# Maths 190 / 190G, 2007 S1
# Assignment 2 solutions.

Q1. (a)

| Price / £ | Orders | # |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1+1, 2 | 2 |
| 3 | 1+1+1, 1+2, 2+1 | 3 |
| 4 | 1+1+1+1, 1+1+2, 1+2+1, 2+1+1, 2+2 | 5 |
| 5 | 1+1+1+1+1, 1+1+1+2, 1+1+2+1, 1+2+1+1, 1+2+2, 2+1+1+1, 2+1+2, 2+2+1 | 8 |
| 6 | 1+1+1+1+1+1, 1+1+1+1+2, 1+1+1+2+1, 1+1+2+1+1, 1+1+2+2, 1+2+1+1+1, 1+2+1+2, 1+2+2+1, 2+1+1+1+1, 2+1+1+2, 2+1+2+1, 2+2+1+1, 2+2+2 | 13 |

(b) Call the number of orders of coins for price $n$, $F_n$.

If the price is $n$, then each possible coin order begins $1 + \cdots$ or it begins $2 + \cdots$.

Case A. If first coin is 1, then the price remaing is $n-1$. There are then $F_{n-1}$ ways to order the remaing coin inserts.

Case B. If first coin is 2, then the

price remaining is $n-2$. There are then $F_{n-2}$ ways to order the remaining coin inserts.

$\therefore$ Total number of ways $\boxed{F_n = F_{n-1} + F_{n-2}}$ $\ast$

Since $F_1 = 1$ & $F_2 = 2$ and since $F_1, F_2, F_3, \ldots$ has the same rule of generating future members in the sequence, $\ast$, the sequence $F_1, F_2, F_3, \ldots$ must be Fibonacci. Q.E.D.

(c) $F_{39} = F_{38} + F_{37}$

$= 63{,}245{,}986 + 39{,}088{,}169$

$= 102{,}334{,}155.$

Q3.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|---|
| $F_n$ | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 |
| $(F_{n+1})^2$ | 1 | 1 | 4 | 9 | 25 | 64 | 169 | . |
| $(F_{n-1})^2$ | . | 0 | 1 | 1 | 4 | 9 | 25 | 64 |
| $(F_{n+1})^2-(F_{n-1})^2$ | . | 1 | 3 | 8 | 21 | 55 | 144 | . |
| $=$ | . | $F_2$ | $F_4$ | $F_6$ | $F_8$ | $F_{10}$ | $F_{12}$ | . |

From these observations we conjecture that $(F_{n+1})^2-(F_{n-1})^2 = F_{2n}$ for $n \geq 1$.

Q2.

$110 = 89 + 21$

$62 = 55 + 5 + 2$

$12 = 8 + 3 + 1$

Q4(a) For each colour there are eight possible configurations, depending on the choice of options:

| option 1 | option 2 | option 3 |
|---|---|---|
| × | × | × |
| × | × | ✓ |
| × | ✓ | × |
| × | ✓ | ✓ |
| ✓ | × | × |
| ✓ | × | ✓ |
| ✓ | ✓ | × |
| ✓ | ✓ | ✓ |

× = option not taken
✓ = option taken

So there is a total of $8 \times 4 = 32$ possible configurations (4 being the number of colours).

Sorting the bicycles according to configuration, and totalling the number in each of the 32 configurations, we must get 100,000. However, if the assertion in (a) is FALSE, then each configuration contains 99 or less bicycles, giving a total of less than $32 \times 99 = 1,188$ bicycles, a contradiction. So the assertion is true.

(b) Since $100,000 = 32 \times 3,125$, $N = \underline{3,125}$

(The previous argument with "10" replaced with "3125" still works. Also, if each configuration contains exactly 3125 bicycles, which is possible, then no configuration will have 3126 or more, showing that $N = 3125$ is indeed maximal.)