1. Show that unit ball in a normed linear space is convex (and hence path connected, and connected).

2. Construct an infinite open cover for $(0, 1] \subset \mathbb{R}$ for which no proper subset is a cover.

3. To the cover in the previous question, add an additional open set so that the resulting collection covers $[0, 1]$. Show that this cover for $[0, 1]$ does have a finite subcover.

4. Show that the continuous image of a compact set is compact.

5. The continuous image of a connected subset is connected and the continuous image of a compact set is compact. Is the continuous image of a complete set complete?

6. We say a subset V of a metric space is **totally bounded** if for every $\varepsilon > 0$, there exists a finite collection of open balls $B_\varepsilon(a_1), \ldots, B_\varepsilon(a_n)$ which cover V. Show that $V \subset \mathbb{R}$ is totally bounded if and only if it is bounded.

7. Show that the inverse image of a closed set under a continuous map is a closed set. Use this and the fact that the norm is continuous function to prove that in a normed linear space X any sphere

$$S = \{x \in X : \|x - a\| = r\}, \qquad a \in X,\ r > 0$$

is closed.

8. Suppose E is a subset of a metric space X. Show that if x is a boundary point of E, then either it is a limit point of E or a limit point of $X \setminus E$. Could it be a limit point of both?