Department of Mathematics

Maths 190 and Maths 190G
Lecture 8 Summary

In this lecture we posed the following questions:

Q1. Are there more rational numbers, or more irrational ones?

Q2. Is $\sqrt{2}$ rational or irrational. Why?

Q3. Can you tell if a number is rational or irrational from its decimal expansion?

Q4. What's the next real number after 0.999999…?

We began by discussing what a rational number is (it's a number that can be written as a fraction).

We then proved, using a lovely argument from the ancient Greeks, that $\sqrt{2}$ is not a rational number. We did this by assuming it is rational, and then deriving a contradiction.

We then showed how any number with a repeating decimal expansion must be a rational number, and vice versa. (This uses the pigeonhole principle in a really cool way!).

Finally, we attempted to convince you that 0.999999999… = 1, and consequently that decimal representations are not always unique.

Before you come to the next lecture: You should spend an hour or two thinking and reading about the ideas presented in the lecture. In particular you should think further about answering Q1 and Q4 above, which weren't fully answered in the lecture. Refer to Sections 2.6 and 2.7 fo the text for help.

You should also:

- Read Section 3.1 of the textbook, *Beyond Numbers*.

Other things to think about:

- Why can you not prove that $\sqrt{4}$ is not a rational number? Where does the proof break down?