

Department of Mathematics
Maths 190 and Maths 190G
Lecture 10 Summary

In this lecture we gave a particularly stark example of the counter-intuitive nature of infinity. The ping-pong ball conundrum (page 149 of the textbook). This is a ‘thought experiment’ lasting exactly 60 seconds. We start with a large empty barrel and a long line of ping-pong balls, numbered in order $1, 2, 3, \dots$. We start the clock.

Step 1. In the first 30 seconds, pour the first 10 balls into the barrel (numbers 1-10), find number 1 and throw it out.

Step 2. In half the remaining time (15 s), pour the next 10 balls into the barrel (numbers 11-20), find number 2 and throw it out.

Step 3. In half the remaining time (7.5 s), pour the next 10 balls into the barrel (numbers 21-30), find number 3 and throw it out.

Continue in this way until 60 seconds has passed, then stop the experiment. Our claim is that at the end of the 60 second experiment there are no balls left in the barrel. Why? Because, for any natural number N , we know that the ball numbered N was thrown out at some time during the experiment.

Several variations of the Conundrum were also presented; these will be discussed further in Tutorial 5.

Finally, we showed another way to systematically list all fractions (rational numbers), and hence to establish that the set of fractions has the same cardinality (“size”) as the set $\{1, 2, 3, \dots\}$ of natural numbers.

Before you come to the next lecture: Spend an hour or two reviewing the material from today’s lecture.

- Read section 3.2 in the textbook.
- Try some of the Mindscapes at the end of section 3.2 in the textbook.
- For the next lecture, review the game of Dodge Ball from Lecture 1 (Story 5 in section 1.1 of the textbook). Think about the *infinite* version of Dodge Ball described at the very end of today’s lecture.

Other activities you could do if you have time are:

- Try to explain the ping-pong ball conundrum to a friend who is not in Maths 190.