Department of Mathematics
Maths 190 and Maths 190G
Lecture 9 Summary

In the next two weeks of the course we study infinity.

Since literally counting to infinity is not possible, one way to grapple with the size of infinite objects is to *compare* them with other infinite objects. This idea led us naturally to the following definition, central to all that will follow:

Definition. Two sets, infinite or otherwise, are said to have the same *cardinality* if and only if they can be put into one-to-one correspondence.

Example. The sets {2,4,6,8,10,…} and {1,3,5,7,9,…} have the same cardinality. A one-to-one correspondence (indicated by the vertical lines) is given by:

$$\begin{array}{ccccccc} \{2, & 4, & 6, & 8, & 10, & \ldots\} \\ | & | & | & | & | & \\ \{1, & 3, & 5, & 7, & 11, & \ldots\} \end{array}$$

($2n$ is paired with $2n-1$, $n=1,2,3,4,\ldots$)

We went on to show that many infinite sets have the same cardinality as the set of all positive integers - even the "infinity of infinities" represented by the squares of an infinite chess board. In a similar vain, we related the story of *Hotel Ad Infinitum,* which is "always full but always has room for you!"

Finally we sketched the idea behind the proof that the set of all rational numbers has the same cardinality as the set of natural numbers (positive integers).

Before you come to the next lecture: You should spend an hour or two reviewing the material from today's lecture.

- Read sections 3.1 and 3.2 in the textbook
- Try some of the Mindscapes at the end of section 3.1 in the textbook.
- Can you think of an example of a collection of objects, infinite in number, which does *not* have the same cardinality as the natural numbers, i.e., one that is genuinely "more infinite"?

Other activities you could do if you have time are:

- Ask a friend or family member what ideas come to mind when they think of infinity. Does the person you talk to have similar ideas to you? Do either of you think there could be different sizes of infinity?
- Read section 3.3 in the text, *The Missing Member.*