1. (a) Show that the five consecutive integers that begin with the number $(1 \times 2 \times 3 \times 4 \times 5 \times 6) + 2$ are all non-primes.

(b) Is it true that for any finite integer N, there exists a run of N consecutive integers, all of which are non-prime? Explain your answer.

2. Australian Tax File Numbers are generated using a check-digit error detection algorithm kept secret by the Tax Office to prevent people creating fake but valid-looking numbers. The algorithm was rather easily discovered by examining large numbers of examples; here are the details: The numbers are 9-digits long; a number $n_1n_2n_3\ n_4n_5n_6\ n_7n_8n_9$ is a valid Tax File Number if and only if it satisfies the equation,

$$n_1 + 4n_2 + 3n_3 + 7n_4 + 5n_5 + 8n_6 + 6n_7 + 9n_8 + 10n_9 \equiv 0 \bmod 11.$$

For example, the number 555 000 555 is a valid Tax File Number, because

$$5 \times 1 + 5 \times 4 + 5 \times 3 + 0 \times 7 + 0 \times 5 + 0 \times 8 + 5 \times 6 + 5 \times 9 + 5 \times 10$$
$$= 5 + 20 + 15 + 0 + 0 + 0 + 30 + 45 + 50 = 165 = 15 \times 11 \equiv 0 \bmod 11.$$

(a) Show that 262 702 231 is not a valid Tax File Number.

(b) With what digit should you swap the 7 to make 262 702 231 a valid Tax File Number?

3. Decide which of the following numbers are rational and rewrite them as fractions:

$$\sqrt{5}, \qquad 1.\dot{9}\dot{8} = 1.989898\ldots, \qquad \sqrt{9}, \qquad 0.1010010001000010000001\ldots, \qquad 3.14159265.$$

4. Let q and r be rational numbers with $q < r$. Find an *irrational* number lying between them on the real number line, i.e., an irrational number I such that $q < I < r$. Be sure to clearly explain why your number I is irrational. You may use without proof the fact that $\sqrt{2}$ is an irrational number. (*Hint.* The number $1/\sqrt{2}$ satisfies the inequality $0 < 1/\sqrt{2} < 1$.)

5. Is it possible to build an irrational number whose decimal digits are just 1's and 2's and only finitely many 2's appear? If so, describe such a number. If not, explain why not.