Challenges and pulsations in undergraduate mathematical education: some personal views from a mathematician's perspective

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Envisioning the Future — Senior Secondary and Undergraduate Mathematical Science
NZIMA Conference (*New Zealand Institute of Mathematics & its Applications*)
University of Auckland
April 12, 2010

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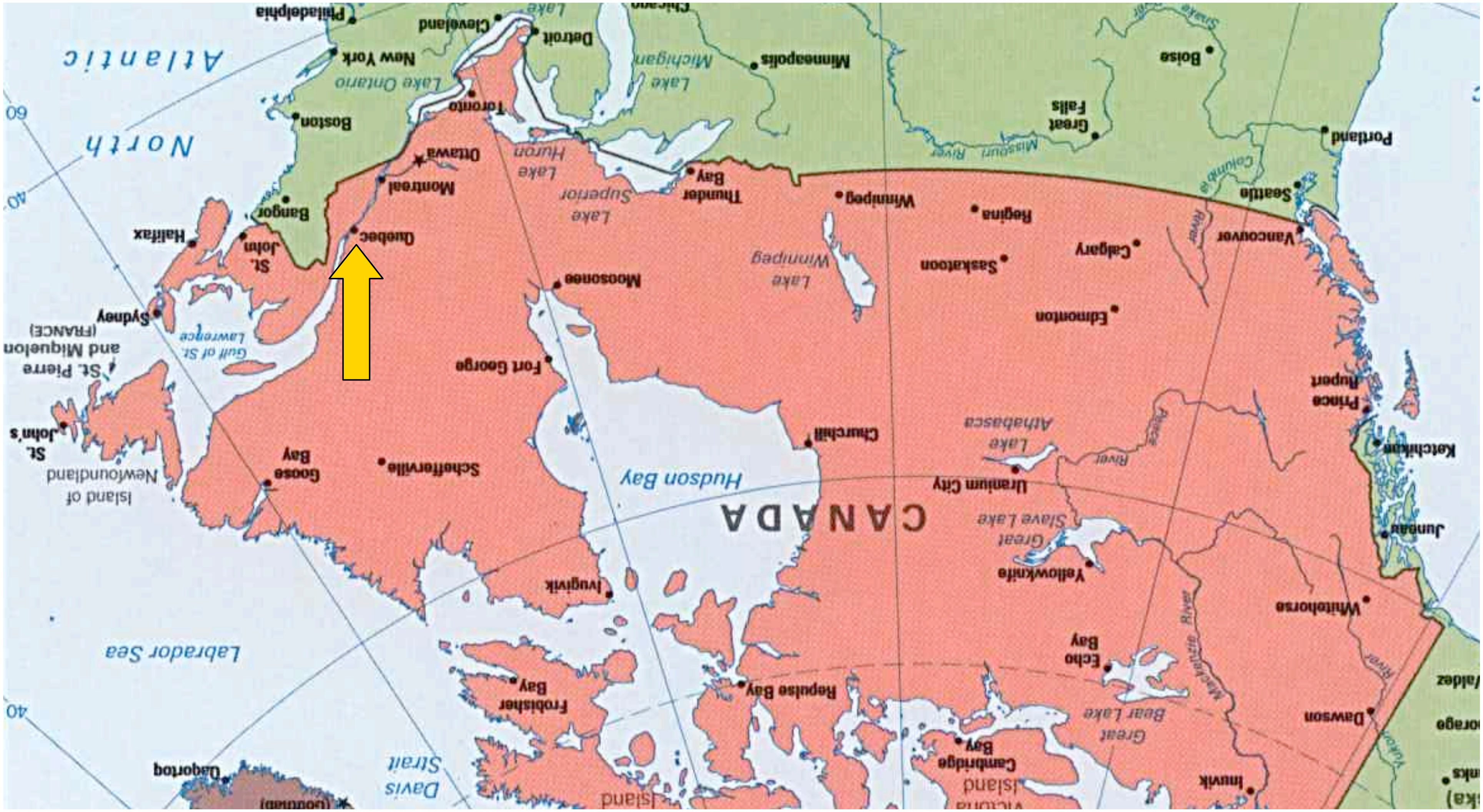
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Great Bear Lake
Island of Newfoundland
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Lake Michigan
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Atlantic
Detroit
Cleveland
Chicago
Philadelphia
Qaqortoq
(Godthåb)

Theme of the conference:
Envisioning the Future:
Senior Secondary & Undergraduate Mathematical Science

- "envision" :
  foresee, anticipate AND *imagine, dream of*

- *Senior Secondary* AND *Undergraduate*
  - issues of collaboration
  - issues of transition (secondary / tertiary)

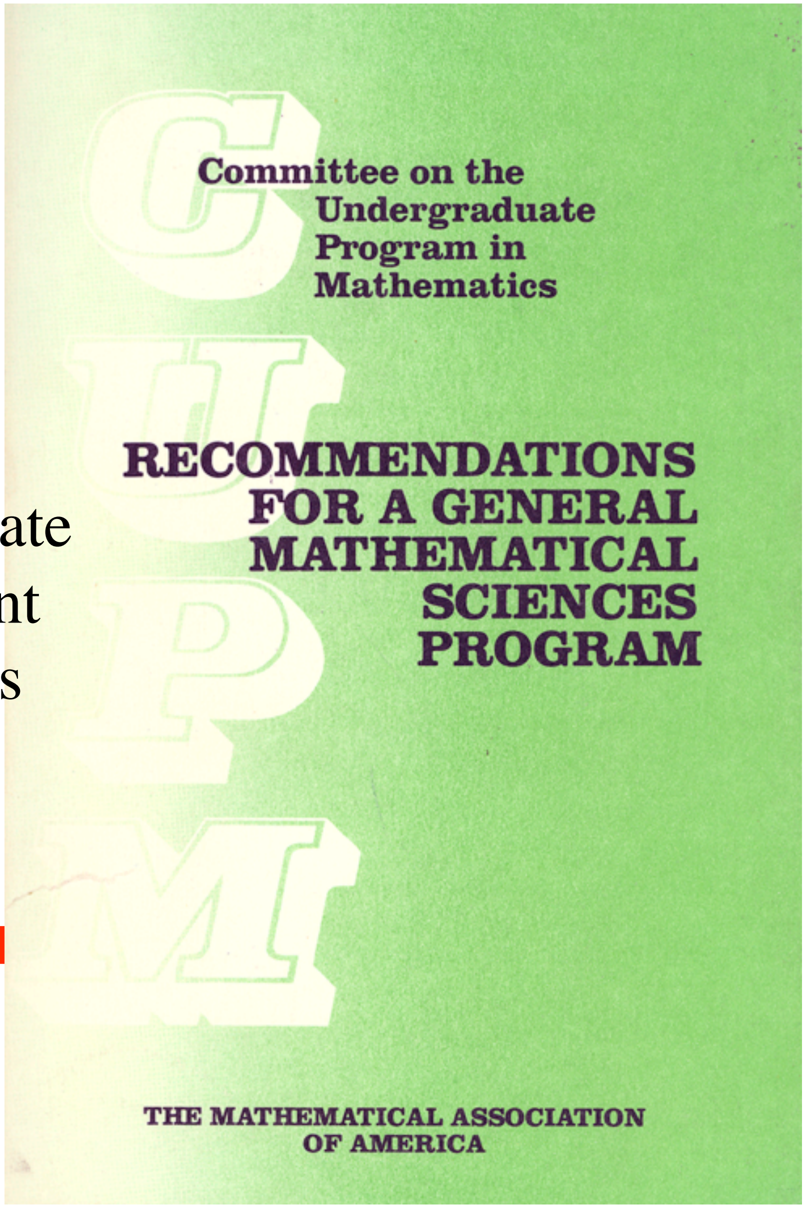
Important to approach these matters from both school and university perspectives ***(SSUMS)***

I will concentrate more on the tertiary level

Envisioning the Future:
Senior Secondary & Undergraduate Mathematical Science

- "Mathematical Science"
  - and not ScienceS
  - MAA (USA)

    CUPM report 1981



"CUPM now believes that the undergraduate major offered by a mathematics department at most American colleges and universities should be called a Mathematical Sciences major." *(Preface)*

Report *The Mathematical Sciences*
Nat. Acad. of Sciences, USA, 1965

My title: *Challenges and* ***pulsations***

- not in the technical sense of Guitard's

  Mathematical Pulsation

  ("paradoxical mental breath"

  inside every mathematical act)

- metaphor

  crucial component of many

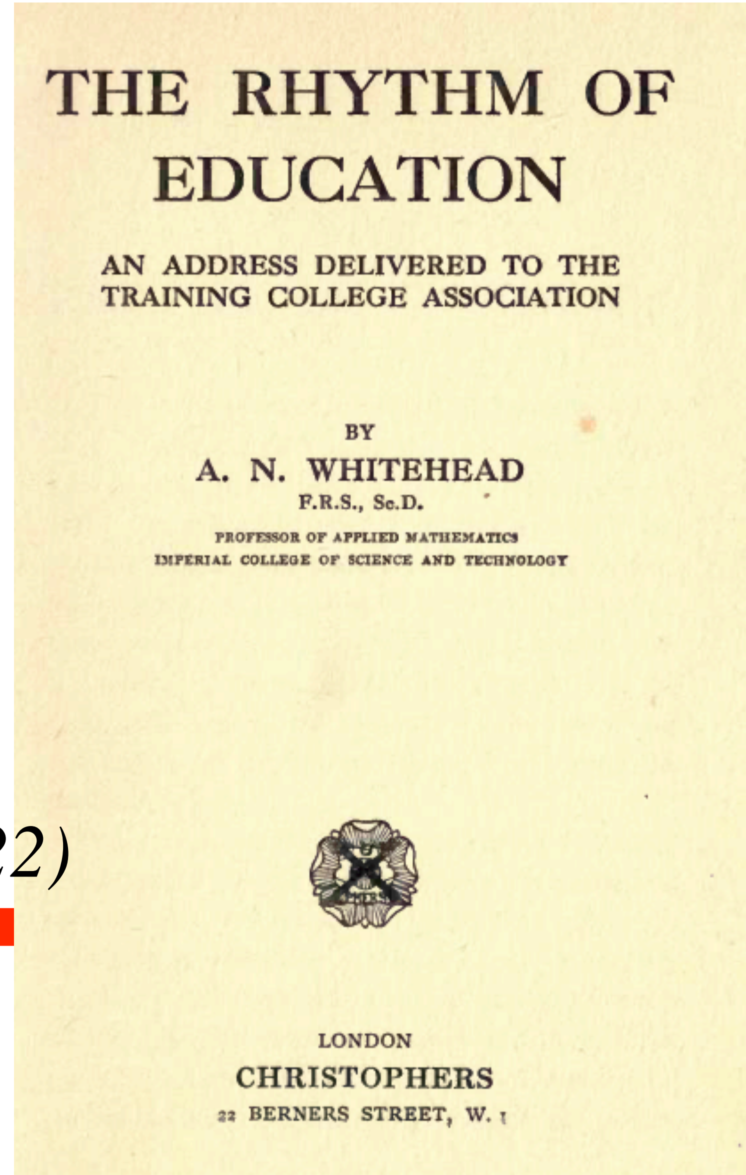
  living organisms

  (alternating movement of the heart)

  "rhythmic pulses of life"

  (Alfred North Whitehead, 1922)

"Le coeur qui bat" – "La vie qui bat"



THE RHYTHM OF EDUCATION

AN ADDRESS DELIVERED TO THE TRAINING COLLEGE ASSOCIATION

BY

A. N. WHITEHEAD

F.R.S., Sc.D.

PROFESSOR OF APPLIED MATHEMATICS

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY

LONDON

CHRISTOPHERS

22 BERNERS STREET, W. 1

Research perspective — Michèle Artigue's talk

The school / university interface

Univ of Auckland (Math Ed Unit), Thursday April 8

Overview of didactical research of the last decades

Diversity of approaches and changing contexts, making it difficult to get a coherent and synthetic view of what we really know, and of how research results can productively guide educational action

Research perspective — Michèle Artigue's talk

Epistemological and cognitive perspectives

concern the nature of mathematical concepts and thinking modes, as well as students' cognitive difficulties (rely on theories of cognitive development)

Institutional and cultural perspectives

approach more globally the transition as a transition between cultures — changes in norms and values from one level to another one (rely on theoretical frameworks such as the Anthropological Theory of Didactics)

More comments later

PLAN OF THE TALK

I- Background 1: Issues of secondary/ tertiary transition

II- Background 2: Reflecting on the undergraduate math curriculum

III- Challenges and pulsations in SSUMS: myths, realities and dreams

I- Issues of secondary/tertiary transition

International Round Table (ICM-1998)

M. de Guzmán, B.R. Hodgson, A. Robert et V. Villani,
"Difficulties in the passage from secondary to tertiary education."

In : G. Fischer et U. Rehmann, dir.,
Proceedings of the International Congress of Mathematicians.
(Berlin, 1998)
Documenta Mathematica, Extra Volume ICM 98,
volume III, pp. 747-762.

Questionnaire "Transition to university mathematics"

Perceptions of fours groups of students (*Université Laval*) about their own transition:

- 1st year math majors (31)
- final year math majors (29)
- 1st year secondary school teachers (72)
- 1st year engineering (118)

- 18 questions (*Likert scale 1-5* — disagreement → agreement)
  - the way teachers present math, organisation of the classroom
  - changes in the mathematical ways of thinking at university
  - textbooks and other materials
- open comments

School Education in the province of Québec

AGE 5 – 11 *Primary school* (K – 6)

AGE 12 – 16 *Secondary school* (1 – 5)

transition

AGE 17 – 18 / 19 "*Cégep*" (1 – 2 / 3)

TRANSITION

MATHEMATICS MAJOR

AGE 19 – 21 *University* (1 – 3)

TEACHER EDUCATION & ENGINEERING

AGE 19 – 22 *University* (1 – 4)

Age as of September (1st month) of schoolyear

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"Transition to university mathematics was difficult for me."								
Likert scale	Group I		Group II		Group III		Totals	
1	7	(12%)	3	(4%)	35	(30%)	45	(18%)
2	17	(28%)	19	(26%)	46	(39%)	82	(33%)
3	14	(23%)	5	(7%)	11	(9%)	30	(12%)
4	17	(28%)	37	(51%)	15	(13%)	69	(28%)
5	5	(8%)	8	(11%)	9	(8%)	22	(9%)
no reply	0	(0%)	0	(0%)	2	(2%)	2	(1%)
Totals	60	(100%)	72	(100%)	118	(100%)	250	(100%)

1 : total disagreement

5 : total agreement

Group I : *MATHEMATICS* (1st and final year)

Group II : *SECONDARY SCHOOL TEACHING*

Group III : *ENGINEERING*

I-) Difficulties linked to the way university teachers present maths and to the organization of the classroom

	Math	*Teaching*	*Engineers*
• *Teachers are often abstract — not enough concrete examples*	agree	agree	disagree
• *Teachers go too fast — they don't check if we have understood*	agree	agree	neutral
• *After the explanations, I am not sure what is important*	disagree	neutral	disagree
• *I am just a number*	disagree	disagree	agree

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II-) Difficulties stemming from the math taught

(changes in the mathematical ways of thinking at the university)

	Math	***Teaching***	***Engineers***
• *Math problems to be solved are substantially more difficult than previously*	agree	agree	disagree
• *I am not used to the abstraction used here*	neutral	agree	disagree
• *Assessment bears upon more abstract math*	agree	agree	disagree

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III-) Difficulties linked to the lack of appropriate learning tools *(books, lecture notes, …)*

	Math	***Teaching***	***Engineers***
• *Note taking prevents me from understanding the explanations*	agree	agree	agree
• *The notes taken in the classroom are not really helpful to understand and do the exercises*	disagree	disagree	neutral

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Criticisms of the teachers

- *Many university teachers do not care whether we understand or not what they are teaching us.*
- *A majority of teachers do not understand that we do not understand.*
- *It is hard for them to make us understand what is evident for them.*
- *Passing from secondary school to university mathematics was not as hard as I was told. But what makes it somewhat hard are the changes in the teachers: many of them are not at all suited for teaching. Here, we have teachers who are topnotch mathematicians. But their pedagogical skills will never outmatch those of my high school teachers.*

Written comments by the students

Students' background, autonomy

- *It seems that I am lacking a lot of prerequisites. It is as if I should know 100% of my high school maths.*
- *In high school, I never learned to do proofs, and now it seems to be taken for granted that we know how to do proofs.*
- *My answers to these questions would vary considerably according to the courses and the instructors. But a general trend is that courses include many many topics which are covered very quickly, so that we need to work a lot on our own outside the classroom.*

Some very positive perceptions of the transition

(it provoked little or no difficulty)

- *I appreciate much more university math, because we try to understand where the results we are using, and were using in high school, come from.*
- *Going from high school to university did not raise special problems for me, as the level of difficulty of high school math prepared us well for that.*

Types of difficulties in the secondary / tertiary transition

- **Epistemological and cognitive difficulties**

  shift in the kind of math to be mastered by the students

  increased depth, more conceptual understanding

  move from “elementary” to “advanced” math thinking

  becoming autonomous, mathematically speaking

this requires some forms of
meta-cognitive knowledge

Research perspective — Michèle Artigue's talk

The school / university interface

Some reasons why the transition is problematic

- the changes in norms and values regarding mathematical knowledge and practices from one institution to another
- the implicit character of most of these norms and values, and the way they are conveyed

Overcoming such difficulties often rests on the sole students' shoulders and is thus more problematic

Not a radical move from intuitive / algorithmic math to the formal world, but rather **an accumulation of “micro-breaches”** hardly visible and not appropriately taken in charge by the institution

- an increasing speed in the introduction of new objects
- a greater diversity of tasks
- much more autonomy given in the solving process
- a new balance between the particular and the general
- objects are more controlled by definitions, results get more systematically proved, so that proofs are no longer “the cheery on the cake” but take the status of mathematical methods

PhD Thesis of Frédéric Praslon (2000)

Types of difficulties in the secondary / tertiary transition *(cont'd)*

- **Sociological and cultural difficulties**

  move from "human-size" high school to the anonymity of a large university campus

  class groups reformed every semester (lack of sense of "classroom community")

  competition

  underestimation of the role and importance of math with respect to their future professional needs

Types of difficulties in the secondary / tertiary transition *(cont'd)*

- **Didactical difficulties related to the univ teachers**

  lack of pedagogical abilities / interest / availability

  lack of sensibility to the epistemology of mathematics (impact on study and work in math)

  lack of innovative teaching methods (technology)

  lack of assessment skills

A few possible actions to help alleviating these difficulties *(including some dreams)*

- Establish a better dialogue between secondary educators and university educators
- Change the context of the transition
- Establish a better dialogue between mathematicians and users of math
- Provide students with individualised help
  (*Students Help Center*)
- Create a context propitious to faculty development and collaboration around pedagogy
- "Less is more"
- …

more dreams to come…

PLAN OF THE TALK

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II- Background 2: Reflecting on the undergraduate math curriculum

III- Challenges and pulsations in SSUMS: myths, realities and dreams

IIa- Selected milestones in the reflection on the undergraduate math curriculum

- CUPM report (1981)
- Anthony Ralston, vg *Amer. Math. Monthly* (Sept. 1981)

  discrete math vs *calculus* in undergraduate math and computer science curricula
- H.S. Wilf, "The disk with the college education." *Amer. Math. Monthly* (Jan. 1982)

  a "distant early-warning signal" about muMATH (on Apple 2!)
- L.A. Steen, *Calculus for a New Century: A Pump, Not a Filter*. MAA, 1987
- "Calculus Reform" movement

Introductory Calculus in 1990

a paper for the I.C.M.I. Symposium on Computers and Mathematics*

B.R. Hodgson (Université Laval)
E.R. Muller (Brock University)
J. Poland (Carleton University)
P.D. Taylor (Queen's University)

1) Introduction

In this article we propose ways in which the introductory Calculus curriculum might respond to the recent and rapidly changing computer resources. Our purpose is not to describe how such computer resources might be used most effectively in the learning of the Calculus but rather to examine the impact of the existence of such resources as computer programs to perform differentiation and definite and indefinite integration.

Our main points are

- it is counterproductive to train our students to perform calculations that they know a microcomputer can do far more accurately and quickly;
- consequently a major reorientation in the style and content of the introductory Calculus course is needed, away from the performance of algorithms and towards a more meaningful and thoughtful experience;
- the spirit of this change calls for presenting the Calculus as one of mankind's finest intellectual achievements, more valuable than ever in its recent applications, and demanding of more interactive classroom teaching.

In a sense, we are entering a golden age of mathematics teaching, in which the deemphasis upon paper-and-pen performance of algorithms frees us to teach in ways that respect what we each feel are the true goals of mathematics education.

ICMI Study 1 (1985)

Bill Barton's Intro (yesterday) — issues of curricula

What if we can start from a clean state???

(But we can't really…)

Anthony Ralston, "A zero-based mathematics curriculum." *ICMI Bulletin* 36 (1994)

Another reason for the frustratingly slow pace of change in mathematics education or, for that matter, in any area of education is that realistic proponents of change must always seek an evolutionary approach because in a system with as much inertia as any educational system, revolution is just not on. It follows from this that proposed reforms in mathematics education emphasize what seems to be (politically) possible rather than being goal-oriented in the sense of proposing where you would like to end up. The result inevitably is incremental change which focuses on the possible rather than on what might be more generally desirable.

The purpose of this paper is to suggest the value of an exercise in mathematics curriculum design which is not constrained by the shackles of reality and which, therefore, can be goal-oriented. The outcome of this purely intellectual exercise would be a curriculum that you would like to be able to implement and some suggestions of how to get from where we are to where we would like to be.

The idea of a zero-based curriculum arises from the notion of zero-based budgeting which is probably familiar to my readers. In zero-based budgeting all items in a budget must be justified *ab initio* and not because there was a similar item in last year's budget. The initial result of such budgeting may be politically or otherwise impossible but it provides, at least, a blueprint for the budget you would like to have. So it is similarly with a zero-based curriculum.

Impact on primary and **secondary school mathematics**

(technology, pedagogy, teacher education, assessment, …)

A. Ralston, *ICMI Bulletin* 36 (1994)

IIb- Reflecting on the undergraduate math curriculum / Some growing consensus

- The main focus of undergraduate math education should NOT be on a procedural / computational understanding of mathematics
- It should aim at a broad understanding based on "*big ideas and processes*" (sense-making)
  crux of the matter: *method* rather than content
  (ways of thoughts, general strategies)

  "Ways of cutting up the mathematical universe, and putting it back together"

  Peter Taylor (Canada), Regular lecture at ICME-9

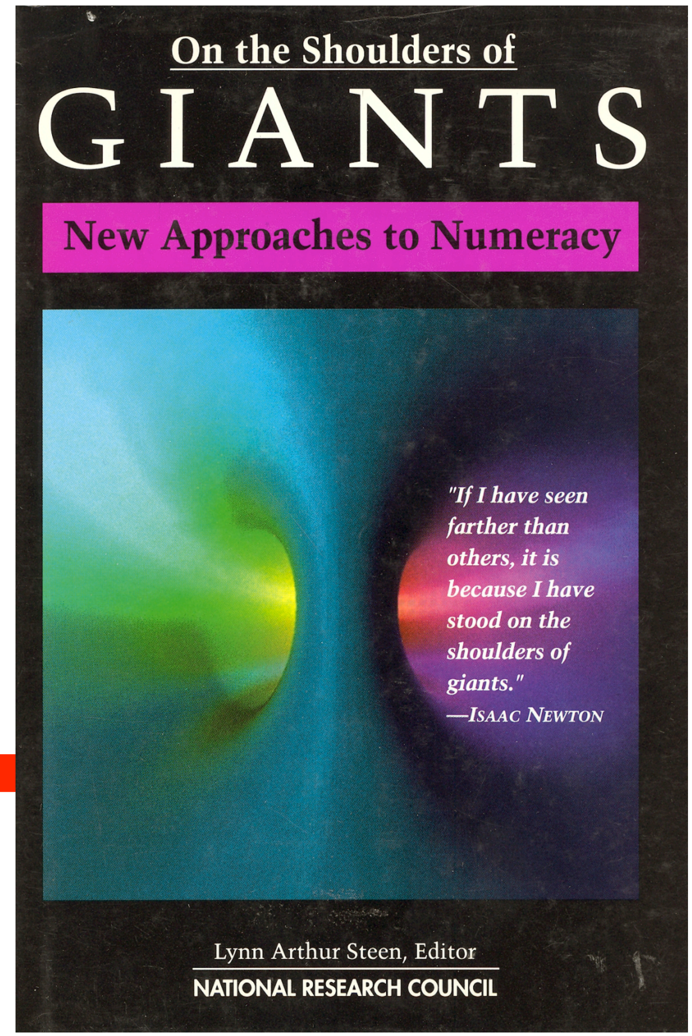
“Deep ideas” of mathematics

Mathematical structures

- Numbers
- Algorithms
- Ratios
- Shapes
- Functions
- Data

Attributes

- Linear
- Periodic
- Symmetric
- Continuous
- Random
- Maximum
- Approximate
- Smooth



L.A. Steen, *On the Shoulders of Giants: New Approaches to Numeracy*, 1990

Actions

- Represent
- Control
- Prove
- Discover
- Apply
- Model
- Experiment
- Classify
- Visualize
- Compute

Abstractions

- Symbols
- Infinity
- Optimization
- Logic
- Equivalence
- Change
- Similarity
- Recursion

L.A. Steen, *On the Shoulders of Giants: New Approaches to Numeracy*, 1990

Attitudes

- Wonder
- Meaning
- Beauty
- Reality

Behaviors

- Motion
- Chaos
- Resonance
- Iteration
- Stability
- Convergence
- Bifurcation
- Oscillation

Dichotomies

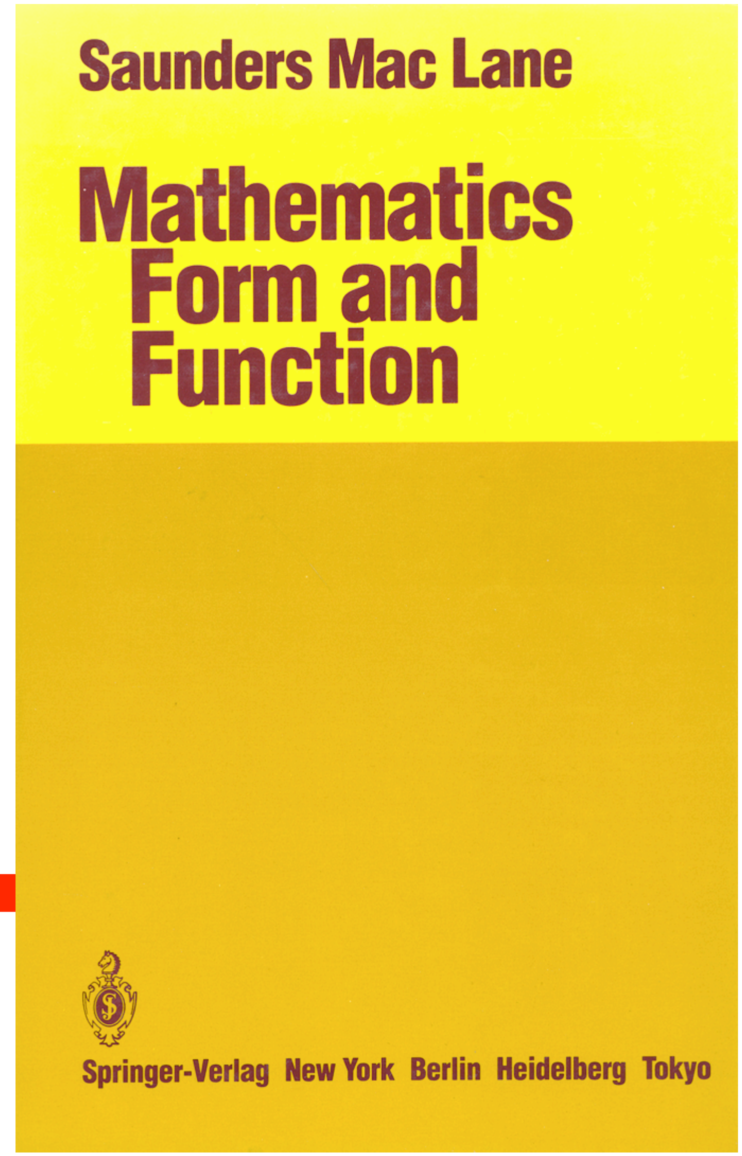
- Discrete vs. continuous
- Finite vs. infinite
- Algorithmic vs. existential
- Stochastic vs. deterministic
- Exact vs. approximate

L.A. Steen, *On the Shoulders of Giants: New Approaches to Numeracy*, 1990

Human activities and “ideas”

from “premathematical concerns”
(*human cultural activities*)
to
somewhat nebulous “ideas”
to
formalization

1986



Activity	Idea	Formulation
Collecting	Collection	Set (of elements)
Counting	Next	Sucessor; order Ordinal number
Comparing	Enumeration	Bijection Cardinal number
Computing	Combination (of nos)	Rules for addition Rules for multiplication Abelian group
Rearranging	Permutation	Bijection Permutation group
Timing	Before and after	Linear order

S. Mac Lane, *Mathematics, Form and Function* (1986)

Activity	Idea	Formulation
Observing	Symmetry	Transformation group
Building, shaping	Figure; symmetry	Collection of points
Measuring	Distance; extent	Metric space
Moving	Change	Rigid motion Transformation group Rate of change
Estimating	Approximation	Continuity Limit
	Nearby	Topological space
Selecting	Part	Subset Boolean algebra
Arguing	Proof	Logical connectives
Choosing	Chance	Probability (favorable/total)
Successive actions	Followed by	Composition Transformation group

See *Mathematics, Form and Function* on Wikipedia

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Boyer Commission — created in 1995 under the auspices of the Carnegie Foundation for the Advancement of Teaching

***Ways to change undergraduate education (general)**, vg*

- make research/inquiry-based learning the standard
- foster interdisciplinary education
- emphasize communication skills
- use technology creatively
- culminate with a capstone experience
- change faculty rewards systems
- cultivate a sense of community

All this clearly applies to math!

Walter Whiteley (York University, Toronto)

Processes (Ontario school curriculum)

- *Problem Solving*, and selecting appropriate problem solving techniques
- *Reasoning and Proving*
- *Reflecting* and monitoring processes
- *Selecting Tools and Computational Strategies*
- *Connecting*
- *Representing* and modelling mathematical ideas in multiple forms: concrete, graphical, numerical, algebraic, and with *technology*
- *Communicating*

UUDLES for math (Ontario universities)

- *integrate* relevant knowledge and pose questions
- *apply* a range of techniques effectively to solve problems
- *construct*, *analyze, and interpret* mathematical models
- *use* computer programs and algorithms: both numerical and graphical
- *collect, organize*, *analyze, interpret* and present conjectures and results
- *analyze data* using appropriate concepts and techniques from statistics and mathematics

University Undergraduate Degree Level Expectations (UUDLES)
(Ontario Council of Academic Vice-Presidents)

UUDLES for math (cont'd)

- *employ technology* effectively, including computer software, to investigate
- *learn* new mathematical concepts, methods and tools
- take a core mathematical concept and '*unpack*' the concept
- *communicate* clearly mathematical and statistical concepts (models, reasoning, explanation, interpretation and solutions)
- *identify and describe* some of the current issues and challenges (professional, ethical, …)

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III- Challenges and pulsations in SSUMS: myths, realities and dreams

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SSUMS as a multi-faceted enterprise

consider some of these components

(focus on Undergraduate)

Toing and froing between myths — realities — dreams

Ways to change undergraduate education — ***Boyer Commission***

Research perspective — Michèle Artigue's talk

- Didactic research shows evident regularities and coherent patterns

- But solutions are local
  Experiments and research show DIVERSITY of:
  - epistemological choices and didactical approaches
  - institutional means and institutional constraints
  - educational contexts and cultures
  - visions developed about technology and its potential role

IIIa- Mathematics and its teaching and learning

1-) Role of technology

(graphic calculators, CAS, spreadsheet, dynamic geometry, …)

- myth *(decision-makers*) : solution to "all" educational problems
- emphasis on an APPROPRIATE use of technology
  (whatever this may mean…)
- in reality: more and more such actions
- however…

"New technology offers invaluable tools to the teacher. The problem does not come from the technology itself, but from the place given to it."

"The teacher is no more encouraged to become a specialist in a given domain of knowledge, but rather an expert in the use of new technologies." (…)

"New technology is not an end in itself, but a mean to reach an end: transmitting knowledge or competencies. Everywhere, in today's school, computer technology tends to replace the living word ("*la parole vivante*") of the teacher."

"Letter" from a "cégep" teacher
Le Devoir, March 10, 2010

Some success stories…

MICA program, Brock University (Ontario, Canada)

Mathematics Integrated with Computers and Applications

"MICA is a cutting edge mathematics program that teaches you how to use powerful combinations of mathematics and computers to solve sophisticated real world problems."

"A four-year honours program that gives you a solid foundation in math and also teaches you the technology you need to know in order to apply what you've learned. You also have the option of specializing in education, pure mathematics, applied mathematics or statistics."

Reports by Muller and Buteau in ICMI Study 11 (*University math teaching*, 2001) and Study 17 (*Technology revisited*, 2010)

Some of my dreams…

- Teachers become fully comfortable with technology
- Teachers are able to remain up-to-date over the years, if only to demystify and integrate new developments
- Math teachers are encouraged to develop as MATHEMATICIANS (including in secondary school!)
  (*cf comments of cégep teacher*)

Technology is used to support a teaching and learning environment where the HUMAN-TO-HUMAN interaction is central

2-) Engagement of students as "mathematicians"

- research / inquiry based learning (Boyer)
- math in the present-day world

Rich activities that develop the habit of DOING math

- "modern" views, such as
  - mathematical modelling
  - math and other disciplines
  - math present in the technology around us
- "traditional" views, such as
  problem-solving
- need for good resources

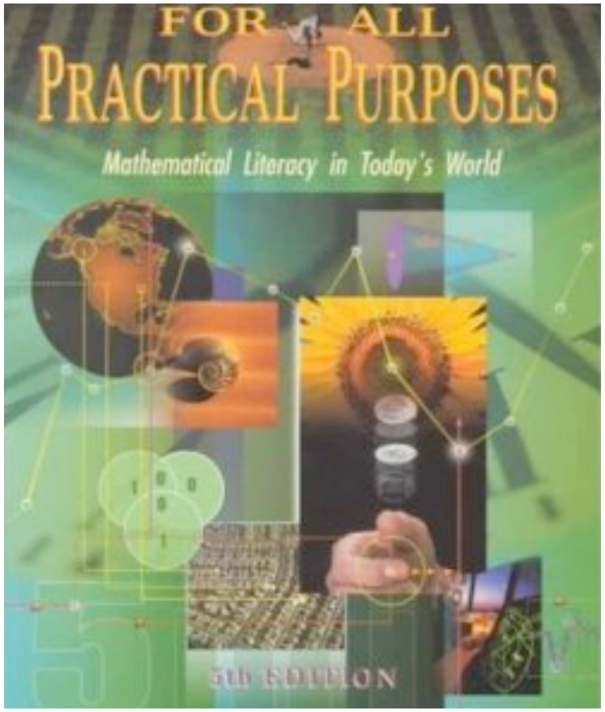
opportunity for team work

(help to develop sense of community)

"My main interest is in the development of what can be called ***investigative problems*** — problems that reach out and grab hold of you, that suggest conjectures you can make and test, that engender discussion and argument among colleagues, that will yield to careful thought and persistence, perhaps with a guiding hand, encouraging mastery of some new skills along the way, and that at the end will display for you a canvas of beauty and power."

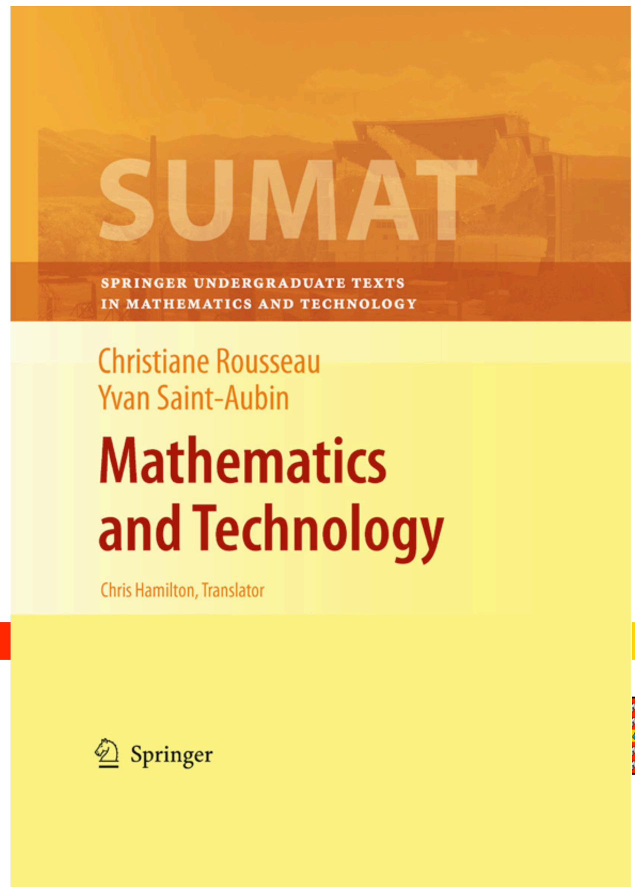
"That's a tall order, but such problems are all around us and the tragedy is that there are so few of them in the school and early university curricula."

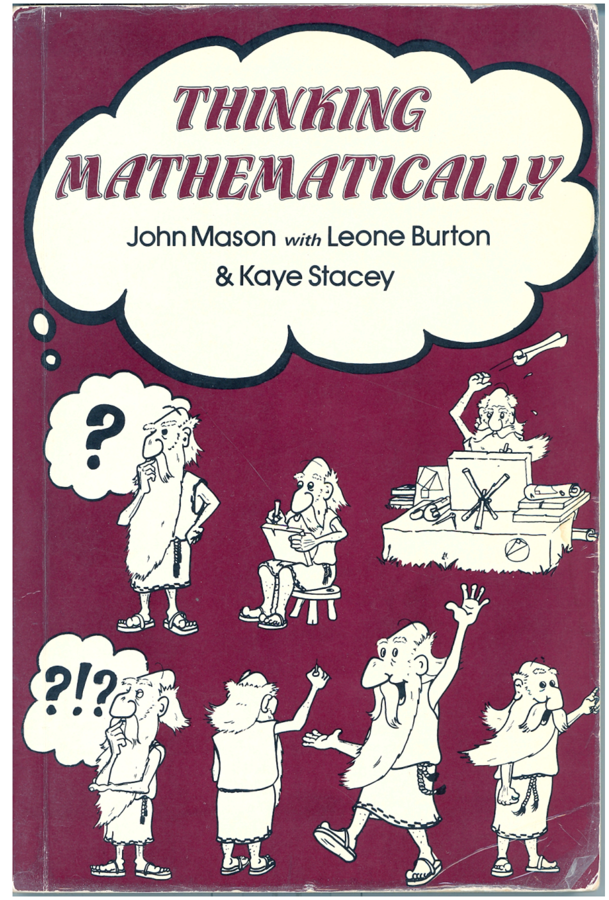
Peter Taylor's website
Queen's University, Canada



Modelling
COMAP

Present-day technology (*developed in a teacher ed course*)





Problem-solving

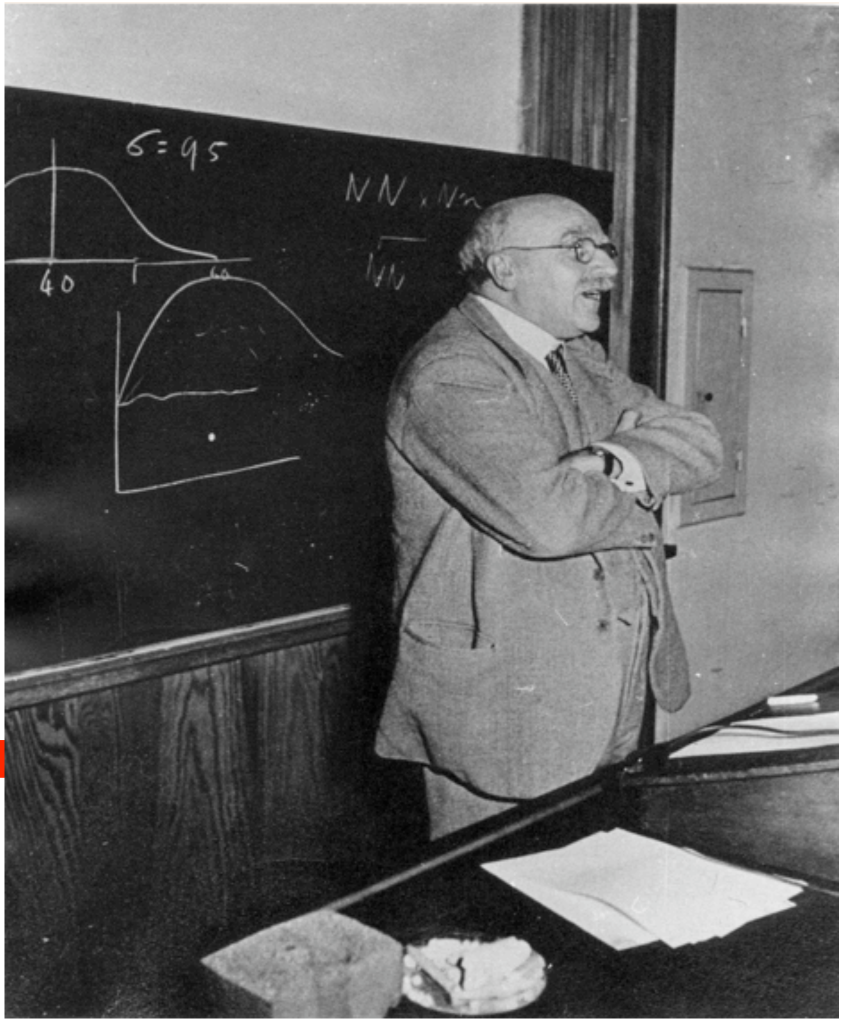
"success stories"

Math and other disciplines

"On being the right size."
Essay (1928) by evolutionary biologist and geneticist
John B.S. Haldane (1892-1964)

Links between the size of an animal and its living functions
(*surface* vs *volume*)

Why there are no small mammals in the Antarctic!



Paper accessible to secondary school teachers

3-) “Meta-visions”

Students need to develop different meta-visions:

- *meta-cognitive processes*

  focus on themselves while doing math

  vg Mason’s framework for problem-solving

  connects to issues of *processes* and *method* mentioned above

  (as opposed to a procedural / computational vision of mathematics)

  helps students develop their autonomy

Students also need to pay attention to

- *meta-content issues*

  focus on the nature of mathematics as a discipline

  (epistemological perspective!)

Such a vision is part of the Danish reform of the school curriculum (2000)

Aim: pupils should develop three kinds of overview and judgement regarding math as a discipline:

- *actual application* in other subjects
- *historical development*
- *the nature of math*

"KOM" — *Competencies and Mathematical learning* (Papers by Mogens Niss)

These components (*applications - history - epistemology*) are central to our secondary school teachers programme (*Laval*)

Two examples (related to history and epistemology):

- ***Nonstandard analysis***

  The existence of the field of real numbers IMPLIES the existence of the ***field*** of “hyperreals” numbers

  (*reals + infinitesimals + infinite numbers*)

  Issue at stake: “existence” in mathematics

Two examples (related to history and epistemology): *cont'd*

- ***Axioms of set theory***

  Matters of "hygiene"

  i.e. practices conducive to maintaining health and preventing disease (*vg* through cleanliness)

  Certain "sets" must be forbidden

  (*cf* division by 0 excluded)

  Teachers to become experts of, say, ZF ???

  - Teachers should be aware that the problem exists…
  - … and that there are solutions to it.

Russell's paradox

$$A = \{X \mid X \notin X\}$$

4-) Social expectations

Important to think of math education

- not only as regards the mathematical knowledge required to have a competent workforce
- but having in mind the challenges raised by the needs of society in general

Basic "Math for all"

but also more sophisticated competencies

"Equip citizens with the prerequisites needed to involve themselves in issues of immediate societal importance."

(Niss, 2003)

Quantitative literacy is part of SSUMS

5-) Mathematics as an "open" endeavour

Have the students play variations in the "math modes":

- imagination
- creativity
- beauty

  (including the beauty of "strange beasts" such as transfinite numbers, fractals and fractal dimension, …)
- intuition
- rigour
- talk — "talking mathematics", "talking about mathematics"

  communications

The object of mathematical rigor is to sanction and legitimise the conquests of intuition.

(Jacques Hadamard)

IIIb- The environment where SSUMS happens

Quality of the environment for supporting the pedagogy

- *School environment* — OK?!
- *University environment* ??? Need to:
  - View teaching as a true and collective responsibility of the entire math department
  - Make teaching a "public" activity, supported by regular discussion and seminars
  - Have the "rewards system" (promotions, etc.) fully supportive of pedagogy involvement

Comments on successful programmes in various MAA reports

Building bridges

- Between university mathematicians and school teachers
- Between mathematicians, mathematics educators and school teachers
- Between *mathematics* and *mathematics education*

Conclusion

I hope that you have heard the *pulsations* of mathematics…

"Perhaps it comes down to life. Being alive means being sensitive to the challenges in our environment. It means responding to these challenges in a flexible and creative, rather than mechanized, way. When we teach mathematics in the conventional manner, as a linear, hierarchical list of algorithms, we give students the false impression that this is an acceptable way to live, at least in school. It is not. We live in a society that is constantly changing and innovating as we ourselves, teachers and students, change and grow. Mathematics is a discipline that is also constantly growing. As such, a curriculum should be a living object open to experience, change, innovation."

Peter Taylor, *Canadian J. of Math, Science and Technology*, 2000

So many dreams …

… and more!