Please submit your solutions in class.

Please show all working.

1. Let $P = (2, 5)$ on $y^2 = x^3 + 8x + 1$ over $\mathbb{Q}$. Compute $[2]P, [3]P, [4]P$.

2. Prove that if $p \equiv 2 \mod 3$ then the elliptic curve $y^2 = x^3 + 1$ over $\mathbb{F}_p$ has $p + 1$ points.

3. Use a variant of Fermat's descent method to prove that

$$x^4 + 4y^4 = z^2$$

has no non-trivial solutions in $\mathbb{Z}$.

4. Determine the class number of $\mathbb{Q}(\sqrt{-35})$.

5. In the ideal class group of $\mathbb{Q}(\sqrt{-41})$ consider the ideal $I = (5, 2 + \sqrt{-41})$. Show that I^2 is not principal and that I^4 is principal. What can you deduce about the class number of $\mathbb{Q}(\sqrt{-41})$?

6. Show that the only solutions $x, y \in \mathbb{Z}$ to $x^3 = y^2 + 200$ are $(x, y) = (6, \pm 4), (9, \pm 23)$ and $(66, \pm 536)$. [Hint: You can assume that $\mathbb{Z}[\sqrt{-2}]$ is a UFD.]