1. Show that (X, d) is connected if and only if ϕ and X are the only clopen sets.

2. What are all the connected subsets of $\mathbb{Q}$ (with the Euclidean metric)?

3. Show that if $f : X \to Y$, then

$$U \subset f^{-1}f(U), \quad U \subset X, \qquad ff^{-1}(V) \subset V, \qquad V \subset Y.$$

When is there equality?

4. Let (X, d) and (Y, ρ) be metric spaces. Suppose that $f : X \to Y$ has the property that the inverse image of every open set is open. By considering the inverse image of the open ball $B_\varepsilon(f(a))$, show that f is continuous at $a \in X$.

5. Is the continuous image of an open set open? Is the continuous image of a closed set closed?

6. Is the inverse image of a closed set under a continuous map closed?

7. Use the sequential criterion for continuity to show that a composition of continuous functions is continuous.

8. Let $V \subset X$, and $\overline{V}$ be its closure. Show that if (x_n) is a sequence in $\overline{V}$ which converges then its limit is in $\overline{V}$.

9. Let V_1, V_2 be connected subsets of X with nonempty intersection. Is their union connected?