

1. Reduce  $7^2 \pmod{3}$ . Reduce  $7^{1000} \pmod{3}$ .
2. Find the missing digit '?' in the following 13-digit European Article Number bar codes:  
9 300?52 200409, 4 00?817 504598.
3. Let  $q$  and  $r$  be rational numbers with  $q < r$ .
  - (a) Show that no matter how close together  $q$  and  $r$  lie on the real number line, there exists a rational number between them. (In other words, no matter how small the number  $r - q$ , there exists a rational number  $R$  such that  $q < R < r$ .)
  - (b) Show that no matter how close together  $q$  and  $r$  lie on the real number line, there exists an *irrational* number between them.
4. Prove that  $\sqrt{3}$  is irrational.
5. **Hand in this question with Assignment 3:** (Mindscapes 22,23, Chapter 2.6 of the textbook). Show that the product of two rational numbers is rational. What about the sum of two rational numbers? What about the product of two *irrational numbers*?
6. Consider the number with the decimal expansion  $0.12345678910111213\dots$ . Is this rational or irrational? What about the number  $0.246810121416\dots$ ? Is this rational or irrational? Make sure you justify your answer in detail.
7. (Looking ahead...) If you had infinite time and an infinite amount of paper, could you list all the rational numbers? What about the irrationals?