1. Let $f\colon [0,1] \to \mathbb{R}$ be a continuous function. Let $\varepsilon > 0$. Show that there exists a polynomial p such that $p(x) = a_0 + a_1 x + \cdots + a_n x^n$, $a_0, \ldots, a_n \in \mathbb{Q}$ and $|f(x) - p(x)| < \varepsilon$ uniformly for all $x \in [0,1]$.

2. Let $f\colon [-1,1] \to \mathbb{R}$ be an even continuous function and $\varepsilon > 0$. Show that there exists an even polynomial p such that $|f(x) - p(x)| < \varepsilon$ uniformly for all $x \in [-1,1]$.

3. Let $f\colon [0,1] \to \mathbb{R}$ be a continuous function with $f(0) = 0$. Let $\varepsilon > 0$. Show that there exists a polynomial p such that $p(0) = 0$ and $|f(x) - p(x)| < \varepsilon$ uniformly for all $x \in [0,1]$.

4. Let $f\colon [0,5] \to \mathbb{R}$ be a C^1 function and $\varepsilon > 0$. Prove that there exists a polynomial p such that $\|f - p\|_\infty < \varepsilon$ and $\|f' - p'\|_\infty < \varepsilon$.
(Hint. First find a polynomial q such that $\|f' - q\|_\infty < \varepsilon$.)