

Department of Mathematics
Maths 190 and Maths 190G
Lecture 5 Summary

In this lecture we illustrate two important observations:

- All natural numbers can be built up as products of prime numbers
- There are an infinite number of prime numbers.

Lecture 5 was based around the following question:

Question: What are the building blocks of the natural numbers?

We started by recalling the definition of a prime number, as one that can be divided only by itself and 1. We then used the Sieve of Eratosthenes to find all the prime numbers less than 100. We did this with a handout consisting of a table of all the numbers up to 100; we then crossed out all those divisible by 2, then those divisible by 3, etc etc. The ones left were the primes.

We also recalled the fact that all natural numbers can be expressed as a product of prime numbers. In this sense prime numbers are “atoms” of the natural numbers.

Finally, we made the claim that despite the increasing rarity of prime numbers, as one looks among higher and higher natural numbers, there is in fact no *highest* prime number. A strategy for proving the infinitude of the primes was outlined. Students are asked to learn the detailed proof by reading over the text.

Before you come to the next lecture: You should spend an hour or two thinking and reading about the ideas presented in the lecture. You should also:

- Read Section 2.5 of the textbook

Other activities if you have time:

- Experiment and see if you believe the following claims:
 - (a) There are infinitely many *pairs* of primes differing from one another by two (e.g., 11 and 13, or 29 and 31).
 - (b) Every even number greater than 2 can be written as the sum of two primes.

Handouts

A table demonstrating the Sieve of Eratosthenes