CULMS Newsletter

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Community for Undergraduate Learning in the Mathematical Sciences

The CULMS Newsletter

CULMS is the Community for Undergraduate Learning in the Mathematical Sciences.

This newsletter is for mathematical science providers at universities with a focus on teaching and learning.

Each issue will share local and international knowledge and research as well as provide information about resources and conferences.

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Editorial

The Full Spectrum

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The University of Auckland

With this issue we are delighted to be able to make an announcement that university undergraduate mathematics is the focus of the major Ako Aotearoa research grant for 2013/4, competing nationwide against all other projects in tertiary education. However, with this comes a significant responsibility—and a call for help.

The project, provisionally entitled LUMOS (Learning in Undergraduate Mathematics and Other Subjects), will focus on the full spectrum of desired learning outcomes of undergraduate mathematics. Not only do we wish to be able to identify content and skills and understanding outcomes, but also we want to find ways to identify, observe, and report on outcomes such as mathematical processes (ability to conjecture, prove, disprove, symbolise, and so on), mathematical habits (inclinations to persist, explore, generalise, abstract, and so on), mathematical attitudes (liking for, confidence in, willingness to invoke mathematics, for example).

We will begin by surveying lecturers, graduates, employers and other recipients of mathematical graduates to find out and categorise desired outcomes for undergraduate mathematics. We will then try to observe these learning outcomes, and finally analyse and report on them at a course level. The object is to produce a Course Learning Profile that describes the overall or average effect of a particular course against these outcomes. We hope to be able to do this well enough to be able to distinguish differential effects of differently presented courses.

The project will not produce “reports” at the individual student level. Many of the outcomes we expect to observe will not be observable during one course for every student. The best we can hope for is a course impact. Similarly, the project will not try to measure an individual lecturer’s impact. We do hope to be able to make statements about the types of learning that certain course designs produce. We expect that different designs will produce a different spectrum of learning outcomes, and, as a result, university departments will want to ensure that students experience a variety of designs in order to produce fully rounded students.

In order to have significantly different course designs available in which to observe learning outcomes, the project includes provision for the development of three innovative courses. One will be a course that is technology intensive, encouraging the use of the web through computers, cellphones or other IT devices during lectures and tutorials, and having technologically open examinations. A second innovation is already in place: Team-Based Learning has been used in two courses in Auckland for some years now. The third innovation will be delivery using only one lecture per week but including intensive tutor-group engagement sessions and considerable web-based components.

The research team is predominantly from The University of Auckland, including researchers both from the Mathematics Education Unit in the Department of Mathematics, but also from two departments in the Faculty of Education. However the team includes researchers from the University of Canterbury and Victoria University of Wellington. These members' mathematical orientations will bring additional insights from the Engineering and Statistical disciplines that are not found in Auckland.

Where are the "Other Subjects"? We believe that a pedagogical analysis of this type will be valuable across the university sector, and have therefore invited groups from other subject areas to be part of at least the initial stages of the project. Whether they will wish to carry that forward into the observation and reporting stage is up to them. The thoughts that lecturers from other disciplines have on the full spectrum of learning outcomes in their discipline are certain to provide us with a broader perspective on our own examination of undergraduate mathematics. We have therefore invited a performance subject (Dance), a humanity (English), a professional discipline (Law) and another science (Psychology) to take part.

And what about our Call for Help? Our first task is to identify and categorise ALL desired learning outcomes of undergraduate mathematical sciences. We would love to hear from anyone who would like to contribute to this task—from full lists to a single email with one idea. Please write to <b.barton@auckland.ac.nz>.

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Which Came First – The Boring Lecture or the Disinterested Students? Mathematics Graduate Students and a Cycle of the Didactic Contract

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If you have a master's degree or PhD in mathematics, when you were a graduate student, when and how did you learn to teach? Were you provided with guidance for different types of teaching, such as lecturing or leading a recitation section? Did you learn how undergraduate students learn mathematics? Were there department-specific or university-wide mentorship programs offered to help you learn about teaching and learning? Or, as in many instances, are your answers to all or most of these questions 'no'?

I had two experiences in graduate programs in departments of mathematics. In the former, I first had to present a mini lesson to a group of mathematics professors, and then I was assigned a teaching mentor to follow for a semester. I attended my teaching mentor's classes and was given the opportunity to teach two lectures during that term, while my teaching mentor evaluated my progress as a lecturer. After a successful semester as a mentee, I was assigned my own sections of calculus to teach and then evaluated by the graduate student supervisor. In the latter experience in a graduate program in mathematics, I was assigned to two recitation sections the day before classes started. I was handed a copy of the book I would need for the course and the list of students. That was the extent of the direction I was given for my teaching. Thus, in the first experience, there seemed to be a system in place to prepare me for teaching. In the second, there was not. My two experiences likely represent the extremes – one where a great deal of explicit attention was paid to my lecturing, the other where no attention was paid to my teaching. In either case, though, no attention was paid to whether the format of my lectures was helping my students to learn and whether I was making mathematics meaningful to my students.

Such formative experiences of future professors have recently come to the fore as a topic worthy of exploration and research. For example, the purpose of the Carnegie Initiative on the Doctorate (Golde & Walker, 2006) in the U.S. was to gain an understanding of doctoral experiences in various disciplines and how future PhDs were prepared for their careers in academia and industry. As part of this work, the preparation for teaching that graduate students receive became a topic of interest, because it was recognized that doctoral programs do little to prepare future professors for their roles as post-secondary teachers (Golde & Walker, 2006; Prewitt, 2006). With regard to mathematics in particular, Bass (2006) concluded that, "Apart from a minimally mentored apprenticeship, through teaching assistantships or graduate instructorships, scant professional development for the work of teaching has been provided to doctoral students in most mathematics departments" (p. 109).

In the U.S., and most likely other countries, mathematics graduate students "often have responsibility for teaching lower-division courses" (Speer, Gutmann, & Murphy, 2005, p. 76), such as first-year calculus. Further, most graduate students in mathematics either are or will become university teachers of mathematics with more than seventy percent of mathematics PhDs finding jobs at institutions focused mainly on undergraduate education (Chan, 2006; Kirkman, Maxwell, & Rose, 2006; Kline, 1977). The National Science Foundation (1996) recommended that, in order to improve undergraduate mathematics teaching, departments need to "provide opportunities for graduate students to learn about effective teaching strategies as part of their graduate programs" (p. 69). Yet, Kyle (1997) concluded "the current professional education and development of future SME&T [science, mathematics, engineering, and technology] faculty members places too little emphasis on teaching and teaching improvement" (p. 547).

At this point, you might be asking whether there is a problem with the system as it currently works – mathematics lectures have been occurring for hundreds of years and students continue to graduate with degrees in the sciences and mathematics. Researchers have found that teaching methods used by university mathematics teachers contribute significantly to the drop-out rate in mathematics and the sciences (Seymour & Hewitt, 1997), as found by Martin (2001): "Studies show that almost half of the students who decide to specialise in a science major switch to a non-science major soon after enrolment. [...] Why the severe drop out? One factor seems to be poor pedagogy. Students who change to non-science subjects cite this as a factor for leaving" (p. 434). Further, when looking at the decline of interest among science majors, the largest drop was in mathematics. Again, one of the reasons cited was the form of teaching in mathematics classes.

The past decade has seen studies concerning mathematics graduate students and their preparation as university professors. In a research project that involved mathematics graduate students in the context of calculus reform, when graduate students in mathematics could speak of teaching using reform-oriented terminology, they also reported rarely using the associated teaching methods and maintained a lecture style form of instruction (Speer, 2001). When mathematics graduate students were offered a course in pedagogy and teaching mathematics, it did not alter their teaching practices (Belnap, 2005; DeFranco & McGivney-Burelle, 2001). In another project, Golde and Walker (2006) found that changes to pedagogy were particularly difficult for mathematics doctoral students. Moreover, it has been concluded by researchers that positive attitudes and beliefs around teaching mathematics did not change graduate students' teaching practices (Belnap, 2005; Speer, 2001).

While previous research reports that mathematics graduate students receive very little preparation for teaching, I would argue that they have essentially received years of implicit instruction in teaching mathematics through their experiences as students. Austin (2002) found that graduate students are "keen observers and listeners," gleaning information from their experiences to understand the emphasis they should place on their different tasks. As well, through their involvement in the routines of a department of mathematics, graduate students' views of the discipline and teaching are shaped (Austin, 2002; DeFranco & McGivney-Burelle, 2001). Further, graduate students in mathematics encounter many situations and structures that have the potential to be interpreted as having meaning and implications for how they should teach mathematics.

With the results of previous research in mind, I conducted a study that investigated experiences of mathematics graduate students and what might have the potential to prevent mathematics graduate students from adopting teaching practices that differed from lectures. What was it about their experiences in mathematics that seemed to keep them rooted in lecturing? For this study, I interviewed six mathematics graduate students in an urban, doctorate-granting university. Their experiences in graduate school ranged from a first-semester master's student to a final-year doctoral student. Over the course of an academic year, I conducted two individual meetings with each graduate students, then two group meetings with all participants, followed by a final individual meeting with teach graduate student. What I explore next in this paper is the notion of the didactic contract (Brousseau, 1997) and the curious case of the mathematics graduate students' various ways of adhering to it.

Brousseau's (1997) didactic contract has been described as the implicit rules or knowledge that "determine the interaction between teacher and students in connection with particular knowledge" (Elia, Gagatsis, Panaoura, Zachariades, & Zoulinaki, 2009, p. 769) and determine "in an implicit way, the expecting behavior and thinking of the teacher and students in a mathematics class" (Elia et al., 2009, p. 769). Some researchers refer to a triad in the didactic contract – the teacher, the students and the contract (e.g., Herbst, 2003), while others use the anthropological theory of didactics to analyze particular classroom teaching and specific interactions between teachers, students, and content (e.g., Chevallard, 1992). For the context of the study described here, the didactic contract is used to name the implicit set of behaviors and expectations that university mathematics teachers and their students engage in; specifically, the understanding that university mathematics teachers communicate mathematics to their students through lectures, which consist of writing mathematics on a chalk board while students remain quiet, copying the mathematics that the teacher writes.

In the setting of the study discussed in this paper, the mathematics graduate students did not teach their own sections of any courses, but were assigned to help undergraduates with mathematics in a tutoring center and the more advanced graduate students were assigned to recitation sections where they would solve problems for students at the board. All but one of the participants in the study were enrolled in graduate-level mathematics courses. In the series of interviews with the participants, I asked them about their experiences as learners as well as their experiences working with undergraduates and their visions of their future teaching practices. During a group interview, one question posed to the participants was "When you walk past a classroom and look through the window, how do you know you are looking into a mathematics classroom, without seeing what is on the chalkboard?" Their first responses included "Everyone is facing the board," "No one is talking except for the teacher," "The students are bored." In follow-up questions about what occurs in mathematics classrooms, the participants responded:

> *Steven*: I can tell you the structure. I think it's just usually – it's like definition, theory, example. Example, definition, theory, over and over and over again.
> *John*: And it's an instructor saying something without much student interaction.
> *Emily*: Very little, yeah.
> *Sara*: Which they don't mind.
> *Steven*: It's sort of worse than less, it's ...
> *John*: But it's efficient.
> *Steven*: You barely have their attention.

When asked what activities might occur in courses other than mathematics, the participants joked about "Sitting in a circle," "Talking about feelings," as well as talking about issues relevant to life. When asked if such activities could occur in mathematics classrooms, Steven said "It's a one-way, whereas in other classes they're expected to like read a journal or a short story and like, then when they go to that class they're going to discuss … That is the difference. That's never going to happen in math… it's just like you're expected to like come and like take up all this information." Thus, the participants' description of university mathematics classrooms is, in essence, an encapsulation of Brouseau's (1997) didactic contract – in university mathematics classrooms there exists and unspoken, yet widely known agreement that students have particular roles in the classroom and, further, there are no alternatives to this agreement.

The impact of such an agreement was felt in their own learning experiences as mathematics graduate students. As learners in their graduate courses, they sat quietly, taking notes, while their professors presented mathematics on the board, with little, if any, interaction among the people in the room – adhering to the didactic contract. However, as learners, they wanted more meaningful learning experiences. They were eager to talk about mathematics, to understand what Sara referred to as “the bigger picture” – where the mathematics they were learning had relevance to their lives and the world. Such opportunities were not presented to them and the lecture format of mathematics learning in which they sat passively listening to their professors had a significant impact on how they felt about mathematics, with Sara saying that “something just drew the love out” of her experiences with mathematics, and that she was beginning to question being in mathematics. For similar reasons, Steven said that he was going to be leaving the field of mathematics after completing his master’s degree.

In turn, despite their lack of meaningful learning experiences as graduate students, they adhered the didactic contract when they described their future classrooms – they would lecture, and their students would be there to sit quietly and take notes. Curiously, the study participants were adamant about the professor’s role in the classroom – it was to present information to the students without any reason to engage students, even though this was a form of learning experience that they found empty. When asked how the format for learning might be changed in mathematics classrooms, John and Steven both mentioned that things couldn’t change because students weren’t interested in mathematics, alluding to the biggest barrier to change as the abilities of the undergraduate students they would eventually teach. The participants interpreted undergraduates’ struggles with mathematics as indications of a lack of motivation or desire to learn mathematics. With the perceived constraint of underprepared and disinterested undergraduates, the mathematics graduate students inferred that the didactic contract was the only way to teach mathematics, not realizing that such a contract might be the very reason the undergraduates were struggling with or disinterested in mathematics.

This brings me to the question – Do uninteresting lectures cause disinterested students or do disinterested students cause uninteresting lectures? The answer to the question doesn’t really matter, and is most likely a matter of opinion. The impact of graduate school experiences is the salient point here. If the preparation, or lack of preparation and support, for teaching causes future professors of mathematics to understand their teaching practices in such ways, as a realization of the didactic contract found in lectures, then something must change. I believe that, as a community that works with and prepares mathematicians for their future work as professors, we must provide significant mentoring and opportunities for mathematics graduate students to learn and talk about teaching. For the graduate student participants in this study, the itinerary of progress through the department did not explicitly address their teaching, but it implicitly promoted a form of mathematics teaching deemed to be problematic by learners and researchers (Martin, 2001; Seymour & Hewitt, 2007). Through their experiences as graduate students, the participants observed that mathematics teaching takes on a particular form.

In thinking of the didactic contract, I am not unaware of the potential for resistance from undergraduate learners. In a course I am currently teaching for undergraduate mathematics majors, during every class period I have the students work on problem-solving activities and the impetus is on them to determine the mathematics within the lesson and what the goals for learning might be with guidance provided along the way. While some of the students have embraced the opportunity to actively think about and engage with mathematics, the transition has not been entirely smooth, with some students asking me regularly when I am going to teach them something. Not only that, though, I feel pressure coming from inside myself to put some mathematics on the board because the group learning activities I ask of my students do not yet feel like a real mathematics lesson. I am in violation of the didactic contract and it is uncomfortable at times.

Pressing on toward pedagogical change is quite important, however, as maintaining the didactic contract produces meaningless learning experiences in mathematics. As Steven said “we’re both [professor and students] going through the motions. And maybe that’s the point, then, of why be enthusiastic because we’re just going through the motions,” and that his enthusiasm for being a mathematics teacher had been “completely smothered” in graduate school. As we imagine change and innovation in undergraduate mathematics education, we must bring mathematics graduate students into the fold. We must realize that, in general, the courses they have taken are taking do not fit this new perspective and their past and current experiences as learners prevent them from seeing how things might be different.

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Earthquake Accelerations: Summative to Formative Assessment

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The Software

Computer-aided assessment (CAA) of mathematics is becoming widely used and was recommended by the American Mathematical Society First-Year Taskforce in 2009. This task force identified issues that mathematics departments are continuing to face in first-year undergraduate mathematics courses, such as student engagement and retention rates. Lack of adequate preparation for first year courses is also becoming an increasing problem. To help address these problems, the mathematics and statistics department at the University of Canterbury is using the CAA software Maple T.A. This article will describe our experiences with this software, particularly in the aftermath of the 2011 Canterbury earthquake.

The mathematics department at the University of Canterbury began using CAA software in 2008, using a software package developed by John Shanks at the University of Otago. This was based on multiple-choice questions and only available on-campus. Maple T.A. was introduced at the end of 2009 as a more flexible option with a web-based interface that can be accessed anywhere and any time with an Internet connection.

Although practice tests were provided for students to work on at home prior to the supervised tests, Maple T.A. was first primarily used as a summative assessment tool, as we became used to the system and developed question banks. Discussions on using the software more effectively, particularly for formative assessment, were well underway when the February 2011 earthquake struck at the beginning of Semester 1. This accelerated our plans to make more extensive use of the software for formative assessment.

With over 1000 first-year students enrolled in mathematics courses, tents going up in car parks, and no guarantee of when computer labs would be available, weekly online quizzes were prepared to support student learning. These quizzes provided formative assessment opportunities with immediate feedback and opportunities for multiple attempts as students revised their understanding.

Our core goal was to increase student success in our courses. We hoped to achieve this by increasing the opportunities for students to engage with the course material and by providing students with a structured means of learning in their independent study time. The challenge for us was to develop questions that supported this aim.

A key ingredient of CAA software is the ability to randomize questions to give different questions of the same type. For a question on a routine technique, the steps to working out the solution should be the same each time, but each problem should lead to a different solution. This can be achieved by using an algorithm editor within the question that can randomize variables, or by writing several different questions for the same method (e.g. integration by parts) and then using the assignment editor to randomize the choice of question. In this way students can have multiple attempts at a quiz and see varying problems of the same type each time.

When generating random problems, it is important to ensure that each version is both possible to solve and of equivalent difficulty. This is not always easy! To aim for invariance of steps in the worked solution, a useful strategy is to “reverse engineer” a problem. For example, Figure 1 shows an example of a Maple-graded question that asks students to find the general solution of a second order differential equation.

In this example, the algorithm editor is used to generate two random integers within set parameters, and these random integers are then used to generate the coefficients of the differential equation and the solution to the problem. This guarantees that the auxiliary equation has two distinct real integer solutions. Examples with one real root and with complex roots are generated in a similar way.

The marking code for a randomized question must allow for equivalent versions of solutions. In this case the marking code checks that the solution satisfies the given differential equation and then checks that it is the general solution. It allows for the students to write the terms in any order, but they do have to use uppercase A and B for their arbitrary constants, as specified in the question.

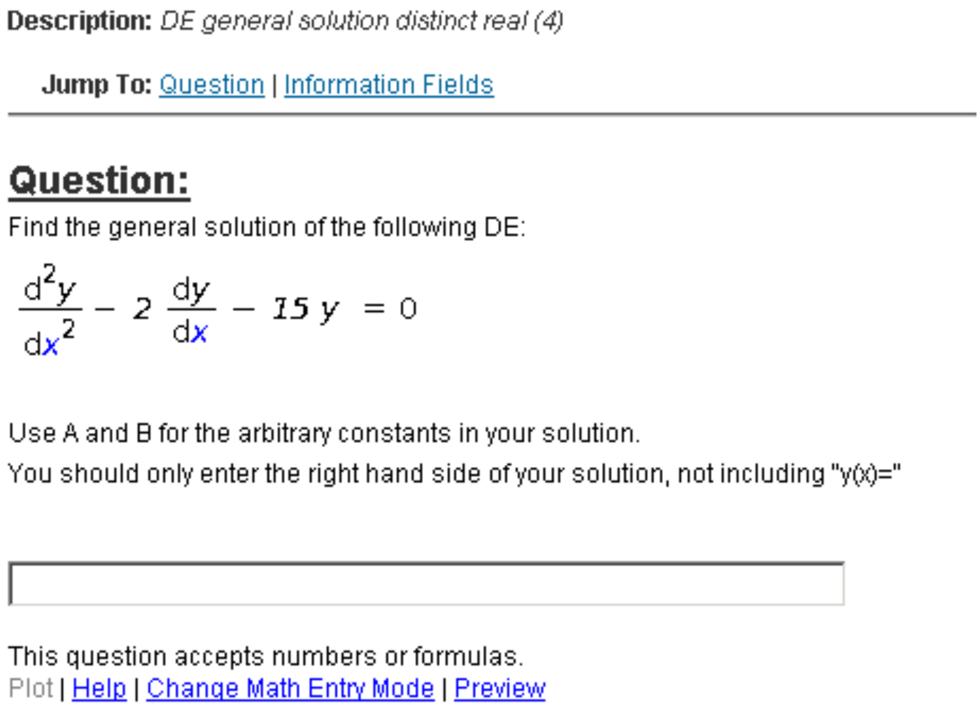


Figure 1: Solving a second order ODE.

Maple T.A. also allows for flexible multipart questions with different response areas, as shown in Figure 2. To provide variation in this case we prepared several questions each with a different differential equation, and then used the assignment editor to randomly select one question out of the question group.

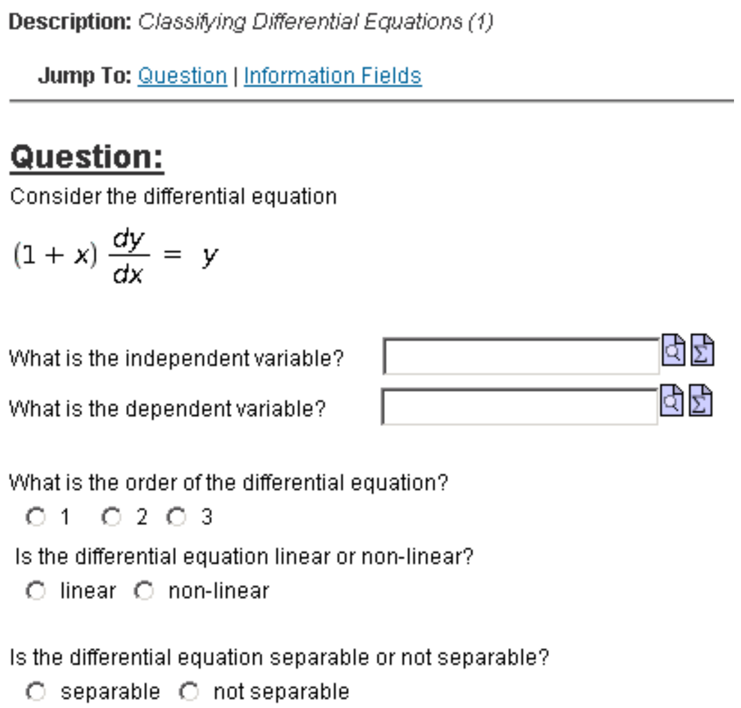


Figure 2: Multiple response areas.

While CAA is probably most useful for assessing the use of routine techniques and the mastery of basic skills, we are also working on developing questions that assess higher order thinking. An example is shown below in Figure 3. Students are asked to construct a quadratic polynomial with a repeated real root. They can use any letter as their variable and write their quadratic expression in any form in their response.

Questions that require students to construct an object lead them to consider the properties needed by that object and how they can be satisfied. This can contribute to a deeper understanding of a topic and hence improved learning outcomes. For example, a question asking students to "give an example of a function with a stationary point at $x = 2$" requires more than a superficial or rote understanding of differentiation, rates of change, and properties of curves.

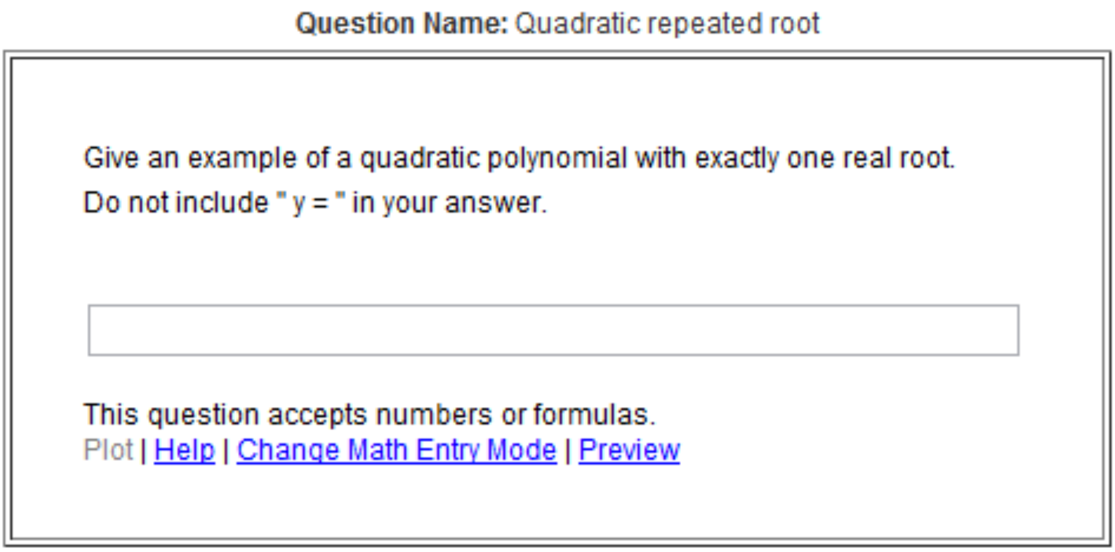


Figure 3: Assessing higher level thinking.

To write questions that assess higher order thinking requires more sophisticated marking than questions with numeric answers or multiple-choice answers. Thus an important feature of CAA software is the ability to correctly grade all possible student responses. CAA software is generally supported by an underlying CAS system. In the case of Maple T.A. this is the Maple software. The programming capabilities of Maple can be used to write a marking script that is able to test student responses using syntactic tests and equivalence tests. However this does come with a rather steep learning curve if you have not had a lot of experience programming with Maple!

The biggest challenge students face when using CAA software is syntax. The most common error is incorrect syntax, but the other problem is correct syntax (that is, syntax that the computer accepts), which does not express what the student means. To try and minimise these problems, we prepared an introductory unit in Maple T.A. made up of 16 questions, each question showing how to enter a different type of expression. We were hoping that students would be more likely to work through an online unit, rather than read a lengthy "how-to" document. We also provided a syntax reference card and a student information document in the course resources.

Although Maple T.A. has more relaxed syntax requirements than Maple itself, there are some areas of confusion. For example, the symbol palette allows the use of the superscript –1 for the inverse sine function, while in other places arcsin is required. Another issue is the use of e^( ) rather than exp( ) for the exponential function, which was accepted in some question types but caused problems in others. However the other side of this issue is that students are learning the need for correct syntax when using computers, which is a skill they will all need.

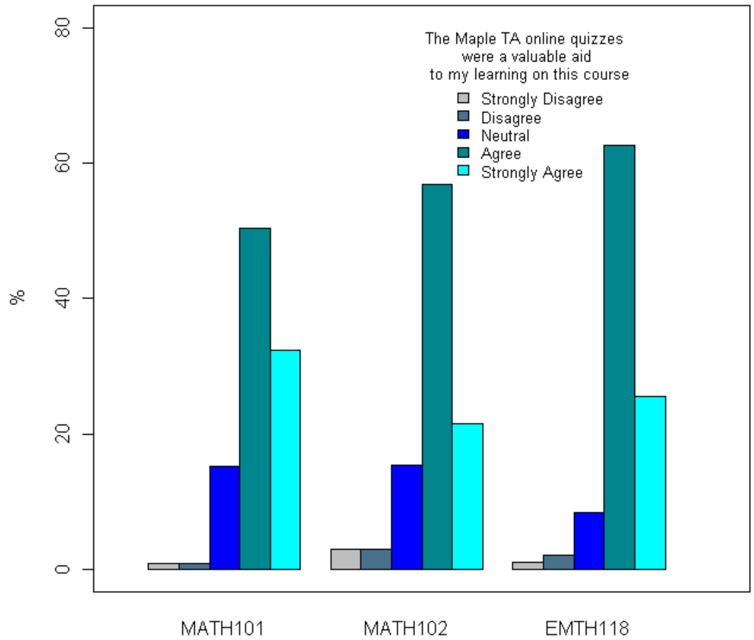
The Students

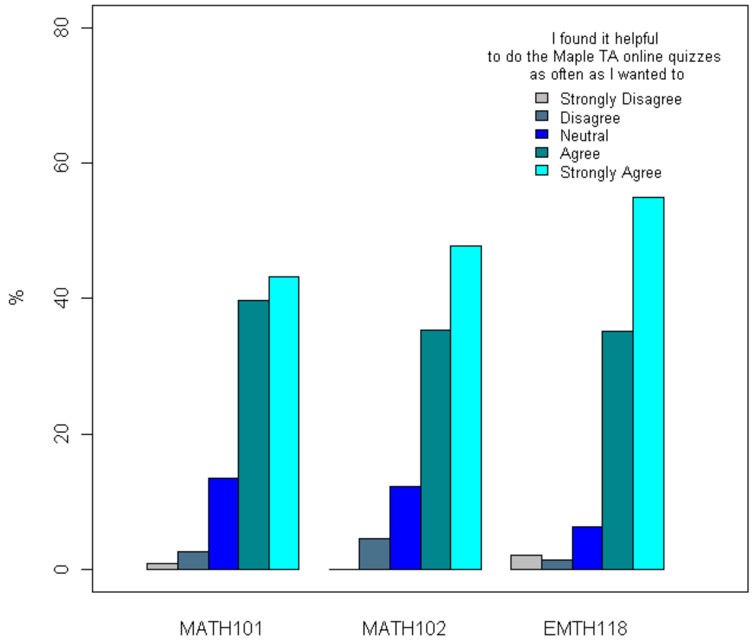
The new formative assessment units were introduced into our core 100-level mathematics courses soon after the February earthquake, and students were quick to start working on them. We received a lot of favourable comments about the modules when speaking to students in person and through emails. Students were very positive about having something to work on that could be done from home.

To obtain more formal student feedback, we conducted paper-based surveys in the last week of lectures for three 100-level first semester courses: EMTH118, MATH101, and MATH102. EMTH118 is made up of engineering intermediate students. MATH101 is our introductory mathematics course, which is primarily made up of commerce students, with some science and engineering students, and a few doing arts or education degrees. MATH102 has similar content to EMTH118, and has approximately equal numbers of commerce and science students, again with a few students doing arts or education.

The survey consisted of seven questions and a free response area. Overall, the feedback was very positive. For all three courses, at least 80% of students thought that the online quizzes were a valuable aid to their learning. More than 80% of students also responded positively about being able to do the quizzes as many times as they wanted to. This went up to 90% for EMTH118, where students are competing for entry into the first professional engineering year. Most students had between 2 and 5 attempts at each quiz, with students in EMTH118 most likely to have had more than one attempt at a quiz.

The survey also asked whether students had completed the Maple T.A. introduction unit. More than a quarter of the students had not done so. Interestingly, this did not show any correlation with the next question, which asked how easy the students found Maple T.A. to use. However in the free response area, all negative comments received were related to syntax issues. While support materials are provided, the problem is to increase their use by students.





One issue that occurs with the sole use of unproctored quizzes is students using websites such as Wolfram Alpha to answer questions that test routine methods such as integration. One way to try and ensure that students are thinking rather than copying is to prepare multistep questions that ask students to give intermediate steps, including specifying the method they are using.

Are we achieving the core goal of increasing student success? Initially results were positive, with improved pass rates in 2011, particularly for students with weaker NCEA Level 3 Calculus results. However other factors unique to 2011 are likely to have played a part in this, with pass rates in 2012 similar to 2010 and prior years. We are working on an overall strategy to improve student success in our first year courses, but CAA will definitely continue to play a part in this.

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Understanding Difficulties in Solving Exercises – Phasing Solutions

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When investigating the solutions of exercises step by step it is often difficult to precisely identify those steps where the difficulties occurred. For an analysis of a solution it is useful to distinguish between the three phases, namely conception, operation and application. First a concept is needed before it can be applied to the exercise. In order to reach the application afterwards there must be an operation. The conception includes a method that is used on an object in the application. A study with 340 students shows that the mistakes in solutions often lie in the methods and the cause is often found in the choice of the method[1].

Introduction

Teachers have been investigating students' solutions of exercises for a long time. They have tried to understand them step by step, so that they might be able to explain the mistakes to the students. There have already been a lot of ideas for the analysis of solutions, for example Schoenfeld (1985) investigated a method for problem solving. In Schoenfeld (2010, p. 3), he wrote

If you want to know why people's attempts to solve challenging (mathematical) problems are successful or not, you need to examine their:

- *knowledge base* – just what (mathematics) do they know?
- *problem solving strategies*, a.k.a. heuristics – what tools or techniques do they have in order to make progress on problems they don't know how to solve?
- *monitoring and self-regulation* – aspects of metacognition concerned with how well individuals “manage” the problem solving resources, including time, at their disposal, and
- *beliefs* – individuals' sense of mathematics, of themselves, of the context and more, all of which shape what they perceive and what they choose to do.

He had a look on the complete solutions. In Schoenfeld (1992, pp. 189, 190), he remarks that the episodes fell rather naturally into one of six categories:

[1] The article is based on Stoppel (2012)

1. Reading or rereading the problem.
2. Analyzing the problem (in a coherent and structured way).
3. Exploring aspects of the problem (in a much less structured way than in Analysis).
4. Planning all or part of a solution.
5. Implementing a plan.
6. Verifying a solution.

A subdivision of a solution of an exercise into *Understanding the Problem*, *Deviding a Plan*, *Carrying to the Plan* and *Looking Back* can be found in Pólya (1945). A comparison of Schoenfeld's stages and Pólya's step is given in Figure 1 (From Rott 2012, p. 3014).

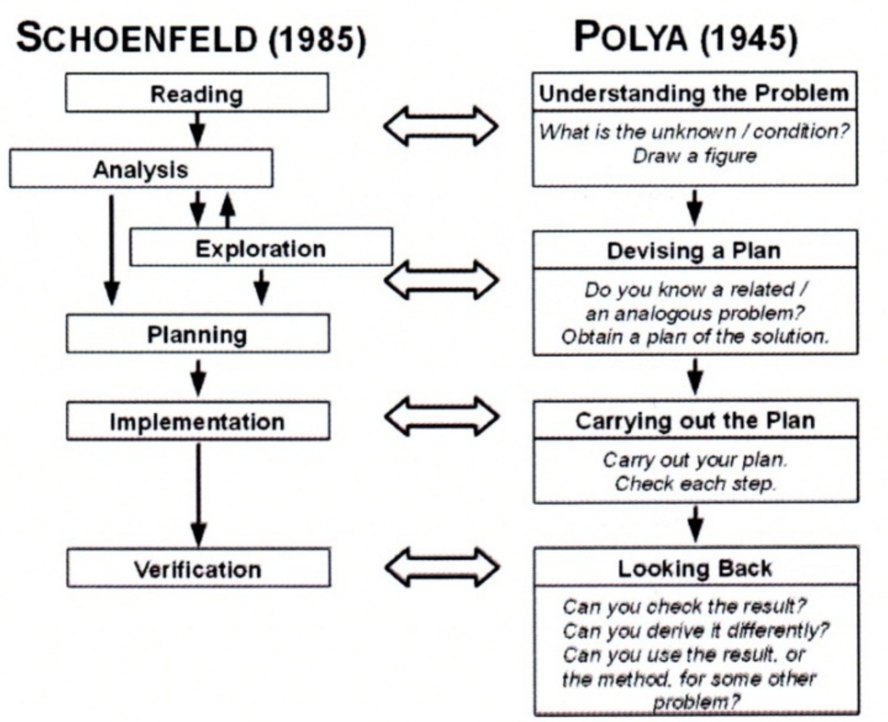

Figure 1: Analogy between Schoenfeld's episodes and Pólya's steps.

Mason and Johnston-Wilder (2004) noticed that mathematical lead processes and activities are described as “exemplifying, specializing, completing, deleting, correcting, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing, refuting”, (p. 109)[2]. Tall (1988, p. 8) distinguishes features of *Advanced Mathematical Thinking*:

- The abstraction of properties to provide concept definitions for mathematical concepts,

[2] A lot of information about the “the literature on the design and use of tasks […] levels” (Breen and O'Shea, 2010, p. 39) of mathematical thinking is given in Breen and O'Shea (2010).

- The use of abstract mathematical concept definitions to ease cognitive strain in thinking,
- The insistence on logical proof rather than coherent justification, which involves:
- The deduction of properties of mathematical concept (from given concept definitions),
- The implication that if certain mathematical properties hold, then others follow.

A question classification scheme is noted in Pointon and Sangwin (2003, p. 5):

> *Factual recall, carry out a routine calculation or algorithm, classify some mathematical object, interpret situation or answer, proof, show, justify- (general argument), extend a concept, criticise a fallacy.*

Such a classification has to be chosen quickly at the beginning to be able to develop a method for the solution. The development or the choice of a method is important for the possibility of a solution. The method needs to be applied to an object, which needs to be chosen in dependence on the method. It seem to be a circle, because it might turn out after the choice of an object that the choice of the method was wrong and must be chosen or developed another method. We will explore the selection and the design of a method and its application to an object inside a solution and try to understand where the difficulties and mistakes mostly are.

Structure and Steps of the Solution

The detection of a problem and the formulation of a solution inside an exercise in mathematics are intensive processes for students. If teachers present a solution to explain it to the students, they often leave out a lot of thinking about the *conception* and just demonstrate the *application* to come to the solution “straight ahead”. The first step of a solution often seems to be quite easy according to teachers, as the following example shows[3]:

Example 1
The octahedron in Figure 2 includes the points A(13|-5|3), B(11|3|1), C(5|3|7) and S_1(13|1|9). The octahedron is included in a cube with edges P_1 and P_2. Determine the coordinates of the points P_6 and P_8 of the cube in the figure above.

[3] It is a part of the Exercise *Abiturprüfung 2008 Mathematik, Leistungskurs, M LK HT 6*, Nordrhein Westfalen

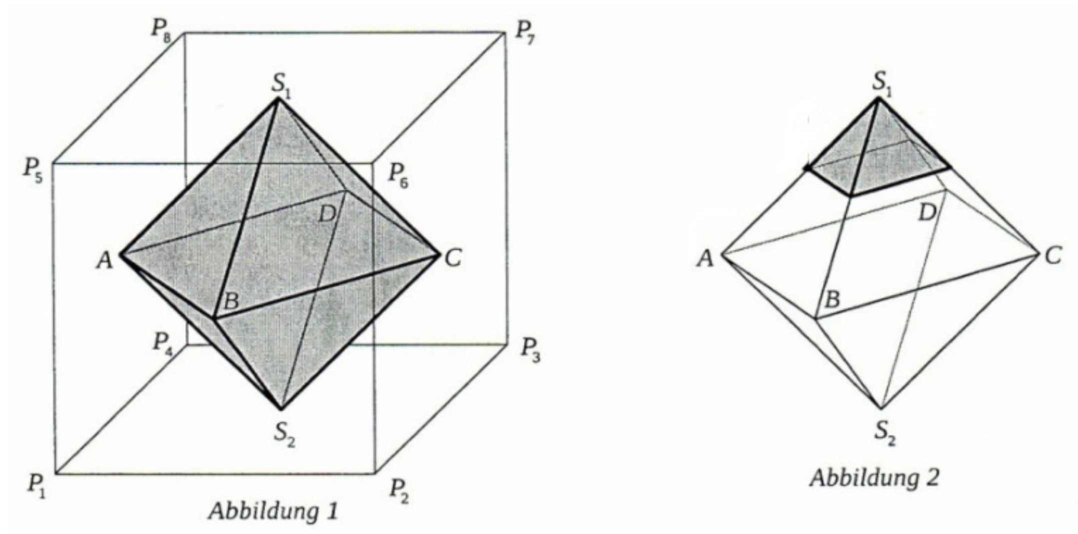

Figure 2: Octahedron of Example 1.

One possibility for a solution is given by:

$$\overrightarrow{0P_6} = \overrightarrow{0S_1} + \overrightarrow{AB} = \begin{pmatrix} 13 \\ 1 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 8 \\ -2 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ 7 \end{pmatrix} \quad \text{and} \tag{1}$$

$$\overrightarrow{0P_8} = \overrightarrow{0S_1} - \overrightarrow{AB} = \begin{pmatrix} 13 \\ 1 \\ 9 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \\ -2 \end{pmatrix} = \begin{pmatrix} 15 \\ -7 \\ 11 \end{pmatrix}, \tag{2}$$

and we have $P_6(11|9|7)$ and $P_8(15|-7|11)$.

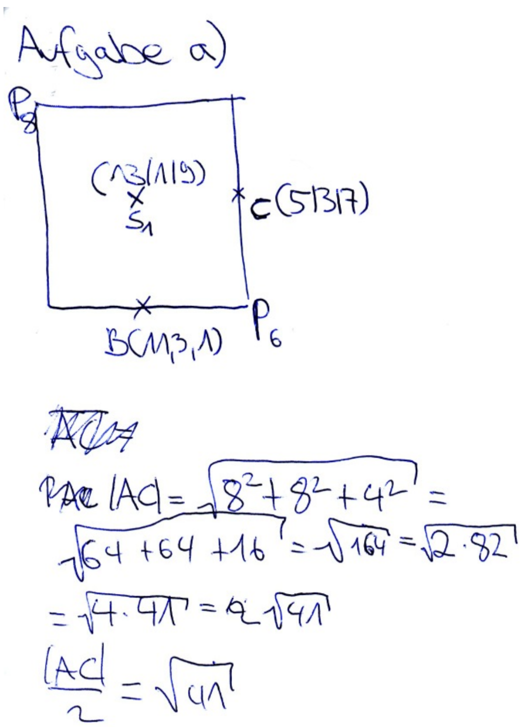

Figure 3: Example of a wrong solution.

The solution seems very easy. We are often unable to anticipate the difficulties students have during a solution of this exercise. The students need to be able to understand the structure of an octahedron and to apply the laws of vector analysis. Figure 3 shows an example of a wrong solution. As is evident there is a wrong approach, the student calculates the length of a vector.

How did the student get to this "solution"? The path from reading the exercise to the beginning of the solution holds a mistake. We need to have a look at this step and investigate the solution in three phases:

Step 1: *conception*, Step 2: *operation*, Step 3: *application*.

Only step 3 is used in the above solution using equations (1) and (2). As will be shown steps 1 and 2 are often neglected[4]. To solve an exercise the development of a *concept* is actually the most important step. With the awareness and understanding of a concept more general exercises can be solved.

Teachers succeed in step 1 of the solution. They recognize processes useful for solving the exercises. They reach step 2 and then start with step 3. In contrast students might not be successful, and we will explore the deeper connections in their "solutions" in the following paragraphs.

Steps inside a solution are given by *skills* and there will be a distinction of two types. Zehavi and Mann (2005) made a distinction between methods during solutions of exercises with or without a computer algebra system and distinguished between *reflective thinking* and *execution*. Following this distinction there will be another between *reflective thinking* like *assume, classify, analyse, generalize, concretize, structure, specialize, form theory, formulate, imagine, remember, imitate* and *execution* skills like *count, calculate, draw, algebrize, apply algorithm, imitate, argument… .*

The following diagrams include steps of a possible solution of the exercise above. It is divided in the three parts *conception*, *operation* and *application*. The concept is created in several steps to be applied afterwards. The *operation* creates links between the *conception* and its *application*[5]. The concept is created in several steps to be applied afterwards. The *operation* creates links between the *conception* and its *application*[6].

Conception

A conception begins at a starting point, which is given by the exercise. A

[4] This raises the question whether, to what extent and with how much awareness the teachers utilize the steps 1 and 2 inside the solution.

[5] Note that the *operations* might be taken the other way round. The diagram here is restricted to the most important operations.

[6] Note that the *operations* might be taken the other way round. The diagram here is restricted to the most important operations.

possibility for the definition of the starting point of exercise 1 is:

"Determine coordinates of P_6 and P_8."

The *conception* needs to be continued until an approach to an *application* of the *concept* is found. The recognition of a possibility of stopping the *conception* and starting the *application* needs some more studies. During the *application* above the first step is the determination of $\overrightarrow{0S_1}$ to start the calculation of the position vector of $\overrightarrow{P_6}$ (or $\overrightarrow{P_8}$).

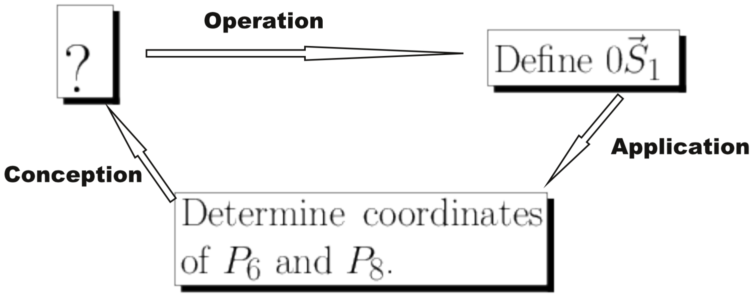

Figure 4: Structure of the diagram.

There might be more than one step inside of the conception and inside of the application. In Figure 4 "?" denotes that the steps during the conception are not yet given. A possibility of a conception is given in Figure 5.

During the steps *C(i,j)* of a *conception* of a solution some *reflective thinking* is needed. (*Conception* is included in reflective thinking in Zehavi and Mann (2005). We cannot be sure about the reflective thinking of students during a solution but we can draw conclusions about them. Examples for the solution above are given by Table 1.

Table 1: Examples of Reflective Thinking in Figure 5

(i,j)	C(i,j)
(1,2)	imagine, assume, remember
(2,3)	concretise, imagine, structure
(3,4)	remember, formulate
(4,5)	imagine, remember
(5,6)	specialise

Table 1 shows that we can only make assumption on relative thinking. Teachers might accept difficulties of the student during the conception and think about possibilities to help the student. During the step-by-step analysis of the solution above, relative thinking and skills first lead to the diagram in Figure 6. Table 1 shows that we can only make assumption on relative thinking. Teachers might accept difficulties of the student during the conception and think about possibilities to help the student. During the step-by-step analysis of the solution above, relative thinking and skills first lead to

the diagram in Figure 6.

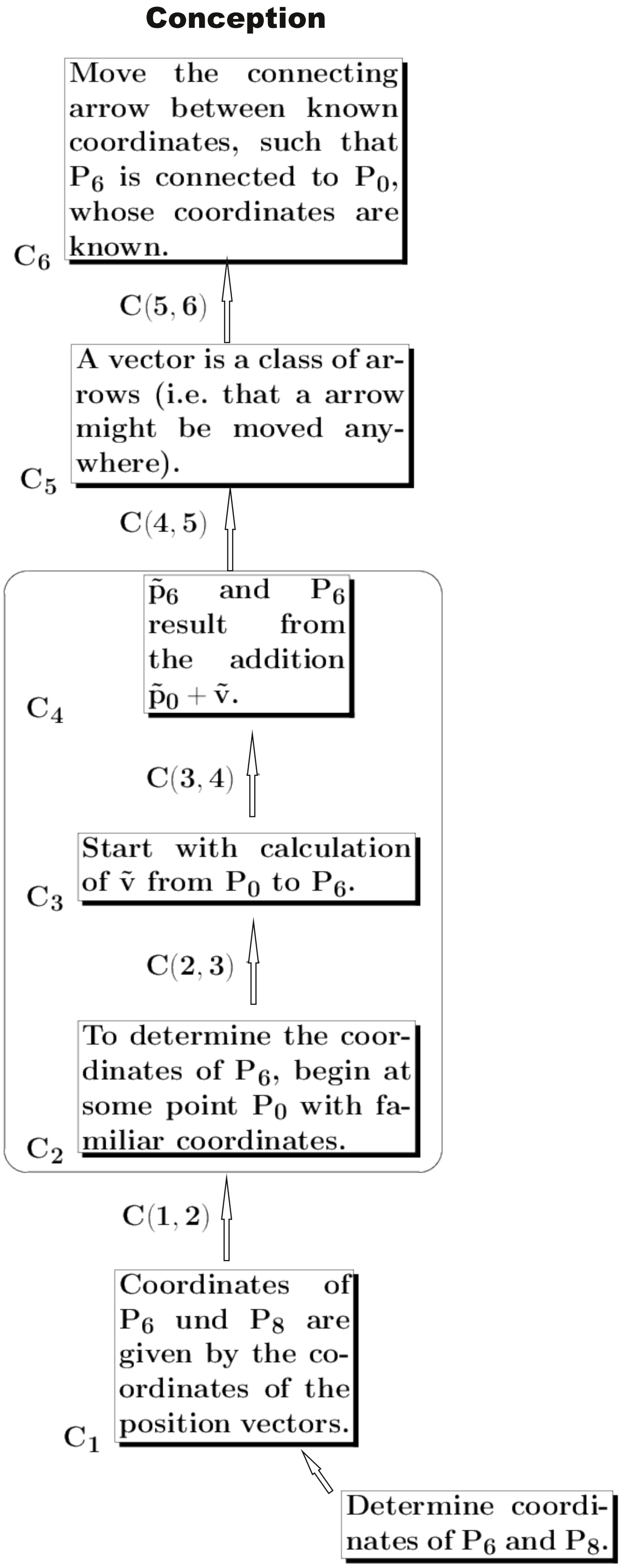

Figure 5: Conception of Example 1.

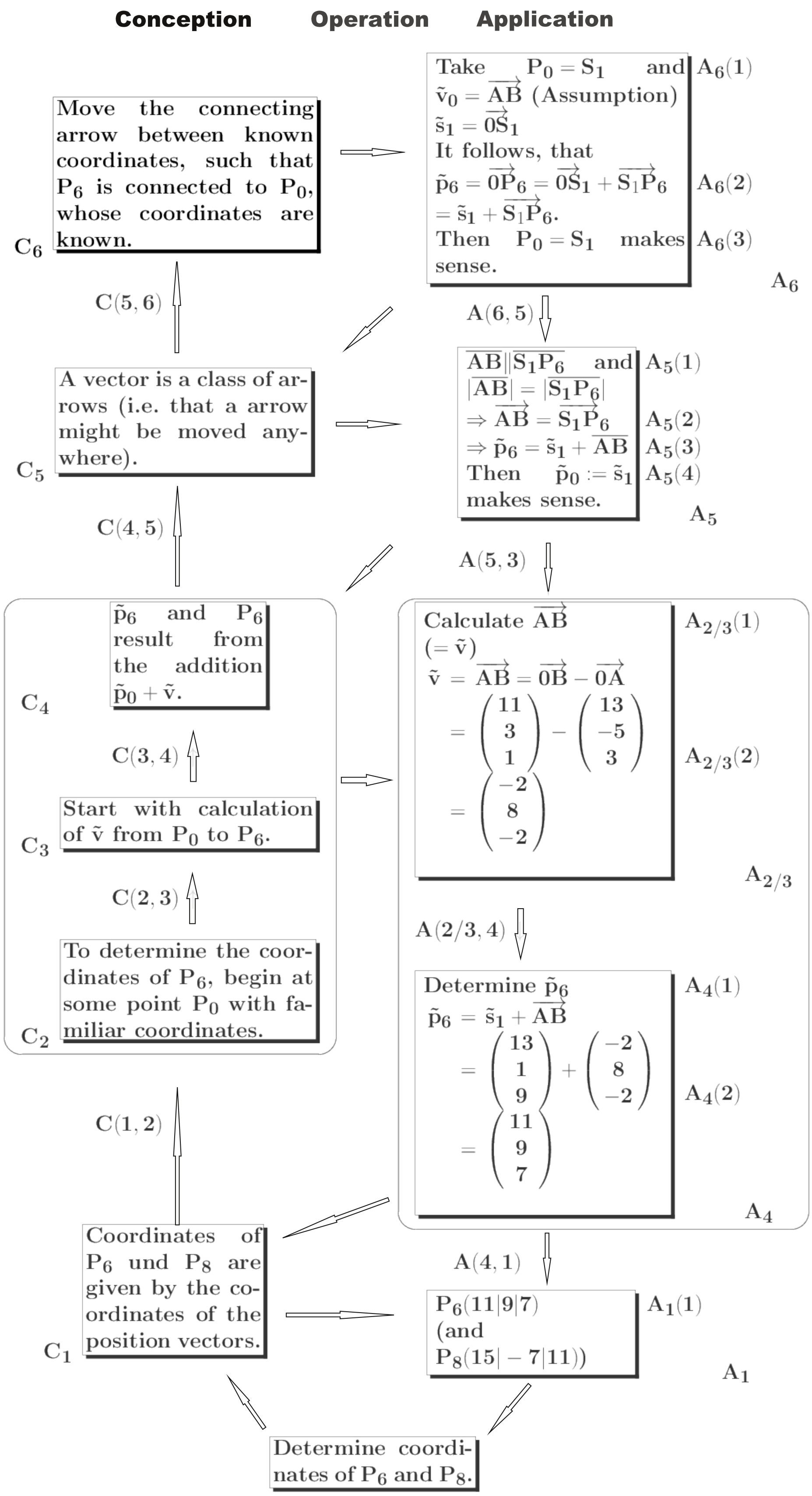

Figure 6: Structure of the solution of Example 1.

Application

When the conception is finished one needs to start with the *application* to come back to the starting point. During the application many steps are taken the other way round compared to the steps that belong to them inside of the conception. This is visible in the column on the right side of the following diagram. A_i denotes the blocks in the right side of the diagram. i denotes the connection between A_i and C_i. $A_i(j)$ are the steps inside of A_i. The arrows indicate the direction of the steps in the application. The diagram in Figure 6 represents the structure of the solution of example 1. (The meaning of *operation* will be explored later.)

For steps $A_i(j) \rightarrow A_i(j+1)$ during the *application execution* skills are needed. We can only make assumptions about the transitions in the application. Examples for the solution above are given by:

Table 2: Possible Transitions of Example 1

Transition	Tool competence
A_6	
$A_6(1) \rightarrow A_6(2)$	calculate, formulate
$A_6(2) \rightarrow A_6(3)$	verbalise, conclude
A_5	
$A_5(1) \rightarrow A_5(2)$	formulate, visualise
$A_5(2) \rightarrow A_5(3)$	specialize
$A_5(3) \rightarrow A_5(4)$	argument, conclude
$A_{2/3}$	
$A_{2/3}(1) \rightarrow A_{2/3}(2)$	formulate, conclude
A_4	
$A_4(1) \rightarrow A_4(2)$	formulate, calculate

Operation

Conceptions and applications are connected by operations, where reflective thinking is used, e.g. the step $C_5 \rightarrow A_5$ from C_5 to A_5 might be imagine or assume. The step $C_6 \rightarrow A_6$ from C_6 to A_6 might be an assumption or an algebraization. It might be an experiment too. We must formulate the approach of the solution and define an object. The selection of the object is included in the formulation. Experiment, assume, imagine, algebraize, specialize are possibilities for the skills in $C_5 \rightarrow A_5$. In analogy it must be formulated which skills one needs in every transition from C_i to A_i. We need a conclusion at the end of every A_i that is necessary to decide whether to take the transition $A_i \rightarrow C_{i-1}$. Most of the time we need connection tools for operations $C_i \rightarrow A_i$.

It is striking that the steps C_2, C_3 and C_5 are united inside a box. The same applies to the affiliated steps $A_{2/3}$ and A_4 of the application. ($A_{2/3}$ means that both A_2 and A_3 are comprehended.)

Steps chosen by students during a solution might not be successful, as can be seen inside the solution in Figure 7 in the colour of the pencil. (The first trial of a solution is written with a blue pencil, the second trial is written with a red pencil.) The solution shows that the student constructed a conception. The first operation of the solution (written with a blue pencil) was not successful, and the student tried a second operation (written with a red pencil) and thus solved example 1.

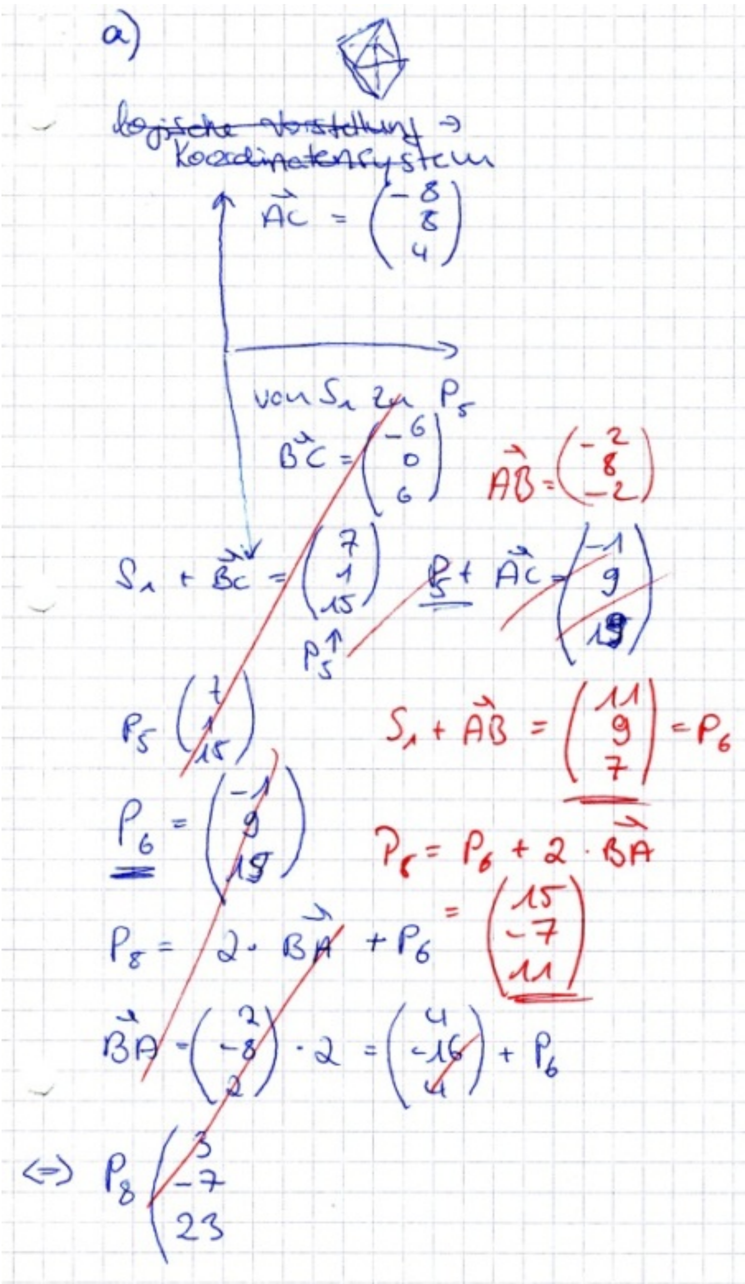

Figure 7: Solution in two operations.

Method

During a solution *methods* are developed and applied to *objects*, e.g. application of the method in the block with the steps (C_2, C_3, C_4) inside to the object $\vec{v}$ in the block with the steps (A_2, A_3, A_4). In contrast to the rest the steps in the blocks are taken in the same order which means that inside of both blocks the steps have the order $2 \rightarrow 3 \rightarrow 4$ $(C_2 \rightarrow C_3 \rightarrow C_4$ and $A_{2/3} \rightarrow A_4)$. This *method* has to be viewed as a unit.

A lot of solutions show difficulties in choosing a *method* and its *application* to an *object*. Because "activity theory starts from the problem and then moves to the selection of the methods" (Kaptelinin and Nardi 2006, p. 72) its use is helpful.

The selection of a *method* and its *application* to an *object* often brings difficulties to students, see above and the following example.

Example 2.

Determine the solutions of $z^8 = 1$ for $z \in \mathbb{C}$.

The student needs to know that the polar coordinates should be used for the solution. As is seen in the example of a solution in figure 8 it might be that cartesian coordinates are used instead.

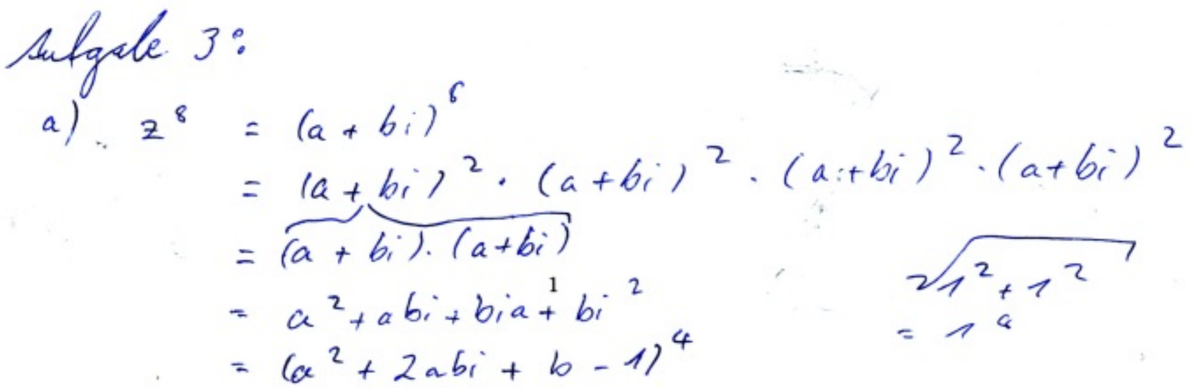

Figure 8: Wrong choice of coordinates.

Execution skills like *application of an algorithm* might be helpful. After an example the *thinking activity imitation* might be conceivable to imitate a *method* by using polar coordinates. The student has chosen an unsuccessful *method*.

Data Analysis

The *method* of a solution is chosen in step $C(1,2)$, and as can be seen below it turns out that this is an important step on the way to the solution of an exercise.

340 students of form 13 were involved in a study of three different grammar schools/high schools, students of the Ruhr-Universität Bochum

(first semester) and RWTH Aachen University (third semester). For the analysis we used ten different exercises and analysed a possible solution for *methods* with consideration of *reflective thinking* and *executive* skills and put the use of *methods* in the solutions into five categories:

Cat. 1. Design/select an appropriate method and apply it to an appropriate object.
Cat. 2. Design/select an appropriate method and apply it with small mistakes (like mistakes in calculation) to an appropriate object.
Cat. 3. Design/select an appropriate method but make mistakes in the choice of the object inside the *application*.
Cat. 4. No success in finding a method during *conception*.
Cat. 5. Design/select a method useless for the exercise and apply it (e.g. memorized methods without comprehension).

Table 3. Results for Octahedron and $z^8 = 1$

	Cat. 1.	Cat 2.	Cat 3.	Cat 4.	Cat 5.	Cat 4 + Cat 5.	Sum of Students
Octahedron	21	9	6	7	17	24	60
$z^8 = 1$	9	10	3	12	23	35	57
Sum	92	66	33	47	102	149	340

The diagrams in Figure 9 show the relative frequencies of the five categories for these exercises:

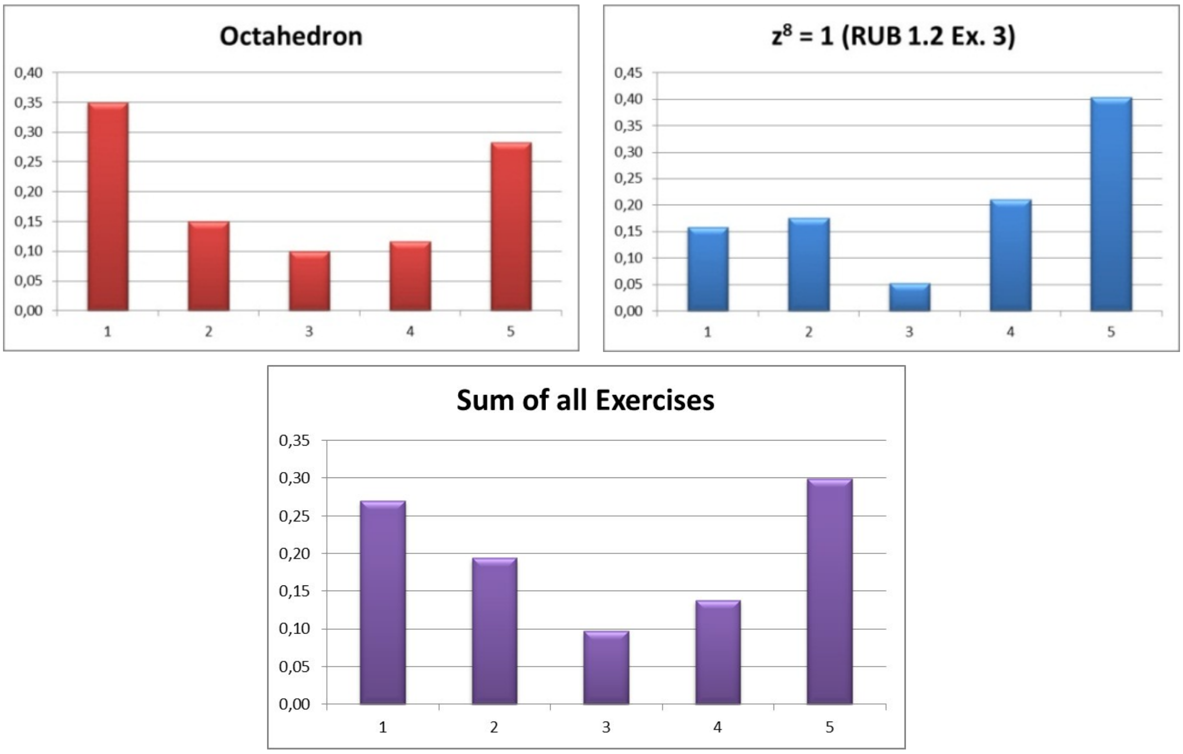

Figure 9: Diagrams of the analysis.

Obviously the relative frequency of type 5 is very high with 30% compared to the other types. Compared to the types many of the students were incapable of finding a proper *method* or any method at all. 44% of the students made a mistake in connection to the method they selected and used.

Conclusions

The analysis of solutions of students gives an insight to the usage of methods. This is a new point of view compared to Schoenfeld (1992) or Lucas, Branca, Goldberg, Kantowski, Kellogg, and Smith (1979).

It seems to be important to analyse problems of students when developing appropriate methods for solutions of exercises and apply them to a wellchosen *objec*t. One should especially have a view on step *C(1,2)* of the *conception*. As is shown in the example of the octahedron exercise the *reflective thinking imagine, assume, remember* might have been used for the choice of a method. It is remarkable that some skills seem to be used very often for the choice of an object for the application, eg. *formulate, verbalise*. To analyse problems it seems to be helpful to concentrate on *reflective* thinking and *execution* skills in connection to *methods* in solutions with a view on *conception*, *operation* and *application*. It will lead to some knowledge about the difficulties students have when solving exercises.

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Passing the Baton

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This article takes a thought provoking look at our teaching methods while analysing the attitudes of those students entering university to become primary school teachers. In particular we look at the attitudes the students have towards the discipline of mathematics. These are the students who will be teaching our children and grandchildren. More importantly these are the upcoming teachers whom we as academics, can influence positively so that negative attitudes towards mathematics do not get passed on to the next generation.

Macquarie University offers one maths subject, MATH106, to prospective primary teachers. This unit is designed with these students in mind, especially given that this may be the only mathematics unit a primary teacher would do in their higher education. We were interested in looking at the attitudes these students had towards mathematics. Surveys were conducted of the 2008 and 2010 cohorts of students during their first week of lectures. Of the 207 students enrolled in this course in 2008, 67 of them completed the survey. There were 416 students enrolled in 2010 and 76 completed the same survey. The results from these groups were very similar and in fact a Chi Squared Test of Significance showed that there was no evidence of a statistically significant difference between the two groups. Therefore we combined the results and in total had a sample of 143 students. While this sample may not necessarily be representative of those studying primary teaching, some of the results can give us an insight into who our students are, and even provoke us to rethink our teaching methods.

Firstly it may be worth looking at the highest level of mathematics these students had done prior to commencing MATH 106. 20% of students said they had done up to a year 10 standard while 55% did year 12 General Mathematics. Only 25% had done HSC Mathematics or higher (High School Calculus based courses). It is worth noting that 75% of surveyed students studying to be primary teachers, had little mathematics in their background before commencing MATH106.

So who are these students who will eventually be teaching mathematics at primary school? 74% of them said that they would like to do more maths (22% were neutral), but 50% said they struggle with maths (31% were neutral), and 23% said they cannot grasp mathematical concepts no matter how hard they try (35% were neutral). Overall there does not seem to be a negative view about maths but rather a struggle with it and a hope that this time round they would understand it better. One student says

> Although I am nervous about maths (again), I am hoping that this time around I will discover the enjoyable practicality of maths. I hope I learn to be excited about teaching maths at primary school level, and as a result create an interest amongst my students.

And another says

> I really want to improve my maths skills so I don't pass my fear of maths on to my students and to give the best possible mathematics education to my students.

While another says

> I struggle with maths despite learning it K–12. I'd like to be able to understand the concepts better.

They also expect that they will need help with the subject.

> Tutoring and extra help may be needed, student only did general maths at school.

These students now want to have a different and better experience doing mathematics and are doing MATH106 in the hope that their nervousness may be alleviated. So the question is, are we teaching it in the right way?

It may be worth noting that previous mathematical experiences have shaped students' views on the subject. 21% of surveyed students said they did not enjoy maths at primary school (17% were neutral). Some students can point to a past experience which turned them off maths:

> Having had bad high school maths experiences I'm keen to get back the positive attitude I had towards maths in primary school and I'm very aware it will affect my students.

Or even attitudes in their family or community which are negative towards maths:

> Many people said to me, math is the most daunting subject to study. When I told my family that I study math unit this semester, they said to me, 'you are on your own' which means I don't ask them about math. I don't have the same impression about math, even though I was made to do so.

Maths seems to be a subject which arouses various emotions, and one which can affect a student's self-esteem. As academics, should we have a greater awareness of the backgrounds of those students studying MATH106? Given that these students will eventually be teachers themselves, should we raise awareness about how they could be affecting their own students one day? We found that 51% of students said they were nervous about teaching maths while 27% were neutral. It is interesting to note that 34% disagreed with the statement "The way I feel about maths will be reflected in my teaching" (14% were neutral).

When it came to studying MATH106, a number expressed views on what they thought should be in the course. Interestingly some commented on what they saw were the relevant things to teach to primary students. One student comments

> I question the reason for the depth of the maths explanations covering topics that not even I covered in high school at ALL. I understand that we may need at times to further our knowledge for particular students ability, however understanding the main topics explained in primary school seem to me as the most important to explore – NOT THE ADVANCED MATERIAL I DID NOT EVEN EXPLORE IN HIGH SCHOOL.

Furthermore in MATH106, comments like the ones below were common.

> Generally I really struggle with maths. I truly don't believe hexadecimal representation and binary is taught in primary schools. I touched on these concepts only in year 8 or 9 maths. I think it is extremely irrelevant.

Students seem to be only interested in topics that they see as directly relevant to their subject area. Rather than dismissing the student's view, should we as academics take a different approach when teaching? Not by changing the content of our courses, but by striving to explain how the material is relevant to them and how it would help them to be better teachers? In this particular case, these students do not seem to realise that learning about binary and hexadecimal representation of numbers would aid their own understanding of place value. In addition they do not see that it would be an important tool for them to use when they themselves are teaching the topic of place value to primary students. Nor do they see that it could be a useful extension for gifted and talented children that they may one day teach. They do not see the need to know more than what they think is necessary to teach and do not seem to be interested in knowing about higher level concepts. The importance of being knowledgeable in the subject area and the importance of being able to extend their own knowledge so they would be respected as teachers seems to be lost. Furthermore the concept of learning for its own sake is something that needs to be emphasised and encouraged.

All of this forces us to look at how we teach mathematics, specifically to those who have some fear of the subject, and especially to those who will be the first ones presenting and introducing children to mathematics. We need to take an approach which is sensitive to the students' past experiences with mathematics and not only show them the relevance of the subject but also pass on an appreciation for learning and good teaching.

If we can change the culture of prospective primary school teachers' attitudes to mathematics, maybe our enrolments will be up and most importantly it may be the start of a revival in society to see the relevance and beauty of mathematics. Furthermore we will also be able to proudly say that we did fulfil the hope our students had in finding maths a more interesting and relevant subject, and affirm comments like

> Maths can be empowering when you attempt a problem and are capable of doing it. Empowerment after successfully completing a challenge.

There are many things we can conclude from this study, however there are three points we would like to comment on. Firstly when we look at those studying to be primary teachers, there may be some worry about the mathematical knowledge of these students and their attitudes to maths. Given that through these prospective teachers, many children will receive their first contact with mathematics, it is imperative that we send the right message to these prospective teachers about how to teach maths and about its relevance.

Secondly, this study forces us to look at how we ourselves teach and present mathematics. We cannot, as teachers, deliver isolated topics without embedding them in a broader context, especially to those students who only take one or two maths subjects during their entire degree. It is time to think about how we present maths to students before it loses its meaning to the majority.

Thirdly, we need to address the issue of learning and acquiring knowledge. We are sure that many of us teaching in the classroom have heard the question from one of our students "Why do we need to know this?" The question itself seems to reveal a lack of appreciation for learning and acquiring knowledge for its own sake. Many students do not seem to be aware that education is of value in its own right. Instead there seems to be greater stress placed on immediate concerns of getting marks and learning for the test. Maybe in Australia, this is an attitude being espoused throughout primary and secondary school education? We as tertiary educators need to reverse this swing, and impart to our students the value of learning.

We do not propose a single solution or one particular method to tackle these issues. We believe as teachers we are all different in our style and have varying qualities to offer. What might work for one person, may not work for another. However, we could all incorporate these three considerations into our own practice and embed them in a way that complements our styles. Our aim should be to bring about a respect for the subject of mathematics, to highlight its relevance, and to impart the value of learning so that we can successfully pass on the baton of scholarly teaching.

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