

Department of Mathematics
Maths 190 and Maths 190G
Lecture 11 Summary

In this lecture we claimed that the collection of real numbers between 0 and 1 is bigger than the collection of all natural numbers, $\{1,2,3,\dots\}$. Here “bigger” is in the sense of cardinality, so that our assertion is equivalent to the following:

*There is no way to **list** the collection of all numbers between 0 and 1.*

We proved this with a proof by contradiction:

Suppose that such a list exists. Use the decimal expansion of numbers in the list as the turns for Player 1, in a game of Infinite Dodge Ball. (We are now playing the game using the digits 0-9, instead of X's and O's.) Then in all cases Player 2 has a winning strategy. Player 2's finished row is therefore the decimal expansion of a number *not* on the list, a contradiction.

By arguments presented in the beginning of the lecture, the claim in italics above additionally implies that there are more irrationals than rationals.

Ideas about different sizes of infinity were developed by a mathematician called Georg Cantor in around 1874. Many mathematicians did not believe these ideas for many years after Cantor described them, and Cantor was ridiculed and belittled for his ideas. You should expect to find these ideas challenging too!

Before you come to the next lecture: Spend two or three hours reviewing the material from today's lecture.

- Read section 3.3 in the textbook.
- Try some of the Mindscapes at the end of section 3.3 in the textbook.

Other activities you could do if you have time are:

- Try to explain why mathematicians believe there are different sizes of infinity to a friend who is not in Maths 190.
- Find out more about Georg Cantor by looking on the internet.