1. (a) Prove that there exists an open set $U \subset \mathbb{R}^2$ with $(0,1) \in U$, and C^2-functions $u, v \colon U \to \mathbb{R}$, such that $u(0,1) = v(0,1) = 0$ and

$$\begin{cases} u(x,y)\,v(x,y) + 2\sin u(x,y) = x, \\ 1 - 3v(x,y)\cos u(x,y) = y, \end{cases} \qquad \text{for all } (x,y) \in U.$$

(b) Calculate $(D_1 u)(0,1)$ and $(D_1 v)(0,1)$.

(c) For fun. Calculate the second-order partial derivative $(D_2 D_1 u)(0,1)$

2. (a) Prove that there exist an open set $\Omega \subset \mathbb{R}^2$ and C^2-functions $u, v \colon \Omega \to \mathbb{R}$ such that $(3,1) \in \Omega$, $u(3,1) = 1$, $v(3,1) = 2$ and

$$\begin{cases} u(x,y) + v(x,y) + x + y = 7, \\ \Big(u(x,y)\Big)^4 \cdot \Big(v(x,y)\Big)^4 \cdot x \cdot y = 48, \end{cases} \qquad \text{for all } (x,y) \in \Omega.$$

(b) Calculate $(D_1 u)(3,1)$ and $(D_2 u)(3,1)$.

3. Let

$$K = \{(x,y) \in \mathbb{R}^2 : x^2 + 4y^2 \le 5\}$$

and define $f \colon K \to \mathbb{R}$ by

$$f(x,y) = (x^2 - 1)(y^2 - 1).$$

(a) Make a sketch of K and a sketch of the zero level set of f.

(b) Find **all** (local!) extrema of f and determine whether there is a maximum or minimum.