that knot to a trivial sphere. We construct some surface-knot diagrams to illustrate geometric properties of the distance between surface-knots.

The Properties of K-subgroup

Kaimin Zhang
Department of Mathematics
The University of Auckland

We introduce a new definition of K- subgroup, research some properties about this group and its generators. For conjugate, normalizer, speciously the author gives the structure of K-subgroup in cyclic group, characteristic, automorphisms and group actions.