Department of Mathematics

Maths 190 and Maths 190G Assignment 2

Due 4pm, Friday March 23rd, 2007

Your completed assignment should be placed in the appropriate box outside the Student Resource Centre in the basement of the Maths building before 4pm on the due date. Marks will be awarded for this assignment based on the **clarity** of your answers. You should aim for well-written, polished answers. The markers will not be concerned with spelling and grammar but will pay close attention to the **logic** of your statements.

If you need help with this assignment, first read the textbook. If you need further help once you have read the textbook, speak to your lecturer. Help may also be obtained in the Stage I Maths Assistance Room. It is located in the basement of the Science Centre, in Room B25 and is open 9-5pm every weekday.

Q1. (6 marks) Imagine that you are buying a ticket to park your car in a London "Pay and Display" car park. The ticket machine takes only 1 and 2 pound coins and it costs 1 pound per hour to park. You have only 1 and 2 pound coins in your pocket. For a specified number of hours parking (=price in pounds), consider the number of ways you can insert coins to buy a ticket covering that period.
For instance, if "+" means "followed by" then we have:

Price £	Coin orders	Number of solutions
1	1	one way
2	1+1 2	two ways
3	1+1+1 1+2 2+1	three ways

Note that 1 followed by 2 is regarded as different from 2 followed by 1.

(a) Add to the above diagram by listing and counting the number of ways to pay for a 4 hour ticket, a 5 hour ticket, and a 6 hour ticket.
(b) Show that the number of solutions in fact form a Fibonacci sequence.
(c) Given that there are 39088169 ways to pay for a 37 hour ticket, and 63245986 ways to pay for a 38 hour ticket, how many ways are there to pay for a 39 hour ticket?

Q2. (4 marks)

The number 38 can be written as a sum of Fibonacci numbers:

$$38 = 34 + 3 + 1$$

Similarly, we have

$$9 = 8 + 1$$

and both 8 and 1 are Fibonacci.

Write the numbers 110, 62 and 12 as sums of Fibonnaci numbers.

Q3. (5 marks)

Let F_n denote the nth Fibonacci number, i.e.,

$F_1 = 1$, $F_2 = 2$, $F_3 = 3$, $F_4 = 5$, $F_5 = 8$, $F_6 = 13$, ...

The rule for generating new numbers in the sequence can then be written

$$F_n = F_{n-1} + F_{n-2}.$$

By experimenting with numerous examples in search of a pattern, determine a simple formula for $(F_{n+1})^2 - (F_{n-1})^2$ — that is, for the difference of the squares of two Fibonacci numbers having just a single Fibonacci number between them in the sequence:

$$F_{n+1}^2 - F_{n-1}^2 = \ ?$$

Q4 (3 + 2 marks)

You want to buy a new bicycle, and you know the model you want. The model has three options, each one of which you can either take or not take, and you have a choice of four colours. So far 100,000 bicycles of this model have been sold.

(a) Explain why, among all the bicycles sold, there is sure to be 100 of them sharing the same options and colour.

(b) What is the largest number N such that we are guaranteed to find N bicycles sold sharing the same options and colour. (There is no need to explain your answer.)

Tutorial write-up: Remember to hand in with your assignment your written solution to the boxed questions in Tutorials 2 and 3, which you would have discussed already during the tutorials. Instructions on how to write up tutorial questions are contained on the tutorial sheets.