CULMS Newsletter

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Foreword

This edition of the CULMS newsletter is the first of two planned for 2013, with the second edition to be published in October. This edition, number seven in the series, marks both a change in the editorial team, and a shift from producing the newsletter in print to an electronic format. We should start by acknowledging the substantial work done by Associate Editor Louise Sheryn, in establishing the CULMS newsletter as a highly-regarded forum for the dissemination of contemporary mathematics education research, practice and developments in undergraduate mathematical sciences from around the world. In addition to developing the CULMS newsletter in its current format, Louise has also developed an extensive distribution network for the newsletter and solicited submissions from a wide international audience. I wish to thank Louise for her work, and the wealth of materials she had made available for me to carry on with for future editions.

Secondly, the newsletter will no longer be available in published-print format. Instead, it will be circulated in electronic format to our CULMS distribution list, and will be available for download from our website at:

www.math.auckland.ac.nz/CULMS

The first article in this latest issue introduces a large-scale multi-institutional project investigating undergraduate learning in mathematics, funded by Ako Aoteoroa and led by Bill Barton and Judy Paterson. This article describes the ambitious scope and nature of the project, and encourages readers interested in the project to contribute in the first phase of the project through to July this year. We expect ongoing reports on the outcomes of this study as it progresses in future CULMS newsletters. Two more of the articles in this issue consider issues associated with first-year students, the first by Kathryn Lenz describes extensive and innovative efforts at the University of Minnesota Duluth to place students in appropriate courses according to their needs and backgrounds; the second by Andy Begg questions the current commonly used approach of *streaming* to place students into courses specifically tailored to their needs. He discusses a range of criteria often used to design such courses, and suggests a number of alternative approaches. The final paper, by Tracy Craig and Anita Campbell, describes an Academic Development Model they have used to improve academic support for students in a second-year engineering vector calculus course in Cape Town. They show how this model contrasts with the standard second-year experience, with early indicators suggesting it is effective in improving the academic performance of students who have struggled in the past.

These four articles present a variety of challenges facing practitioners in undergraduate mathematical sciences, and suggest ideas for facing these. We welcome submissions of articles that consider new developments, research and practice in the teaching and learning of undergraduate mathematical sciences, including those that address the transition from secondary to tertiary levels. Please email submissions to the Associate Editor.

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Capturing Undergraduate Learning

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The Holy Grail of educational research is to link student learning to pedagogical interventions. It is an impossible quest because the goal is never clearly defined, and the myriad of variables and external influences mean causal links are obscured. Nevertheless, Sir Galahad pursued the chalice relentlessly, and so must we. We are a group of eager knights who join the quest, learn as we traverse the path, and return with a few treasures, if not the chalice itself.

Undergraduate mathematics pedagogy has evolved through tradition adapted by necessity—for example lecture theatres have become media centres, but not sites of interaction; examinations have been augmented by coursework, but remain the prime criterion for the award of grades. The result is a discontinuity between student outputs sought by lecturers (and, indeed, by employers and the students themselves), and the outputs that we conventionally measure in undergraduate learning (Speer, Smith, & Horvath, 2010).

New Zealand research has explored the edges of this question. In specific contexts for specific subjects questions have been asked about, for example, "soft skills" required in vocational activity (Ferguson, 2010); affective factors (Shepard, 2008); and self-efficacy (Dalgety & Cole, 2006).

The problem in undergraduate mathematics is that we do not have comprehensive tools with which to evaluate student learning against espoused desired outcomes. Learning is assessed in assignments and examinations, where knowledge of content is the main focus. However, students' mathematical thinking, their mathematical processes, their understanding of the field in broad terms, their ability to work together and communicate mathematically, and their attitudes towards the subject, are also regarded as important attributes by lecturers and employers. We do not evaluate the impact of their undergraduate experience on these characteristics, and hence we are in no position to evaluate the impact of course design on students' learning.

We need to create a framework in which the learning goals of undergraduate mathematics courses can be comprehensively specified, and then produce a practical scheme by which we can observe, analyse, and effectively report student learning for a particular course. Such a framework and scheme (which together we call a Course Learning Profile (CLP)) needs to be trialled in a variety of contexts where different outcomes are anticipated, so that it can be tested for robustness, and build its credibility amongst those who design and deliver these courses, and those who receive the graduates. Only then can we claim to have practical research-based evidence for comparing one type of course design against another.

There is some evidence that existing, traditional course delivery approaches need to be modified to achieve some of the learning outcomes desired by employers and graduate courses (Ferguson, 2010; Hart Research, 2010). This is particularly in the case of engineering and mathematical sciences education. Innovations need to be proposed and trialled, and some NZ work has been done, for example, Klymchuk, Zverkova, Gruenwald, and Sauerbier (2008) and Sneddon (2006). But we also need a basis on which to judge the result and adopt, modify or abandon these approaches.

Our Project

At the end of 2012, we secured significant funding from the NZ government through Ako Aotearoa and the Teaching and Learning Research Initiative to investigate undergraduate learning in mathematics. A large, multi-institution and multi-disciplinary team has been formed to investigate the outcomes of undergraduate education—not just the conventional outcomes, but all desired outcomes.

We hope to include outcomes that capture learning of mathematical processes, advanced thinking, habits, and modes; outcomes in the affective domain; and broad outcomes such as those described in university graduate profiles. We will be canvassing lecturers and students, as well as searching the literature (for example, Burton, 2004; Cuoco, Goldenburg, & Mark, 1996; Harel, Seldon, & Seldon, 2006; and Watson & Barton, 2010). In particular we will be asking those who receive mathematics undergraduates, for example lecturers of more advanced courses, lecturers of courses in other subjects requiring mathematics, and prospective employers.

In this way we seek to develop a comprehensive approach to observing undergraduate learning in mathematics. We aim to work at the course level. That is we wish to develop a Learning Profile for particular courses. We hope to be able to catalogue, observe, and report on the outcomes at a course level with sufficient discrimination that we will be able to distinguish different course types or course delivery design. Our expectation is that different courses will have different outcome spectra; Learning Profiles will not be able to be used to evaluate course delivery in an overall fashion.

The Learning Profile of a course will include:

- broad statements of desired learning particular to that course, encompassing content knowledge, mathematical processes, mathematical thinking, affective outcomes, and broad academic characteristics;
- qualitative evidence of learning from groups and individuals;
- quantitative evidence of learning, including conventional assessments.

Part of the project will be to develop three course innovations so that we have some distinctly different courses on which to test the discrimination of the Learning Profiles. Learning profiles will also be developed for parallel existing courses. The innovations are as follows.

- Intensive technology. An entry level course will be taught with intensive use of technology including on-line texts, exercises, and testing; use of web-links during lectures by both lecturer and students; full technology access during final examination; and encouragement for students to use any devices they choose as much as they can.
- Team-based learning. This well-defined teaching technique has a large literature and has been used extensively in several other disciplines. Two 300-level courses (and a graduate course) in mathematics have been trialling the technique for three years (Paterson & Sneddon, 2011).
- Low lecture delivery. Rather than three lectures and one tutorial per week, this delivery design will have one lecture, one tutorial, and a fortnightly Engagement Session where a group of eight students work on an open-ended problem with a lecturer and then individually write a report on their work. Learning of routine facts and skills will be done by students using on-line tutorials and self-monitored with on-line tests (Barton, 2010).

Request for Input

The first phase of our project (to July, 2013) involves collecting and categorising "desired learning outcomes" for undergraduate mathematics.

We invite readers to contribute to our project by contributing their desired outcomes—either from the point of view of a lecturer of undergraduate mathematics courses, or as a receiver of students who have been through undergraduate mathematics courses.

This may be done by email, responding to the questions below, or on-line using the Google Scholar response system at:

https://docs.google.com/forms/d/1mvQCN6Hr_UoInw6v5vAA9q-vbKeecxf46JTH6UKW3oU/viewform

We welcome interest in the project, not only from lecturers in the mathematical sciences, but also from lecturers in other disciplines. A side activity of the project is to work with colleagues in English, Law and Dance, to compare desired learning outcomes for undergraduate courses in our different disciplines, and thereby obtain a broader context within which to examine mathematical outcomes.

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Entry-level Mathematics Placement Study at the University of Minnesota Duluth

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The purposes of this study were to identify factors available within UMD student information records that correlated with subsequent student success in entry-level university mathematics courses and to give recommendations for using these factors to determine math course placement for matriculating students, minimal prerequisites for the courses, and student advisement guidelines. Logistic models of student-success were constructed and used to create an Excel advisement tool. Placement requirements based on a combination of high school mathematics grade and standardized test score were recommended to UMD administrators. Subsequent changes to standardized test score prerequisites were implemented, but without the high school grade requirements. A follow-up extension investigated model-predicted effects of the implemented changes on course passing rates. Both the primary and follow-up investigations included undergraduate student research.

Introduction

The University of Minnesota Duluth (UMD) strives to be accessible to a wide range of high-school graduates while also increasing student retention rates, increasing four-year graduation rates, maintaining high-quality programmes, and producing well educated graduates. To further these goals, UMD expends much effort in: curriculum development, faculty development, research and internship opportunities for students, improvements to student support services and updating advisement resources and practices. The results of these efforts can be compromised by inaccurate placement of matriculating students into entry-level mathematics courses.

Success in university mathematics is necessary for achievement in STEM fields. At UMD, most science and engineering bachelors programmes require at least a year of university-level, single-variable Calculus as prerequisite for further course work in the major discipline. The standard first year of mathematics for these students comprises Calculus 1 and Calculus 2. However, students who are not prepared for Calculus 1 may take the prerequisite course Precalculus Analysis in their first semester. Or they could start another semester further back in College Algebra and then take Precalculus the following semester. These options can present a dilemma because if a student starts with a mathematics course that is either too elementary or too advanced, then his/her progress toward graduation can be delayed or even derailed.

Can we raise passing rates from 75% to 85% by improving course placement?

Course failure wastes money, time and effort and adversely affects student retention, class size and morale. In 2009, UMD administrators had determined that in some UMD first year mathematics courses the percentages of students who received at least a C were around 75%. Both administrators and instructors wanted these percentages to rise, but not at the expense of student learning.

Potentially, students' high school records, national university entrance exams, in-house university placement exams and academic advisement can all be used for course placement. High school courses and grades give an indication of matriculating students' educational background and performance records. However, non-uniformity in curricular offerings and grade inflation among high schools blunt the precision with which this information can be interpreted.

In the United States, Midwestern university applications generally require students' scores on the ACT standardized entrance exam. Many students also take standardized advanced placement (AP) exams. See information about the ACT at www.act.org, and the AP exams at www.collegeboard.org. At UMD, current placement based on AP Calculus exam scores has been reliable. However, we have not found the ACT math component score (ACT_M) to be as definitive. Prior to 2006, in addition to ACT_M, UMD used in-house mathematics placement exams. However, these were expensive to administer and their usefulness was undermined by faculty advisers and students who ignored the placement exams results.

Since 2006 UMD has been used ACT_M alone, or in combination with individual-student advisement, to place matriculating students into their first mathematics courses. In 2009, UMD's Swenson College of Science and Engineering (SCSE) administrators wondering if raising the ACT_M requirements for entry-level mathematics courses would improve passing rates. UMD statistics professor Kang James, undergraduate mathematics and statistics major Jenalyn Wright and I discussed the problem with SCSE associate dean Penelope Morton and Janny Walker, head of SCSE's advisement office.. Their input and aide were essential to this study.

There were two undergraduate research students participating in this study, Jenalyn Wright in 2010 and Aaron Shepanik in 2012. Ms. Wright's research work began in the spring semester of 2010 with her Undergraduate Research Opportunity Program (UROP) project, funded by the University of Minnesota and co-advised by Kang James and myself. At that time, Ms. Wright was a senior undergraduate who had earned top marks in discrete mathematics, abstract algebra, real analysis, probability theory, modelling, and regression. She also had prior experience using software packages Excel [office.microsoft.com/en-nz/excel/] and SAS [http://www.sas.com/].

Ms. Wright worked with three semesters of UMD student information records (Fall 2008, Spring 2009 and Fall 2009), stripped of student identifier information. For each of the UMD courses College Algebra (Algebra), Precalculus Analysis (Precalc), Finite Mathematics with Introduction to Calculus (Finite Math) and Calculus 1 (Calc 1) she investigated the relationship between a matriculating student's ACT_M and his/her passing the course. Passing meant receiving a C or better in the course and was represented in our statistical analysis as a binary variable y that was 0 for not passing and 1 for passing. ACT_M was an integer x between 0 and 36, with increasing ACT_M supposedly corresponding to increasing mastery of high school mathematics skills.

Ms. Wright was unable to find a satisfactory linear regression or logistic model fit for the probability of passing based on ACT_M for any of the four courses considered. However, she was able to model the probability of passing Algebra, finite math, and Calc 1 as logistic functions of both ACT_M and gender. For each course, the associated chi-squared statistics indicated that both ACT_M and gender were strongly predictive of the probability of passing. She did not, however, find a good model for the probability of passing Precalc.

Based on her model fits and associated data summaries, we determined minimum values of ACT_M for which a student, depending on gender, had an 85% probability of passing Algebra, finite math, or Calc 1. These are given in Table 1, with a few additional points for comparison. Note that for each course, a male student would need an ACT_M at least four points higher than a female student in order to have an 85% chance of passing!

Table 1. Model-predicted Minimum ACTM Math Scores, by Gender, for 85% Probability of Passing

UMD course	Female ACT_M (chance of passing)	Male ACT_M (chance of passing)
Algebra	23-24 (84-86%)	28 (85%); 23(74%)
Finite Math	23 (85%)	29 (85%); 27 (80%)
Precalc	No score for > 81%	No score for > 70%
Calc 1	26-27 (84-86%)	31 (85%); 26 (69%)

Although these results were intriguing, we did not recommend that gender to be one of the criteria used for course placement because we suspected that gender was surrogate for some more actionable factor. Instead we obtained SCSE funds for investigating what other variable(s) in student records could be used along with ACT_M to predict passing rates.

Following up on spring 2010 work, in the summer of 2010 we obtained UMD student information records data for students who had taken Algebra, Finite Math, Precalc or Calc1in fall, 2006 through spring 2010. This data, scrubbed of all student identifiers, included gender, ACT composite score and subject sub scores, high school grade point average, high school and university math courses taken and grades, college of enrolment, and major field of study.

We used data from four semesters, Fall 2008 through Spring 2010, to identify factors correlating with students' achievement for each of three different measures of success. These were achieving at least a C in the course (C success), achieving at least a B- (B- success) and achieving at least a B (B success). For each course Ms. Wright found that ACT_M and high school math grade (HS_M) could predict each of the three measures of success, except for C success in Precalc. She also found that ACT_M alone could predict B- and B success for all four courses and that HS_M alone could predict C success in Precalc. It was curious that ACT_M was not significant for C success in Precalc, while it was significant for all other success/course combinations considered. We also determined that several other variables were not significant in combination with ACT_M including ACT English subject score, whether the course was taken in a spring or fall semester, the student's college, and the student's major.

Probability of Success Based on ACT_M and HS_M

In order to consider high school math grade as a second independent variable for modelling probability of success, we placed letter grades into bins and used grade-bin number (HS_M) as a numeric variable. For models based on both ACT_M and HS_M, the high school math letter-grade bins were {A, A-}, {B+, B, B-}, and{C+ or less} and were numbered 2, 1, and 0 respectively. Ms. Wright used SAS and data of the form (x_1, z_1, y_1),...,(x_n, z_n, y_n), where x_i was ACT_M for the ith student, z_i was HS_M for the ith student and y_i was 1 (success) or 0 (no success). The models for probability of success, p, were of the form

$p = \frac{1}{1+e^{-(a+bx_1+cz_1)}}$, with regression-specified parameters a, b and c.

Such models were fit for each of the three success measures and for each of the four courses. For each model except the one for C success in Precalc, the associated chi-squared statistics indicated that both ACT_M and HS_M were strongly predictive. For the model of C success in Precalc, only HS_M was predictive.

We constructed an Excel advisement tool that incorporated all twelve models of success. The tool is a dynamic alternative to using the models' graphs or look-up tables. The models are hidden behind a user interface that provides the probability of C success, B- success and B success for each of Calc 1, finite, Precalc and Algebra based on user-provided ACT_M and high school math grade selected from draw-down menus. We gave this tool to the SCSE advisement office for use as an aide when discussing a student's mathematics preparation and placement.

Will Raising ACT_M Prerequisites Alone Raise Passing Rates to 85%?

Starting in the fall semester of 2012, UMD raised prerequisite minimal ACT_M for each course as shown in Table 2. A prerequisite high school math course grade was not added, even though our study indicated that this was highly significant. Table 2 lists ACT_M and HS_M combinations for which our models predicted at least 85% probability of C success, at least 70% probability of B- success and at least 65% probability of B success. Our findings indicate that UMD's new ACT_M prerequisites align with these target goals for students with an HS_M A but are too low for students with an HS_M B. We found that no ACT_M suffices for these high probabilities of success for HS_M C.

Table 2. Changes in Prerequisite Minimum ACT_M Implemented for Fall, 2012, Compared to Recommended Prerequisites.

Course placement	Calculus I	Finite Math & Intro. Calc.	Precalculus Analysis	College Algebra
implemented ACT_M	from 25 to 27	from 23 to 24	from 22 to 24	from 18 to 21
recommended ACT_M if HS_M A	27	24	27	20
recommended ACT_M if HS_M B	33	31	none	26

In the summer of 2012 we investigated whether the new ACT_M prerequisites would be sufficient to raise course passing rates to 85%. Undergraduate research assistant Aaron Shepanik, whose salary was funded through a UMD Chancellor's small grant, participated in this part of the study. At the time, Mr. Shepanik had experience with Excel and had earned top marks in discrete mathematics, multi-variable Calculus, and introductory probability and statistics. However, he had not yet taken any senior-level courses in mathematics or statistics.

Mr. Shepanik organized the data into three groupings of four semesters each: Fall 2006-Spring 2008, Fall 2008-Spring 2010 and Fall 2010-Spring 2012. For each model except the one for Precalc C success, which did not depend on ACT_M, he compared the model's probability of success with the actual percent of success in the data at each ACT_M value. His graph in Figure 2 for Fall 2010 – Spring 2012 Calculus 1 data was typical. Note that for each ACT_M value shown, the passing rates for students with HS_M A were significantly higher than the passing rates for all students with that ACT_M.

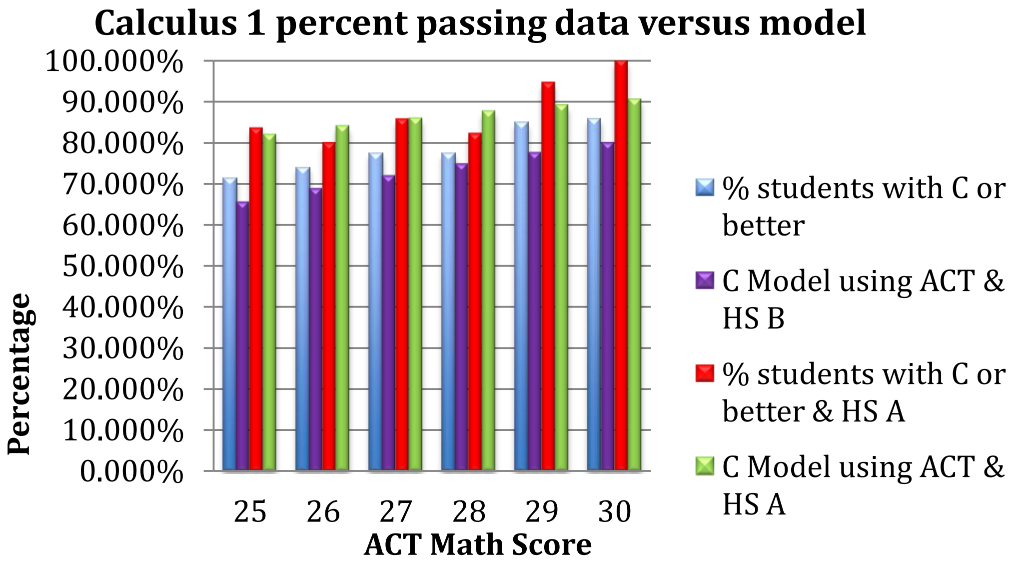

Figure 2. Comparison of C success data with C success model-predictions at each ACT_M from 25 to 30.

While Figure 2 shows the information important for advising individual students, from an administrative perspective we also want to focus on course-level passing rates. We investigated these based on the ACT_M prerequisites in place from fall, 2006, through spring, 2012 as well as the ACT_M prerequisites that would be in place starting in fall, 2012. For each ACT_M value at or above the course prerequisite value, Mr. Shepanik multiplied the number of students with that ACT_M by the model's probability of success at that ACT_M. The sum of these products divided by the number of all students within the ACT_M range was the model's predicted success rate for the course. We compared these predicted success rates with the corresponding actual success rates, as shown in Figure 3.

The graphs in Figure 3 for Fall 2010–Spring 2012 Calc 1 data are typical of what Mr. Shepanik found. These graphs indicate that raising the ACT_M prerequisites to their new values will raise course passing rates but not to the 85% target rate. The graphs also indicate that requiring HS_M A would raise the passing rates to at least 85% even with the current ACT_M prerequisites. Thus, if entry-level math course passing rates do not improve sufficiently with the new ACT_M prerequisites our findings can be used to argue for additional preparatory requirements or intervention for matriculation students with an HS_M of B or less.

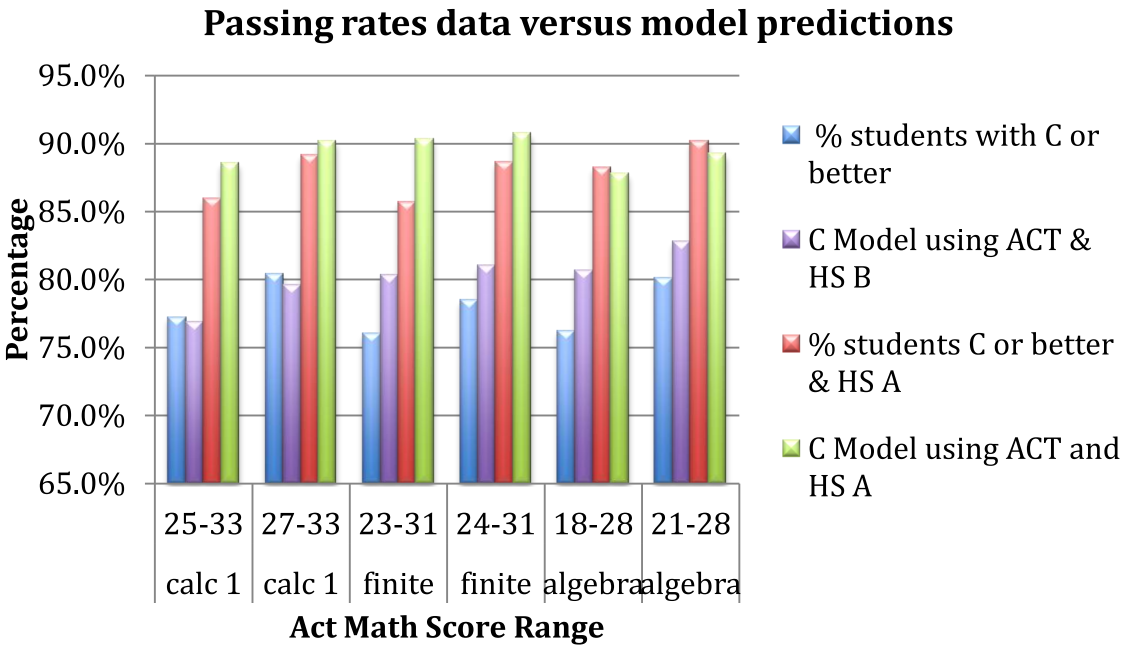

Figure 3. Comparison of data with modelled C success rates for Calc 1, finite, and Algebra from fall, 2010 through spring, 2012, based on current and future ACT_M prerequisites.

Discussion

Studies similar to the one reported here can be conducted at other institutions. Investigations can focus on modelling student success as in this study or could include variations such as the following. The student-population could be broken down demographically. Or the focus could be on monitoring ongoing student success in entry level mathematics courses given evolving factors regarding mathematics education, standardized exams, student population profiles, advisement practices and university student support services. Or the focus could be on the success of university students progressing through a sequence of mathematics courses or progressing from a mathematics course to a subsequent science or engineering course.

Universities keep extensive student-records data bases and analytics software packages are becoming more broadly accessible. As a result, administration staff monitoring the health of their institution's degree programmes do not seek the involvement of mathematics and statistics faculty in order to compute passing rates and other statistics for university courses. However, mathematics and statistics faculty involvement in such analysis is important for influencing the queries made, the directions that investigations take, and the subsequent conclusions drawn.

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Mathematics—Bridging, 101, 201, or What?

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Many universities have introduced mathematics bridging courses to enable students who have not yet reached an 'appropriate' standard to study mathematics at university. This paper questions whether streaming (Bridging mathematics, Math 101, Math 201, …) is the best approach to this issue, and suggests other ways in which this situation might be approached.

Introduction

Traditionally universities admitted a cohort of mathematically-able students into first-year mathematics, but times have changed. Now prospective mathematics students display a much broader spectrum of abilities. In response to this change many universities have introduced bridging courses to either prepare less-able students for mathematics classes or to help them arrive at the conclusion that they are inadequately equipped to cope with the mathematical study.

This 'streaming' approach seems flawed, research does not support it, and I wonder what the fundamental problem is. Some questions that have arisen for me are:

- Is this problem of readiness for university study unique to mathematics?
- Is this a staff or a student problem?
- Is the issue caused by the desire to maintain standards?
- Does the problem relate to the curriculum content of mathematics?
- Is the issue related to the pedagogy employed?
- Is the problem exacerbated by Math 101 or by bridging courses?

While the problem could well result from a combination of causes, I will explore these six issues separately.

Issues

Readiness?

There is no doubt that universities need to cope today with a much broader ability group than they did in the past. The increased percentage of the population attending university means a broader range of abilities; and the social and cultural backgrounds are now more varied than they were in the past. Additionally, many vocational training courses have become university courses rather than technical institute or community college courses, the percentage of students leaving school and wishing to enter universities has increased significantly, and many students were used to being streamed at high school. These factors mean that university teachers in nearly all subjects face an increasingly diverse range of entry-abilities.

Who's problem?

Streaming was a solution designed by staff who were genuinely concerned about standards, but in my opinion, they have not considered alternative solutions. It becomes the student's problem when they are either not able to take mathematics courses or when they are forced to do non-credit bridging courses and/or spend an extra year reaching the level they wish to reach.

Maintaining standards?

While standards are often cited as the main reason for streaming and excluding students, limiting an intake to only more able students is hardly an academically honest way to maintain standards.

Content?

I believe that the content of mathematics courses is not the problem. Mathematics is an academic subject, but that does not imply that it must be taught in a very formal way. Mathematics is a subject where all content can be approached in many different ways. It seems to me that mathematics lecturers are too often concerned only with theoretical results rather than the 'discovery' process that mathematicians went through to reach these results.

Pedagogy?

Universities have a philosophy of enquiry/research but in mathematics (and other subjects) this is not usually part of the approach taken with undergraduate courses. Too often lecturers teach as they were taught; which reflects the lack of tertiary tutor/teacher/lecturer education provided by universities.

Multiple options?

My belief is that while universities exclude students with 'inadequate' background in the subject, and offer bridging courses, normal courses and extension courses, they are not taking their educational responsibilities seriously. They may blame high school teachers but if they listened to their own students who did not carry on with mathematics, they might rethink the blame game. All teachers need to remember the mantra, *"Start where the learner is,"* and this for me suggests approaching the subject from an alternative perspective when students are all at different starting points.

Other subjects?

While some university courses restrict entry because of imposed financial restraints, it is interesting to note that many courses accept students whether or not they have previously studied the topic. They seem to believe that students coming into their courses can be accommodated and want to study the subject; and in asking myself, 'what, if anything, is different with mathematics?' I have come to the conclusion, 'very little!'

Concluding issue

I believe that streaming, in schools and universities exacerbates, rather than solves, the challenge caused by the diversity of students' mathematical backgrounds. I believe that all subjects need to be approached in ways that allow all students to increase their understanding of the topics within the subject; and that as university teachers we have the responsibility to facilitate learning rather than lecture.

Thinking About Mathematics Differently

Mathematics education involves both curriculum foci and pedagogical approaches. I will now briefly describe some assumptions I am making, some axioms I think are important, and a number of alternative approaches that may help address the issue.

With each of these alternatives I make the same assumptions, these are:

- that in every class students will have a broad range of relevant prior knowledge
- that every topic is capable of extension in terms of breadth and depth
- that technology has its place, but does not replace understanding
- that the focus should be on learning rather than teaching
- that pure and applied content, and mathematical thinking are all important.

I accept that we cannot start with a clean slate. We, both teachers and students, have been conditioned by years of experience in education from which we have constructed our understanding of mathematics, and of how learning occurs. However, if things are to improve then we need to rethink our fundamental assumptions and begin to experiment with different ways of working. As mathematicians we are usually quite logical so perhaps we need to consider our educational axioms.

Some Mathematics Education Axioms

Axiom 0:

University (and pre-university) education is primarily concerned with filling the heads of learners with the accumulated knowledge of society.

> *But, this notion is no longer relevant (if it ever was) in our society where so much knowledge is readily available through the available technologies. Thus, from my perspective, axiom 0 is no longer relevant.*

Axiom 1:

That education (at schools and undergraduate levels) should focus on thinking, enquiry and research.

> *This fits with the school-level curriculum documents that emphasize thinking, and with universities that emphasize research. However, in my experience I have found that while many teachers pay lip-service to this, it hardly influences their approaches to their work.*

Axiom 2:

Mathematics is often thought of as a logically-structured subject, but this is not how the subject is learnt by most students.

> *The logical structure reflects how mathematicians have formally repackaged what they have come to know. Learning is a messy process in which ideas are slowly constructed in rather haphazard ways and then linked together as a network of ideas as the learner comes to see links between the elements. The formal logical organization of knowledge only makes sense after experience is gained as the learner 'plays with' and reorganizes the discrete elements of their learning.*

Axiom 3:

Learning mathematics is usually assumed to be a rational (logical) thinking process, but this is only one of many relevant forms of thinking.

Other forms of thinking required within mathematics education are:

- *concrete and abstract thinking*
- *generalizing and specializing*
- *critical thinking (in particular, making explicit all assumptions made)*
- *narrative thinking (talking about ideas)*
- *contextual thinking (relating examples/applications to theories)*
- *visual thinking (geometric and graphical visualization)*
- *creative thinking (imagining and considering alternatives)*
- *relational thinking (seeing connections, both causal and random)*
- *conceptual thinking (thinking big/thinking small)*
- *symbolic thinking (understanding symbol use)*
- *interrogative thinking (posing and solving problems)*
- *modelling thinking (multiple representations—words, symbols, models, …)*
- *intuitive thinking (resting with an idea)*
- *meta-cognitive thinking (monitoring one's thinking)*
- *ethical and caring thinking (related to others and to society).*

Alternative Approaches

For me there are at least six alternatives to our traditional way of approaching first-year mathematics classes at university that deserve consideration; no doubt others also exist, and these six can be combined in various ways. They are:

integrated	*relational*	*visual*
applied	*multidimensional*	*technological*

1. Integrated approach

Is mathematics a series of disconnected topics (arithmetic, algebra, geometry, trigonometry, calculus, statistics, and so on); or is it a unified subject which these six topics forming parts of the subject? It seems to me that the current topic structure that many universities use splits mathematics into 'silos' rather than emphasizing the connections between topics and a unified view of mathematics.

For example, when one discusses $y = x^2$ in algebra, the parabolic shape of the graph is mentioned but its geometric connection as a locus related to the ellipse and hyperbola is rarely touched upon, applications (such as projectiles) are hardly ever mentioned, and graphical solutions to problems are often ignored in favour of algebraic/symbolic approaches. The student learns that the parabola is simply an algebraic graph rather than something that is useful, can be defined as a locus, can be extended to more than two dimensions, and has many interesting applications.

There seem to be three important foci in mathematics—content, processes, and thinking; but most of the emphasis is on content to be learnt. The mathematical processes which were introduced by various reports on school mathematics education (e.g., National Council of Teachers of Mathematics, 1989) have listed the following as 'mathematical processes' that supplement the traditional content:

- reasoning and logic,
- problem solving (including modelling, investigating, and problem posing)
- communicating
- making connections
- using tools (technology).

Mathematical thinking has been discussed by writers such as Mason, Burton & Stacey (1982) and while I believe they do not include enough forms of thinking, they certainly push far beyond what I have observed both in first-year mathematics classes at university and in the final years of high school. For many mathematics teachers it seems that thinking only means logical reasoning and proof; problem solving is often limited to theoretical problems and trivial applications rather than modelling or investigation tasks; using tools only implies calculators and computers; and both communicating and connecting are totally ignored.

Similar deficiencies occur with geometric topics, and geometry is not even taught in some first-year university courses. The typical approach involves coordinates and sometimes transformations, but hardly ever compares these with the historic Euclidean approach or with a more practical technical drawing approach. In addition, geometry is usually limited to two dimensions with many students and teachers not even appreciating the properties of a parallelepiped or a spherical triangle.

To confirm the value of using an integrated approach within mathematics education I suggest one draws two Venn diagrams, showing three intersecting sets {number, algebra, and calculus}, {geometry, and trigonometry}, and {statistics and probability} and then listing the skills needed within each. Secondly, on another Venn diagram with the same three sets, list the mathematical process and mathematical thinking skills within the appropriate areas of the diagram. I believe that the most densely-packed area of each Venn diagram will be where the three sets intersect.

2. Relational approach

An alternative approach to mathematics involves what I see as the fundamental concept of mathematics—namely, the notion of a 'relation', or, 'a set of ordered pairs'. It was not until I read the textbook *"Mathématique moderne 1"* (Papy, 1963) that I saw the importance and unifying nature of relations. After this, in discussion with colleagues and the Maori Language Commission, when asked to say what mathematics was, we were able to say 'the study of relations', and hence the Maori name for the subject, *pāngarau* (*pānga* meaning relationships or connections, and *rau* meaning 100 or many).

Many teachers assume relations are mainly algebraic sets such as:

- "has as square" = $\{(x, y): y = x^2\} = \{\ldots, ({}^-1, 1), (0,0), (1,1), (2,4), (3,9), \ldots\}$

but relations involve not only algebra, but also:

- arithmetic e.g., "+" = $\{((x, y), z): z = x + y$, and x, y, z are numbers$\}$
- geometry e.g., "reflection in y-axis" = $\{((x, y), ({}^-x, y))$, for all x and y$\}$
- trigonometry e.g., $\{(x, \sin x): -180^\circ < x < 540^\circ\}$
- calculus e.g., $\{f(x), f'(x): f'(x) = d(f(x)/dx)\}$
- probability e.g., {(E, P(E)): P(E) is the probability of event E}

One advantage of a relational approach is that so much can be done in a simple way with arrow graphs which appeals to students' visual thinking and provides simple concrete examples. In my own education I found university mathematics was taught rather abstractly, and the following two incidents with similar lessons from my own experience suggest that an abstract approach is not always the most sensible.

Incident 1, Third-year University:

In an abstract algebra lecture in third-year university pure mathematics many years ago; we were taught about reflexive, symmetric and transitive properties of relations on sets of elements, and that if a relation had these three properties then it was an equivalence relations, and partitioned the set. This was done with a blackboard full of symbols that I still vaguely recollect as:

Consider relation *R* defined on set *S*,

(i) For all *x* belonging to *S*, if (*x*, *x*) belongs to *R*, then *R* is reflexive.
(ii) For all *x* and *y* belonging to *S*, if for all (*x*, *y*) belonging to *R*, (*y*, *x*) also belongs to *R*, then *R* is symmetric.
(iii) For all *x*, *y* and *z* belonging to *S*, if for all (*x*, *y*) and (*y*, *z*) both belonging to *R*, (*x*, *z*) also belongs to *R*, then *R* is transitive.

Then: If *R* is reflexive, symmetric and transitive
then *R* is an equivalence relation;
and R partitions S.

I was able to parrot back these definitions of the reflexive, symmetric, and transitive properties and the fact that all three implied equivalence and partitioning, but I had virtually no idea what the lecture was about.

Incident 2, Four years later with 14-year old students:

I was teaching a gifted (yes, streamed) high-school class of year-10 students and was using the textbook *Mathématique moderne I* (Papy, 1963). Inspired by this text I had students consider a set of twelve students and a number of relations such as:

- ‘is the same age as’, ‘is the same gender as’, ‘is the brother of’,
- ‘has the same parent as’, ‘is taller than,’ ‘weighs more than’

I put a chart on the board listing the details of 12 students including their names, ages, gender, heights, and weights, The students drew six Venn diagram with each student represented by a labelled point then drew an arrow graph for each relation.

All the students quickly found what an arrow graph looked like if the relation was reflexive, symmetric, and transitive; and the notions of equivalence and partitioning were immediately obvious.

From my perspective, no matter how capable students are, most topics can be approached in ways that use thinking that is concrete (rather than abstract) and visual (rather than symbolic). However, in my experience, apart from the increased use of technology, many university teachers still teach as they were taught.

3. Visual approach

The saying, “a picture is worth a thousand words’ has much to commend it. How often do we use pictures (diagrams, graphs, flowcharts, photographs, and so on) in mathematics? And how often do we suggest to students that they draw pictures whenever they can and in particular to explain their arguments. Visual thinking is an important component of mathematical thinking but seems often to be ignored.

In geometry we usually draw a diagram, and in algebra we may draw Venn diagrams and/or Cartesian graphs, but we hardly ever use arrow graphs, and many students have difficulty with three-dimensional diagrams be they graphs or simple 3-d solids. Perhaps they need to read “*Flatland: a romance of many dimensions*” (Abbott, 1884) to understand the problem that two-dimensional beings have with three-dimensional thinking and to understand their own difficulties with three and higher dimensions.

It seems to me that mathematics requires visual and graphical literacy from all our students, and I remember hearing, perhaps apocryphally, of an exasperated geometry lecturer who himself was blind, saying to the students, "*the trouble is you just can't see*", then after a hushed silence, "*well, what does a sphere look like, from the inside?*"

4. Applied or Modelling approach

Is the pure/applied split in mathematics desirable? I believe that most students studying mathematics are doing so because they will use it in their future work and only a small number are studying it for its own sake; thus for motivation, applications of topics need to be made explicit. I believe that students who have a 'pure' perspective of mathematics will not be deterred by applications, rather, they will have concrete examples which can be visualized easily and make the mathematics seem more useful. (However, I do remember an analysis lecturer who, when told of an application of a topic he had just taught, walked away mumbling that he would have to change this topic in his 'pure' mathematics course.)

Most of us find it easier to visualize ideas when applications or modelling situations are provided instead of merely abstract or theoretical justifications—thus for pedagogical reasons, this approach is desirable. But, it can create problems, when teaching calculus the commerce students do not want engineering problems and vice-versa; so ideally most practical situations will be chosen with either very high or very general interest. I would suggest that in first and second year mathematics virtually every concept should be introduced in a concrete way—applications should not be add-ons, but rather, real situations that help understanding by providing a contextual frame for the concept.

Working at one university I was aware of the calculus lecturer carefully choosing both a commercial and an engineering example using the same (or at least very similar) differential equations to provide real-world situations that resonated with the two largest groups of students in the class. However, in many classrooms we have more than two special interest groups and thus need to find applications involving very general situations that appeal to nearly all students.

5. Multidimensional approach

Mathematics in many high schools is a rather 'flat' subject and visualizing in three dimensions is a challenge for many students. However, engineering, architectural, and numerous other students need this capacity in three dimensions, and in algebraic programming *n*-dimensional thinking is needed.

Teaching elementary topics in 3-dimensions provides excellent revision for prior work in 2-dimensions and the challenge of broadening their thinking means that the concepts do not have to be more difficult, the context will be the challenge. For example, what is the gradient of a plane? Or, what is the angle sum for a spherical triangle?

6. Technological approach

There is no doubt that technology, computers and calculators, are part of our students' worlds, but technology is only a tool—a text-book substitute, a lecturer alternative, and a tool for doing mathematics; however, technology does not guarantee that learning occurs. Basic concepts still need to be understood, thus, while technology may take the pain out of many complex tasks, for me it implies that teaching and learning needs to be more conceptually based.

Consequently, when using technology within mathematics education one must carefully discriminate between conceptual understanding and procedural understanding, and the assessment of students needs to balance these two aspects of understanding.

Conclusion

I believe that a curriculum review needs much more than ‘shifting the deck-chairs on the Titanic’; we need to have a major rethink of our curriculum and how it is taught and learnt.

I have not attempted to suggest with this paper that there is one best way to move forward, perhaps the revolution might involve more than one of my suggestions, but I am hopeful that I might have stimulated some people to look at the structural problems as we get closer to the icebergs!

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Vector Calculus for Engineers – The Academic Development Model

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We describe a model for a second year engineering vector calculus course designed to improve the support for students in an academic support programme where the historical pass rate for these students is typically below 50%. The model adopts some features of the support given to these students across multiple subjects in their first year. Academic development is described in the South African context. We describe the support experienced by our students in their first year, the contrast with the standard second-year experience and how our model aims to smooth the transition. Early indicators suggest the model is effective.

Introduction

The current and expected demand for engineers in South Africa has put pressure on universities to increase the number of engineering graduates. The University of Cape Town has chosen to achieve this goal not simply by accepting more students into their degree programmes but by improving teaching and learning so that a greater proportion of students graduate. A problematic, compulsory second-year course is Vector Calculus for Engineers, with an average pass rate over the past three years of only 62%. However, students in this course from a reduced-pace academic support programme have pass rates typically 20% lower. The relatively lower pass rate of this particular cohort of students motivated the model described in this paper, which is aimed at the academic development students.

To give some background to the development of the model, we first describe the existing academic support structures in the Faculty of Engineering and the Built Environment at the University of Cape Town and then more specifically in the Academic Support Programme for Engineering in Cape Town (ASPECT). We then describe the traditional vector calculus course and the proposed academic development model of the same course. We conclude by making some early comparisons between students' performances in the traditional course and the academic development course.

The Academic Development Context

The University of Cape Town, South Africa, offers so-called academic development programmes in all faculties except Law. The principle aim of the programmes is to offer access to tertiary studies to students from a previously disadvantaged educational background and thereby play a role in national and institutional transformation. South Africa's apartheid history left generations of coloured, African and Indian students without access to adequate education. While the country tries to reform the school system, thousands of potential university students would be denied access to tertiary institutions unless efforts are made to offer redress (Scott, Yeld & Hendry, 2007).

In the Faculty of Engineering and the Built Environment (EBE) the academic development programme (ASPECT) offers an extended curriculum programme. The students who are accepted to ASPECT narrowly missed the points requirement for entry into mainstream engineering; however they have excellent mathematics and science results and their educational records suggest they might be able to succeed at university. For many of these students, immediate placement into mainstream studies would not be advisable as they struggle with the simultaneous challenges of language, work load, cognitive demand as well as having to deal with cultural and social dislocation (Kapp et al, forthcoming; Craig, 2011). In the ASPECT model, students are allowed to complete their first two years' of study over three academic years. The subjects completed are identical to those in mainstream studies apart from a small entry-level support course in communication – there are no bridging courses – the load is simply somewhat lighter (Pearce, Campbell, Craig, le Roux, Nathoo and Vicatos, 2012).

Until 2013, ASPECT academic support was only offered in first-year, thereafter the students joined much larger mainstream classes. In 2013 we are investigating the worth of extending academic support to the first semester of second-year mathematics, in an effort to alleviate the shock of the transition from first to second year (Pearce et al., 2012). In their first year, the students receive extensive support which is traditionally not extended to any of their second-year courses.

ASPECT First-year Mathematics Support

First-year ASPECT students enrol for three and a quarter courses: mathematics, either physics or chemistry, an introductory engineering course and a quarter course on communication. This is in contrast to their mainstream colleagues who mostly enrol for six courses in their first year, for instance mathematics, physics, chemistry, engineering drawing, statics and an introductory engineering course. (This stated curriculum does differ across engineering departments). ASPECT lecturers teach the mathematics, physics and communication. In mathematics, the syllabus is identical to the mainstream course; however the assessment differs to allow for different timing, a larger number of smaller assessments, and a variety of assessment tasks. The differences between ASPECT and mainstream are:

- Double period lectures (105 minutes) instead of single periods (45 minutes)
- More lecture times per week
- 3 hour tutorials instead of 2 hours (in mathematics)
- Greater number of hours of lecturer availability
- Additional resources
- Regular and varied assessment tasks with fast feedback
- A small class (60-90 students) instead of mainstream's 450-500 students split across three venues and three lecturers.

The students in general do well in first-year ASPECT mathematics. The pass rates are high (typically between 75% - 90%), as are student approval ratings. We strive to maintain a sense of community, with a community outing once a year, a group photo, an indispensable mother-figure secretary and continual positive reinforcement from all the staff.

Vector Calculus

It is from this supportive atmosphere that the ASPECT students would enter second-year mathematics with the mainstream students. The first semester covers vector calculus, a course which is notoriously challenging. The students enter the course already intimidated by its reputation, the number of classroom contact hours decreases from 11 to 6, the lecturers teach multiple courses and unavoidably have limited consultation time, the resources are limited, the classes are large, the assessment tasks are few, lengthy and difficult. The pass rates for vector calculus are low for the entire cohort but even lower for the ASPECT students (see Table 1).

Table 1. Pass Rates in Vector Calculus

	Pass rate in %	
	Entire cohort (*N*)	ASPECT (*N*)
2010 semester 1	67.7 (589)	41.4 (99)
2010 semester 2	54.9 (226)	42.9 (56)
2011 semester 1	73.1 (465)	53.4 (58)
2011 semester 2	44 (302)	38.7 (62)
2012 semester 1	71 (594)	51.6 (126)
2012 semester 2	60.9 (233)	46.9 (96)

The issue of raising the pass rate in vector calculus overall is a topic of much animated discussion in the halls of our institution, but the issue of how to raise the pass rate of ASPECT students is what concerns us here. The course is challenging for any student. The shift from single variable to multivariate calculus is more than simply a matter of the symbolic demand of calculus with more variables. Worsley, Bulmer and O'Brien (2008, p. 143) suggest that multiple integration can be seen as a 'threshold concept' (Meyer and Land, 2006) with specific areas of difficulty identified as the changing between coordinate systems, working out limits of integration and changing the order of integration. Hesterman, Male and Baillie (2011, p. 622) describe troublesome topics in vector calculus as *conceptually difficult,* for example understanding the distinction between vectors and scalars and between velocity and acceleration, and *alien,* for example using the axis of rotation to represent angular motion, understanding the physical interpretation of dot and cross products, understanding that a vector with constant magnitude can have a non-zero derivative, and using different notations for vectors interchangeably. In our experience, the demands of 3-dimensional visualisation and the many conceptual challenges around continuity and differentiability in these contexts challenge all students.

Do ASPECT students, with their often weaker school grounding, find greater trouble with these visual and conceptual contexts? Or is it more a case of having had the rug pulled out from under them with the shift from extensive academic support to very little? In 2013 we are implementing and investigating whether continuing ASPECT involvement into the first semester of second year might increase the success of ASPECT students in the course.

The Vector Calculus Academic Development Model

It is neither possible not advisable to mimic the first-year support in second year. The students' timetables do not allow for double periods, nor for much increase of the number of lectures. In addition, the students need to adjust to the demands of mainstream tertiary study, the provisions made in their first-year having been always planned to be decreased in their second year when they found their feet. The idea is to make the shift from first-year to second-year mathematics less like a step function and more like a steep curve.

The ASPECT vector calculus timetable has more classroom contact hours than mainstream vector calculus: 5 single lecture periods per week instead of 4, and 3 hours of afternoon workshop instead of 2. In addition, we have small tutorials dotted throughout the week for students to complete assignments or to ask general work-related questions. Because the ASPECT lecturer is dedicated to this course alone, rather than the two or three courses usually taught (and prepared for, and marked, and so on) by the three mainstream lecturers, more “open door” time is possible for students to ask for help. The class is small (85 students) in contrast to mainstream’s 470 students (divided in to three classes of approximately equal size). A small amount of time is available during class time for the students to work interactively, rather than more passively observe the lecturer. Beyond that, the courses are identical. All lecturers work from the same notes, to the same timetable. The weekly tutorial assignments and homework are the same and the assessment tasks are the same.

Discussion and Conclusions

What causes the ASPECT students to fare so much worse than their so-called mainstream counterparts? ASPECT students, on the whole, have weaker university entry level results than mainstream students, but not in mathematics and science. The school-leaving mathematics and science results of ASPECT students are comparable with those of the majority of mainstream students. The ASPECT model is predicated on the understanding that students with school-leaving results necessary for success in engineering studies, but coming from backgrounds with historically weak support for the demands of an English language institute of higher education, will achieve success if supported through initial cultural, language and academic literacy difficulties. Some students fail to thrive in the ASPECT environment, but the majority do well. Is the poor performance in vector calculus due to these students lacking the mathematical skill to deal with the course? We in ASPECT consider this not to be the case. These students are academically competent - as competent as the majority of the mainstream cohort.

If the students are sufficiently mathematically competent, then perhaps their impediment to success is greater than that of the mainstream students due to the sudden change in academic support, a change with which the mainstream students do not have to contend (at least in the first- to second-year transition). It is this abrupt change which we seek to ameliorate with the ASPECT vector calculus offering. So far, the signs are good. One class test has been written. The overall pass rates are disappointing, but what is notable is that the pass rate of the ASPECT vector calculus cohort is identical to that of the mainstream cohort: 55% in both cases.

The students who are repeating the course (and hence have experienced the mainstream system) report favourably on the ASPECT vector calculus system. The weekly tutorial assignments, meant to be completed in the 2-hour tutorials (3 hours in ASPECT) continue to be worked on constructively in the extra (short) tutorial periods throughout the week, making it clear that the extra class time is advantageous. We shall continue to monitor both the class's absolute progress and their results relative to the mainstream cohort, but evidence to date suggests that the academic development offering of second-year vector calculus is effective. Should the model raise the performance of academic development students to that of the mainstream students, this pilot vector calculus course will continue to be run for future ASPECT classes. Further improvement may involve finding ways to improve absolute performance of the entire cohort as well as relative performance between groups.

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