CULMS Newsletter

Number 5

May 2012

Community for Undergraduate Learning in the Mathematical Sciences

CULMS Newsletter

Number 5 May 2012

Editorial

Mike Thomas
The University of Auckland

One thing that we cannot afford to do as university educators is to make assumptions about, or ignore, student mathematical knowledge, in all the forms that Mason discusses below, or the manner in which they build it. The question of how students want to learn, as Todd reminds us, is often not considered. We see from his paper that students are attracted to learning that is multimodal, natural, manageable, organised, rewarding and less formal. Certainly if we want to retain students in our courses then we need to make learning both enjoyable and productive for them. As Barton notes, the report of the President's Council of Advisors on Science and Technology (PCAST) (2012) claims that introductory mathematics courses for those entering science, technology, engineering and mathematics do little to encourage students to continue in these fields, and that preparation for teaching undergraduate mathematics is behind that in engineering, technology and science. It also concludes that that in the USA alone there is a need to produce, over the next decade, around 1 million *more* college graduates in Science, Technology, Engineering, and Mathematics (STEM) fields than were previously expected. One of the report's five overarching recommendations is to "launch a national experiment in postsecondary mathematics education to address the mathematics-preparation gap".

One way in which university faculty address issues in student learning is through the construction of novel courses. Two of these initiatives are discussed in this issue, with Craig describing how writing explanatory paragraphs in mathematical problem-solving has been shown to improve student understanding of the problems being addressed. In turn Martinez-Luaces and Velázquez present details of an innovative Experimental Design course based on real-life problems that increased student (and teachers') learning and satisfaction.

One of my recent roles has been leading an International Congress of Mathematical Education (ICME – to be held July 8-15, 2012, Seoul, Korea, see http://www.icme12.org) survey team, ST4, considering Key Mathematical Concepts in the Transition from Secondary to University (see ST4 at http://icme12.org/sub/sub02_03.asp). One issue that arose in this review was a consideration of the way in which we prepare students to build knowledge of proofs and proving, both in schools and universities. One conclusion is that to guide students in a way that is rewarding, enjoyable and satisfying for them we should be helping them with proof construction rather than simply presenting proofs to students and getting them to try to reproduce them. It appears from our investigation that around half of university departments have a course that explicitly teaches methods of proof construction, mostly in the second year. Using such a course to help students build knowledge of how to read proofs like a mathematician does, and to construct counter-examples, conjectures and definitions, among other aspects of proof construction, seems an excellent idea, and one I would like to commend for consideration by those who do not yet have such a course.

Apart from this CULMS Newsletter, there are other forums that are explicitly concerned with issues such as the building of undergraduate student knowledge. Oates describes many of the interesting papers presented at one of them, the 2011 Delta Conference on Undergraduate Mathematics and Statistics Teaching and Learning (the next conference is 24-28 November, 2013 in Kiama, New South Wales, Australia), and another is the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education (SIGMAA on RUME) (see http://sigmaa.maa.org/rume/Site/News.html), which has a yearly conference. The next one will be held in Denver, Colorado in February, 2013. I hope we will all take advantage of these and continue to be actively involved in the discussion of issues related to undergraduate learning and teaching of mathematics.

References

President's Council of Advisors on Science and Technology (PCAST) (2012). *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics*. Washington, DC: The White House.

Author

Mike Thomas, Department of Mathematics, The University of Auckland, NZ.
Email: moj.thomas@auckland.ac.nz

Explanatory Writing in Problem-Solving: Understanding Through Reflection

Tracy Craig
University of Cape Town

An intervention was piloted in a first-year university mathematics class requiring the writing of explanatory problem-solving strategies. The aim of the intervention was to advantageously affect problem-solving behaviour by encouraging reflection. Data was collected in the form of students' written submissions, assessment tasks and interviews. Improved understanding of the underpinning mathematics was observed and explained using Piaget's constructivist theory of learning.

Introduction

Writing explanatory paragraphs in mathematical problem-solving has been shown to improve student understanding of the problems being addressed. An in-depth exploration of this process of deepening understanding was the focus of the author's PhD research (Craig, 2007). While facets of the research have been presented elsewhere (Craig, 2011; Craig, 2012) this article serves as an overview of the project. The project required students to write about their problem-solving processes, either before or after engaging with the problems, in the hope that such reflective activity would advantageously affect problem-solving behaviour. Recalling Pólya's (1945) four steps of successful problem-solving, (1) understand the problem, (2) devise a plan, (3) carry out the plan, (4) look back, it was notable that the first crucial step of "understand the problem" was the step most influenced by the reflective writing process.

The author is a lecturer of first and second year mathematics at university level. It was observed in the classroom and in assessment tasks that many students either did not possess much skill in problem-solving or possessed low levels of self-confidence in their ability to solve problems. It was simultaneously observed that the first year course, the vehicle for the writing project, did not explicitly teach problem-solving, instead it taught many algorithms and mathematical recipes. In conflict with the content of the course, the lecturers occasionally set problems that demanded a high level of problem-solving ability from the students. It was the aim of this study to address the occasional imbalance between what was taught and what was assessed by creating a course activity that would improve problem-solving skills. The research question guiding the project was "What effect does the writing of explanatory strategies have on mathematical problem-solving?"

A mathematical problem can be defined as "a task that is difficult for the individual who is trying to solve it. Moreover, that difficulty should be an intellectual impasse rather than a computational one" (Schoenfeld, 1985, p. 74; Stanic & Kilpatrick, 1989) and mathematical problem-solving simply as the solving of such problems. Problem-solving is distinct from using well-worn algorithms for solving exercises, the former being much more difficult to teach than the latter.

Specific areas within the field of problem-solving which have received focussed interest include metacognition and expert-novice distinctions, both issues of interest in this project. Metacognition can be defined both as knowledge about cognitive phenomena, and monitoring and regulation of cognitive phenomena (Brown et al, 1996; Garofalo & Lester, 1985; Schoenfeld, 1985) with rather less frequent attention being given to a further definition of metacognition as beliefs and affects and their effects on performance (Schoenfeld, 1992). The writing exercises used to develop problem-solving behaviour required the students to reflect on their problem-solving processes, and in so doing to invoke metacognitive processes in their manifestation as declarative knowledge of cognitive processes. The observed positive effect of the activity was that reflection compelled deeper engagement with the mathematical material than might have been the case without reflection. Metacognitive control was not enhanced so much as active cognitive engagement with mathematics deeper than simply carrying out algorithms.

There is widespread support for the usefulness of writing in mathematics from various viewpoints. There is the school of thought that writing and problem-solving involve exactly the same steps, essentially Pólya' problem-solving steps, and therefore combining the two activities simultaneously brings advantage to both (Kenyon, 1989). There is the constructivist viewpoint that writing about mathematics requires the writer to form associations and construct cognitive knowledge structures in order to communicate thought processes in an understandable form (Ellerton & Clements, 1992). Then there are the supporters of the metacognitive advantages of writing through processes of reflection, monitoring and reaction (Pugalee, 2001; Kenyon, 1989. All three views played a role in the project described in this article.

Research Methodology

The course within which the writing experiment took place was a year long course, divided into two semesters. The writing initiative was carried out in the second semester. The class was large, approximately 500 students divided into two smaller classes of approximately 250 students. The class further divided into afternoon tutorial groups of approximately 30 students, which even so are larger than some groups used in problem-solving teaching experiments (Schoenfeld, 1985). The course was a preparatory course for second year mathematics, applied mathematics, first and second year physics, chemistry and economics. It was a compulsory course for all Bachelor of Science students as well as Actuarial Science students. It was the aim of the study to design an intervention which could be added to the existing course without a need for content changes, and which would in some way enhance students' problem-solving abilities.

The writing exercises were introduced in the tutorial classes, of which three were run with the author as tutor. Two of the three tutorial classes were experimental groups, running different versions of the writing experiment, and the third was the control, identically run to the remaining non-experimental tutorial groups. One of the experimental groups (the A, After, group) were required to write about the problem-solving processes after having carried out a problem calculation, and the other group (the B, Before, group) were required to write about the planned problem-solving process before carrying out problem calculations. The After group had one extra element, which was to make a brief statement of expectation (a few symbols, a short phrase) on what form the problem solution was anticipated to take. Data was collected in the form of interviews, the written exercises submitted over the course of a semester, the author's journal of observations and quantitative analyses of the students' assessment tasks throughout the academic year.

As shall be elucidated, the data revealed that the facet of the problem-solving process which the writing initiative particularly supported was Pólya's first step of *understand the problem*; more specifically *understand the mathematics underpinning the problem*. The process by which the writing encouraged understanding was modelled using Piaget's stage-independent learning theory.

Data Analysis

Three approaches of data analysis were taken. The first was analysis of carefully chosen problems within the standard course assessment. The second approach was to categorise the written submissions by their stance towards knowledge using Waywood's (1992) categorization scheme (Craig, 2011). The third approach was to view all the data through the theoretical lens of Piaget's theory of learning.

As much as was possible, each course assessment activity included a problem carefully chosen by the author and approved by the course convenor (also a co-supervisor) to be a 'problem' rather than an exercise and to have potential for revealing, through the students' attempted solutions, evidence of understanding and metacognitive control. The one measure on which the student solutions did show measurable improvement was evidence of understanding.

The writing exercises were categorized using Waywood's (1992) categorization scheme, which categorises written submissions (originally structured journal entries) into three categories: (1) Recount, exhibiting a passive approach towards knowledge, (2) Summary, exhibiting a utilitarian or functional view of knowledge and (3) Dialogue which exhibits a creative view of knowledge, something to be shaped through enquiry. Finally, all the observations in the data were viewed through the lens of Piaget's three-pronged theory of learning involving alpha, beta and gamma behaviour.

Theoretical Framework – Piaget's Theory of Learning

Piaget's theory of constructive learning begins with the learner encountering a novel item. The student will exhibit one of three kinds of behaviour, categorised by Piaget (1985) as alpha, beta or gamma behaviour. Alpha behaviour encompasses both no learning and poor learning, with the subject either denying that the item is novel at all, or constructing unstable knowledge structures that will not stand up to scrutiny. Alpha behaviour requires less cognitive effort than beta behaviour (true learning) and can be undertaken either consciously or inadvertently (Piaget, 1985).

When the student encounters a novel item there is an attempt to assimilate it into the existing cognitive structures. The subject applies a scheme, a package of cognitive items and actions, to the item, chosen for its similarity to items previously encountered. If the item is indeed novel then the scheme, properly applied, results in an unexpected outcome for the subject, termed a *perturbation*. The student's cognitive structures are said to be in disequilibrium, and the process of reacquiring equilibrium is termed equilibration. Robust reaction to perturbation involves a cycle of assimilation and accommodation. Assimilation is the act of applying to the item the subject's already existing cognitive operations, and accommodation is the modification of cognitive structures. During the process of accommodation, if the item involves or impacts upon logico-mathematical reasoning (as mathematics does) a process occurs called reflective abstraction. The difference between alpha behaviour and beta behaviour particularly resides in the absence or presence of reflective abstraction.

Gamma behaviour is epistemically more desirable than beta behaviour, yet it is not the type of learning that is usually expected by the teacher in the classroom. Gamma behaviour occurs when the item is indeed novel, yet the existing cognitive structures are sufficiently well developed that they allow for the possibility of variation on items previously encountered.

A particularly notable feature of Piaget's system, beyond the two pathways of alpha and beta, is the perturbation that is required to push, “force”, one might say, the subject into exhibiting beta behaviour. In order for beta behaviour to occur, there has to be a perturbation, a surprise or a disturbance, shunting the subject down the beta pathway instead of the much more easily accessed alpha pathway.

Discussion

The overview of the problem-solving writing project provided in this article is necessarily cursory, given the constraints of page count. A concise account is given here of the results of the project as evidenced by the different modes of data collection; (1) quantitative data, (2) the submitted writing exercises, (3) the author’s journal and (4) the interviews.

Quantitative data in the context of this project refers to the frequency data accumulated from observations of problem-solving behaviour in students’ assessment tasks. The data were collected for the full year, not only the experimental second semester, in order to be able to observe students’ problem-solving progress (if any) both before and during the experiment. Both understanding and metacognitive control evidenced potential improvement, in comparison to the Control group. The effects were not significant, but provided a suggestion that the writing exercises might be having an advantageous effect.

The weekly writing exercises submitted were analysed using the adapted form of Waywood’s journal categorization scheme. The results show a trend away from Recount and towards Summary and Dialogue. Over the course of the semester there was movement of relative frequencies away from simple recounting of facts and towards explanations. There was a change in the stance towards learning, away from the student as passive observer of objective knowledge and towards the student as active engager in the creation of knowledge (Craig, 2011).

The journal kept throughout the project, recording observations made largely during the tutorial sessions, was of limited use except for one particular feature. One of the secondary research questions of the project was *Are any observed effects of writing in problem solving different for students with differing main languages?* The journal came in useful in answering this question, particularly with respect to the mathematics register and how speakers of English as an additional language might be intimidated by the idea of using that register in explanatory writing (Craig, 2012).

The interviews revealed evidence of two levels of engagement with the problems encountered through the writing exercises. It would be possible to describe the two levels of engagement as a surface approach and a deep approach (Chi et al., 1981) or, similarly, Piaget’s (1985) modes of alpha and beta behaviour. Piaget’s stage-independent theory of cognitive development was found to be particularly useful as a lens for analysing the data as it not only recognized the two modes of engagement, but provided a mechanism for how those two modes came about.

A surface approach (as identified through the interviews and student responses) to the tutorial questions is characterised by being more likely to be invoked than the deep approach (that is, calculation without understanding requires less cognitive effort), not leading to understanding, often resulting in "dead ends" and providing only one view among many of a mathematical process. A deep approach is characterised by being more difficult to attain than a surface approach and is often non-spontaneous in that the writing exercises forced the students (so they reported in the interviews) into processes they would not otherwise have carried out.

In Piaget's learning theory, in the absence of a perturbation, alpha behaviour is more likely to be invoked than beta behaviour. Beta behaviour, with its reflective abstraction and accommodation, represents successful learning (Dubinsky & Lewin, 1986) while alpha behaviour represents either unstable or incomplete learning or no learning at all. The observations made by the students during the interviews support the observations made by the author during the study project of the existence of the two levels of engagement, shallow and deep, and their parallels with Piaget's alpha and beta behaviour. In addition, the requirement of a perturbation to invoke beta behaviour resonates with the aggressive language used by the students in reporting that the deeper engagement was "forced".

Conclusions

The requirement of the writing exercises was that participants describe and justify solution processes. In the After group, in which students wrote about problem-solving processes after carrying out calculation, the students were expected to look back (Pólya, 1945) over their solution, to reflect on what procedures had been carried out and why, and to reflect on what their solution expectations had been and whether their expectations had been fulfilled. In the Before group, in which students wrote about problem-solving processes before carrying out calculations, the students were expected to devise a plan (Pólya, 1945), reflecting on the problem requirements, possible procedures for solving the problem and why those procedures were justified. In both cases the demand to provide explanations and justifications required the students to engage more deeply with the mathematical requirements of the problems than might be expected through straightforward symbolic solutions of the problems. The demand for reflection on the problem solution processes encouraged the practice of reflective abstraction apparent in beta behaviour and not in alpha behaviour.

Understanding of the mathematical content of a question is more likely to be achieved if a deep approach is taken, that is if Piaget's beta behaviour is practised. The writing exercises encourage the students to take a deep approach both by encouraging beta behaviour from the first moment of working on a mathematical question and by challenging unstable knowledge structures created during alpha behaviour. There is evidence that continual practice of the writing exercises gradually deepens students' engagement with mathematical content, corresponding to a changing stance towards knowledge as a creative process in which the student can be actively engaged.

There is some evidence to suggest that non-English main language speakers experience greater difficulties with the mathematics course delivered in English than English main language speakers, and, in addition, experience greater difficulty with the writing exercises than the English-speaking students. There is no significant evidence that a student's level of mathematical preparedness impacts on their performance in or on their perception of the writing exercises.

Acknowledgements

The research discussed in this paper was supported by a grant from the Spencer Foundation. John Webb, of the Department of Mathematics and Applied Mathematics, and Tim Dunne, of the Department of Statistical Sciences, University of Cape Town, provided feedback on earlier drafts of this work.

References

Brown, A. L., Metz, K. E., & Campione J. C. (1996). Social interaction and individual understanding in a community of learners: The influence of Piaget and Vygotsky. In A. Tryphon & J. Voneche (Eds.) *Piaget-Vygotsky: The social genesis of thought*. Hove, UK: Psychology Press.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, *5*, 121-152.

Craig, T. S. (2007). *Promoting understanding in mathematical problem-solving through writing: A Piagetian analysis*. Unpublished PhD thesis, University of Cape Town.

Craig, T. S. (2011). Categorisation and analysis of explanatory writing in mathematics. *International Journal of Mathematics Education in Science and Technology, 42*(7), 867-878.

Craig, T. S. (2012). Experiences of non-English students in writing for learning in mathematical problem-solving. *Proceedings of ICME12 – 12th International Congress on Mathematical Education,* Seoul, July 2012 (forthcoming)

Dubinsky, E., & Lewin, P. (1986). Reflective abstraction and mathematics education: the genetic decomposition of induction and compactness. *Journal of Mathematical Behavior*, 5, 55-92.

Ellerton, N. F., & Clements, M. A. (1992). A research agenda for the "Writing Mathematics" movement. In M. Horne & M. Supple (Eds.) *Mathematics: Meeting the* *challenge*. Melbourne: Mathematical Association of Victoria.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, *16*(3), 163-176.

Kenyon, R. W. (1989). Writing is problem solving. In P. Connolly & T. Vilardi (Eds.) *Writing to Learn Mathematics and Science* (pp. 73 – 87). New York: Teachers College Press.

Piaget, J. (1985). The equilibration of cognitive structures – The central problem of intellectual development. Translated by T. Brown & K.J. Thampy from *L'Equilibration des Structures Cognitives* (1975). Chicago: University of Chicago Press.

Pólya, G. (1945). *How to solve it* (2nd ed.). Princeton, New Jersey: Princeton University Press.

Pugalee, D. K. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics, 101*(5), 236-245.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, Florida, USA: Academic Press, inc.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.) *Handbook of Research on Mathematics Teaching and Learning* (pp. 334-370). New York: Macmillan.

Stanic, G. M. A., & Kilpatrick, J. (1989). Historical perspectives on problem solving in the mathematics curriculum. In R.I. Charles & E.A. Silver (Eds.) *The teaching and assessing of mathematical problem solving*. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Waywood, A. (1992). Journal writing and learning mathematics. *For the learning of mathematics, 12*(2), 34-43.

Author

Tracy Craig, Academic Support Programme for Engineering in Cape Town, Centre for Research in Engineering Education, University of Cape Town
Email: Tracy.Craig@uct.ac.za

How do Undergraduate Mathematics Students Want to Learn?

Simon Todd
University of Canterbury

This article is based on an oral presentation given at Volcanic Delta in November 2011 and addresses the question of what students perceive as an ideal learning experience in an undergraduate mathematics degree. To do this, opinions are presented which were obtained through open discussion with undergraduate mathematics students at the University of Canterbury in late 2011, as well as reflections on my own recent years of undergraduate study. These opinions and reflections cover the wide topics of motivation, structure, communication, technology, and assessment. Ultimately, they create an image of the student's ideal undergraduate mathematics degree, one which aims to produce larger amounts of buy-in, and correspondingly higher degrees of success, from students.

In the editorial of the fourth issue of the CULMS newsletter, Kevin McLeod posed the question, "what do we want students of mathematics to learn?" In this article, I want to shift the focus of attention from a student-external view of 'what' to a student-internal view of 'how'; that is, as a student, I want to address the often unanswered question of how students want to learn.

While McLeod's question and the one presented here are ultimately different, it should be clear that they are not independent, and that therefore answers to the latter can provide valuable insight into ways of implementing answers to the former. After all, it is all very well to know what you want students to learn, but, in order for them to care enough to actually learn it, they must be willing to buy into the whole process.

It seems natural that such buy-in should be greatest when courses reflect the desires of the students, allowing them to do things the way they want. Furthermore, it also seems natural that students with large amounts of buy-in should be successful in their studies. Therefore, if it can be determined what students perceive as an ideal learning experience in an undergraduate mathematics degree, then this can be used to shape undergraduate mathematics degrees accordingly, leading to more enthused, successful students who will happily and keenly learn whatever their lecturer wants them to.

Sources of Information

To answer the question of how to promote buy-in amongst students, this article uses information gleaned from opinion; specifically, the opinions of students who were (primarily) in their third year at the University of Canterbury in 2011, of which I am one.

These opinions were collected through open discussion, where students from a second- and several third-year classes were invited to meet and talk informally about their mathematical experiences over pizza. In total, 12 students attended the discussion, all of whom were male. Of these 12, two were mature students who had returned to study after spending time in the workforce and three were first-years studying at advanced levels. Only two students were studying just mathematics, with one in the applied and one in the pure field. The other students were mostly studying mathematics in conjunction with engineering or a physical science. Additionally, four third-year students (3 male, 1 female) who were not able to attend, provided feedback via e-mail.[1]

I do not wish to imply that the views of this sample are representative of the views of students as a whole, due to obvious limitations and biases. However, the methodology of open, unstructured discussion in a relatively small group of peers is likely to elicit reflections which might be lacking in the results of a survey or other large-scale formal investigation which balances for various factors. For this reason, it is extremely difficult to see if these views generalise, so, to allow for discussion, I have assumed parts of them to be common to most students; if this meets with discomfort, the resultant broad judgements I make should be treated as merely relevant to the case at hand.

While there was no specific structure to the discussion, it centred around five main areas related to undergraduate mathematics education: student motivation; programme and lecture structure; mathematical communication; the use of technology; and methods of assessment. These areas will form the basis of this article.

Motivation

Clearly, one of the prime concerns when trying to get students to buy into their studies is what motivated them to pursue those studies in the first place and what has kept them engaged since. Such factors are relevant at any level and should be considered foundational for any endeavour that aims to capture

[1] Full summary notes from the discussion and e-mails can be found online at http://www.scholcalc.co.nz/downloads/Summary.pdf

student interest.

When discussing reasons for which they study mathematics, students often made reference to its relationship with a wide variety of other disciplines, stating it as the underlying "language of science". As the backgrounds of all but one of the students present revolved around applications of mathematics, this is to be expected. However, going beyond the applied-pure division, this idea has important ramifications that were seen in other comments – mathematics students appreciate physical implementations of their subject matter and see them as useful ways to build understanding of abstractions. By appealing to such implementations, lecturers may be able to better appeal to the interest of their students while also illustrating the wide-ranging power of mathematics.

Students also promoted the logically deductive nature of mathematics as a reason for studying it, saying that it built their reasoning skills in both educational and everyday senses. There was agreement that such use of logical deduction created objectivity in mathematics, as opposed to the subjectivity that students perceived to be prevalent in areas such as the humanities. One student stated that this objectivity gave mathematics a "purity", in that mathematical proof is unambiguous and holds forever, which he found highly attractive. Other students echoed this standpoint in a more practical way, stating that they enjoyed the fact that there is always a right answer in mathematics. Of course, this is not to say that there is only one *way* to approach mathematical questions; on the contrary, students were appreciative of the way in which logical deduction could interact with creativity to allow innovative approaches, in both solving problems and proving propositions. With this in mind, it seems that perhaps student interest could be bolstered by exploring how (and why) multiple mathematical approaches are valid and all lead to the same result.

Interestingly, it was not until specifically asked that students made any reference to mathematics being fun. However, there was a good response to this idea once prompted, with a number of students stating that the challenging nature of mathematics made progress rewarding and addictive. One student even likened solving mathematics problems to golf: it can be extremely frustrating hacking out of the rough when progress is not forthcoming, but that only adds to the satisfaction of a good shot down the fairway and onto the green when everything falls into place. It is this satisfaction, he argued, which triumphs over all the hard work and keeps him going back for more. In the absence of a struggle, however, any satisfaction is hollow, and so student buy-in can be better targeted with genuinely enjoyable problem-solving over slavish drill-type exercises.

Structure

Simply appealing to the things which motivate students in their mathematical studies may not create the optimum educational experience, however; student buy-in is affected not only by what is done, but also by the way in which it is done. This idea leads onto the discussion of structure, both within and outside of lectures.

At UC, undergraduate mathematics courses are typically packaged as a number of lectures each week complemented by an hour-long tutorial. On top of this, most lecturers hold office hours and invite students to approach them then. Large first-year courses are supported by additional drop-in sessions close to the time of exams, but, apart from this, the pattern does not vary widely.

When my classmates and I were in our second year, however, a general 200-level drop-in programme was trialled, and the feedback from students who attended this was positive. Conversely, none of the students who attended my open discussion had ever been to see their lecturers during office hours (and some even admitted they did not know when the office hours were). From this, it seems apparent that students want their formal lecture-based learning complemented by more informal learning outside of lectures, which office hours do not allow and which standard tutorials may not encourage. The view of the students at the discussion was that drop-in sessions should be more widely available, particularly due to the flexibility and social environment they offer.

For within lectures, students advocated the use of an initial 5-minute recap of previous work and of an organised structure consisting of clear sections and subsections. A number of the contributors also indicated that they enjoyed courses that closely follow a textbook, both because it provides a clear structure and because it offers a secondary source of information with alternative examples and methods of explanation. In-lecture exercises were seen to be valuable at lower levels, but infeasible at higher levels since no progress could be made in the small amount of time allowed without sacrificing the complexity of the problem. However, the feedback link that such exercises provide was seen as useful, leading one student to suggest that the task could be simply to *start* a problem at higher levels, as this is often the most conceptually difficult and important.

Communication

Structure and motivation are quite broad concepts, which may not have that much of a direct impact on students. One thing that does have a direct impact is communication. Communication in a mathematical context can be used by the lecturer to encourage student buy-in and by the students to reflect this buy-in.

When talking about communicative efforts by lecturers, students were able to identify a number of areas that they find helpful and others that should be avoided. For instance, as previously mentioned, students indicated that they find real-life examples and analogies preferable to immediate abstraction, and, as a flow-on from this, prefer the use of informal speech when describing formal written statements such as mathematical definitions. Such informal speech is well accompanied by diagrams and even physical demonstrations, such as the use of pens and boards to represent vectors and planes in linear algebra. Students also indicated that interaction and questions play an important part in both keeping their interest and keeping the lecturer grounded with respect to what is actually being understood, but that "babying" questions, which have obvious answers and seem to be asked simply as a means of interrupting the lecturer monologue, are counterproductive. The views collected indicate that there should be two types of questions asked by lecturers: intelligent ones which aim to extend understanding, such as "what if…", and (not necessarily mathematical) ones which simply check that the messages being delivered are being properly received. The latter category should occur frequently and could include the opportunity for students to ask their own questions, encouraging more of a dialogue in the classroom. Ultimately, students experience the most buy-in when the communicative efforts of lecturers treat them as interested laypeople with some background knowledge, rather than as anything more.

Communication is not just something for lecturers, however; students have a relationship with it as well. For me, communication is key to mathematics, and the extent to which I use it (in the sense of verbalisation) reflects the extent to which I understand and engage with the subject matter. This seems to be a viewpoint which is shared by my lecturers who constantly write "use full English sentences to explain your working" on assignment instruction sheets, but is it also shared by other students? I asked the discussion participants whether or not they actively put effort into being communicative in their assignments, and the majority answered positively. One student responded with "definitely", justifying his position by saying, "Just because maths is fundamental to science does not mean it's fundamental to being a human being – we're not scientific in nature – communication adds that human aspect." Other students were less absolute, answering that they recognised the importance of communication and wanted to communicate effectively, but did not know how; nobody had ever told them what 'mathematical communication' meant and how much was expected of them. There were also reports of mixed messages, where some lecturers would stress the importance of verbalised communication while others would completely deny it. I argue that, given that students seem to want to communicate, they can be made to buy into their studies more if lecturers consistently respect this, giving both opportunity and guidance for it. This should bolster confidence and, consequently, success, not only in academic situations but also in everyday ones, where communication skills are equally as important.

Technology

A major talking point of the discussion was the use of technology, given its heightened prevalence at UC following the Canterbury earthquakes. It was the opinion of most third-year students present that technology offered some good ways of supporting study, particularly when its use reinforced the structural and communicative concepts already discussed. While the younger students were less enthusiastic about the use of technology, they attributed this to low self-discipline which would create problems for the increased level of self-management it promotes, rather than to any flaw in the technology itself.

In alignment with communicative concepts, the idea of using clickers as a means to obtain instant class feedback was positively received, though no students present had actually been in lectures where clickers had been used. Students also commented on how helpful they had found online discussion forums (either on Moodle or on Facebook) as an informal, convenient way to ask and answer questions with both peers and lecturers. One student highlighted the necessity of an environment like this by commenting that "the majority of students doing this subject do not talk to each other [face-to-face]". However, there was recognition of the fact that such efforts do not always work, especially when they are not actively promoted and easily accessible. Personally, I think it is important for lecturers to give students opportunities to respond to their peers' queries before jumping into the discussion themselves, which is difficult to manage without letting the forum lapse into silence or unanswered questions. If everything works out, though, it is clear from student attestations that the use of technology can enhance communication and thus increase buy-in.

Some discussion was also devoted to online lectures, in the form of pre-recorded videos and video conferencing. Most of the students present had experienced learning through pre-recorded videos and indicated that they found them helpful in the absence of face-to-face contact, particularly for revision purposes. Students also indicated that the use of video lectures was most effective when combined with a more informal style of contact such as drop-in help sessions. Two flaws identified in pre-recorded videos were the onus on self-managed learning and the inability to interact with peers and the lecturer. Though these flaws are not present in video conferencing, it was agreed upon that neither of these forms of delivery could replace personal contact. Furthermore, it was argued that any form of online lecture should be delivered in smaller pieces according to content, rather than following the traditional time-based model. Such an approach would be healthier for students watching videos, would allow focus to be sustained more easily, and would reinforce the structural concepts which promote student buy-in.

Assessment

One of the easiest ways to lose student interest is through assessment: nobody wants to be put under large amounts of pressure for little gain or feel like they have failed. It therefore follows that ‘student-friendly’ assessment is crucial to building buy-in. What, though, does such assessment look like?

The students I gathered opinions from indicated that they do not mind regular assessment and in fact find it helpful as a means of staying on top of their studies, provided it does not cause them undue stress. Such stress could arise from assessment being too frequent or too large. Students indicated that fortnightly assignments worth up to 10% each were preferable, while any weekly hand-in tutorial problems should be smaller, worth less and have flexible due dates. Some students expressed the opinion that any larger projects should be optional, for extra credit, so as not to cause unnecessary overloading. This could tie in well with the idea of having variable weighting for pieces of assessment, where doing an optional larger project could cause the final exam to be worth less. Students were in favour of this idea because it would give them control over how they demonstrated their knowledge: some disliked assignments because of the extra workload, preferring to load everything into the exam instead, while others disfavoured the artificial constraints of exams compared to the more ‘real-world’ nature of assignments. The overall consensus was that a low-stress assessment programme that allows flexible control based on the student would be well received.

It is not just the structure of the assessment which can be used to promote student buy-in, however; effective marking can also lead students to feel more confident and successful, even when they get things wrong. The discussion identified two main ways in which this can be done: by giving feedback and by allocating marks holistically to reward both ability and hard work. As a student, I feel cheated when I put hours of work into something and get only ticks and crosses in return. I want feedback to acknowledge what I have done well, what needs improvement, and how that improvement could occur; only with such feedback can I feel that my work has been respected, take pride in the things I’ve done well, and feel confident that I know how to improve for next time. Similarly, I feel better if the allocation of marks rewards not just the final answer, but all the work I’ve done to get there. If it is clear that I have put in a lot of work and made good progress with a problem, despite not knowing how to finish or making a small mistake along the way, recognition of this with partial marks also serves to give pride and confidence. These views were echoed by students in the discussion, who also indicated that consistency of feedback and marking, both within a course and across different courses, is important in making them feel positively disposed towards their studies.

Concluding Remarks

This article has attempted to answer the question, “how do undergraduate mathematics students want to learn?” by using opinions to paint a picture of attractive learning for students. According to the students I spoke to, attractive learning is multimodal, natural, manageable, organised, rewarding and less formal. While the combination of these aspects may be somewhat of an idealisation, each of them taken individually is valuable in promoting student buy-in. It is hoped that you, the readers of this article, will take these aspects and the opinions presented here (or opinions collected from your own students in a similar manner) into consideration in your educational practices, creating an environment that allows students to enjoy, and therefore succeed in, whatever you want them to learn.

Author

Simon Todd, University of Canterbury. Email: simon.todd@canterbury.ac.nz

PCAST on USTEM: What on Earth is That?

Bill Barton
The University of Auckland

Recently, in America, a report emerged from the President's Council of Advisors on Science and Technology (PCAST) on Undergraduate STEM education. David Bressoud, a mathematics professor and past president of MAA, has a blog in which he gives a summary (and link to the full report): http://launchings.blogspot.com/2012/03/on-engaging-to-excel.html

The report claims that introductory mathematics courses for those entering science, technology, engineering and mathematics do little to encourage students to continue in these fields, and that preparation for teaching undergraduate mathematics is behind that in engineering, technology and science. Quite a challenge to us if we accept that such a statement is true (or even partly true).

The report worries about the poor preparation of students entering undergraduate courses, and recommends bridging and remedial courses for students using technology. Unsurprising.

What is new is the suggestion to boost College and high school mathematics teaching using people from “mathematics-intensive” disciplines other than mathematics. Actually, it recommends a national experiment to do this.

The response in America to this suggestion has been varied — those suggesting we as a mathematics/mathematics education community should take the positive aspects of this, encourage learning from experiences in other fields, welcome the acknowledgment of the central role of mathematics, and welcome a move to active learning; and those worried about teachers with a less than comprehensive mathematical perspective on the world.

Much of the debate swings around what “mathematics-intensive” means. Is this what I call mathematical sciences: mathematics, statistics, computer science, engineering science, physics; or does it include all of engineering and chemistry and economics; or is it even broader including biology, psychology and geological sciences?

We need to pay attention – I feel sure that these suggestions, and the accompanying debate, will spread to Australasia and elsewhere.

Author

Bill Barton, The University of Auckland, NZ, Email: b.barton@math.auckland.ac.nz

2011 Delta Conference on Undergraduate Mathematics and Statistics Teaching and Learning: Teachers' Day and Issues of Mutual Interest

Greg Oates
The University of Auckland

The 8th Delta Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics was held in Rotorua from the 27th November to the 2nd December 2011, with 155 delegates from many countries, especially the constituent Delta communities of Australia, South Africa, New Zealand and South America. It was jointly organised by the Universities of Auckland and Canterbury, with grateful support and sponsorship from many other institutions, including AUT, Unitec, Waikato University, Victoria University, NZIMA, CULMS, NZMS, Statistics NZ, HRS and Ako Aoteoroa. The theme of the conference was "Te Ara Mokoroa", or "The Long Abiding Path of Knowledge", with presentations focusing on the path students take between university and school, both in bridging courses & adult education through to teacher training, emphasising the links between the different mathematical domains of pure and applied mathematics, statistics & engineering. An innovation at this year's conference was a Teachers' Day for upper secondary Calculus and Statistics teachers. The following is a brief report of the three invited presentations at Teachers' Day, with a further twenty two papers selected from the remainder of the conference, of common interest to secondary and undergraduate teaching.

Prof. John Mason (Open University, UK) opened Teachers' Day with a discussion of the confusion that students experience with definitions and the multiple ways in which mathematicians and teachers use the term *definition.* Mason noted how mathematicians sometimes use the word *define* to make global definitions which are supposed to be stable over time and place, other times they use it when they define a new concept. He believes we should distinguish between *extractive* definitions (usage is described and reported, as in a dictionary) and *stipulative* definitions (specifying required properties, as in mathematics). Students are culturally immersed in and familiar with the former, but in order to succeed in mathematics they have to become used to the latter. He suggests ways in which secondary teachers may prepare students for the emphasis on stipulative definitions at university (Hannah, Thomas & Sheryn, 2011, pp. 267-280).

Caroline Yoon (University of Auckland) considered the notion of inverse problems, and noted how these are often more difficult than their direct counterpart, for example factorising is more difficult than expansion. She observed that mastering a mathematical process in the inverse direction often leads to deeper mathematical understandings, and illustrated this with examples from her case-study of eight undergraduate students working on inverse problems in calculus and ratios. Although the students were all adept at working in the forward direction (e.g. differentiating), they struggled when asked to solve the problem in the inverse direction (anti-differentiating). The students adapted their knowledge of the problems in the forward direction to make the inverse problem easier, helping to reveal the depth of their mathematical understanding (Hannah et al., p. 487).

Chris Sangwin (Invited speaker, University of Birmingham) considered the increasing use of computer-aided assessment (CAA) in mathematics, specifically the use of computer algebra systems such as the STACK system designed by Sangwin, to support online assessment. Typically in such systems, a student will be asked to answer a mathematical question through a web browser. Their answers take the form of a mathematical expression, which the system then assesses, and provides feedback in the form of a numerical mark or a text-based formative response. Sangwin gives the example of systems of polynomial equations, with a particular focus on establishing when two sets of polynomial equations represent the “same” situation. He described how the system establishes whether the student's system is correct, and whether their system of equations is inconsistent, underdetermined or overdetermined. In each case, the equation/equations responsible can be isolated, and feedback provided. Students may receive valuable insights into mathematics, and teachers a greater appreciation of the subtle issues associated with mathematics assessment (Hannah et al., p. 473).

The remaining part of this report briefly describes eleven presentations for each of calculus and statistics, by nature of the classes they report on, or the content examples they consider. Other papers of a more general educational nature that may interest teachers can be found in the two sets of conference proceedings (Hannah et al., 2011; Thomas & Hannah, 2011). These include the use of tablet PCs and recorded lectures in undergraduate courses (Holgate; Yoon, Oates & Sneddon; in Hannah et al., pp. 437; 475); the use of online aids to support large classes (Harding, Engelbrecht & Verwey, in Thomas & Hannah, pp. 847-856); and academics’ perceptions of software in mathematics, statistics, econometrics and finance (Kyng, Tickle & Wood, in Hannah et al., p. 444). Paterson and Sneddon examined the use of team-based learning techniques in advanced mathematics courses (in Thomas & Hannah, pp. 879-890), and two papers by Kensington-Miller, Greenwood and Afamasaga-Futai, and Sheryn and Greenwood discussed issues for Pasifika students (in Hannah et al., pp. 159-169; pp. 345-354).

Calculus

CAS-technology in the teaching of calculus is examined in two papers using different computer-software applications (*Derive 6.0, GeoGebra, Scientific Work Place 5.5, Mathematica 8.0, Wolfram Alpha*), and two more using CAS-calculators, the *TI-Nspire* and the *TI-89*. Lin and Thomas (in Hannah et al., pp. 216-227) describe how integrated programmes such as the open-source mathematical software GeoGebra may be used to improve students' understanding of Riemann integration and the Fundamental theorem of Calculus. Ponce-Campuzano and Rivera-Figuero (in Hannah et al., pp. 303-313) used a variety of CAS-software to compute antiderivatives of functions, and the examples they provide support the important role of technology in observing and exploring mathematical concepts and theorems related to the Fundamental Theorem of Calculus. Tobin and Weiss (in Hannah et al., pp. 375-385) describe research on the use of CAS in teaching a traditional differential equations course, and the response of students given the opportunity to use CAS in their examinations. Their results suggest that CAS use by students is patchy, and that students have yet to use the CAS to be able to backtrack to solve aspects of a problem other than the direct DE solution. Ng (in Thomas & Hannah, pp. 925-938) describes a design experiment with a class of 35 students from a secondary school in Singapore using the TI-Nspire™ with the aid of TI-Nspire™ Navigator, a wireless classroom network system that enables instant and active interaction between students and teachers. The TI-Nspire™ enabled students to better visualize the concepts and make generalizations about relevant mathematical properties, as well as linking multiple representations (especially algebraic and graphical representations) to improve their conceptual understanding and problem-solving skills. These four papers suggest both a significant role for technology in aiding student understanding of calculus concepts, as well as ongoing research into problems such technologies may pose for students' procedural skills and technological proficiency.

Three papers consider the use of projects and investigations with practical models to motivate and extend students learning. Schott (in Hannah et al., pp. 336-344) describes the effects of students working in teams on a variety of interesting projects involving growth processes and first-order differential equations. He notes that the reaction of students and staff to such projects was generally highly favourable. Dagan and Satianov, and Satianov and Dagan (in Hannah et al., p. 424; p. 461), describe two approaches using real-life models and investigations that they have used with promising results to help students overcome their common perception of calculus as purely an abstract subject. In the first, they observe that while students can often perform formal symbolic operations successfully, they have difficulties in their interpretation. They describe the use of non-formal metaphoric schemes of function investigation, which assume answers on certain non-formal questions, but require students to give proper visual interpretations at each stage. They found that students using this approach were more self-confident in their results and made fewer mistakes, not only in graphic representations, but also in formal operations such as the calculations of limits. In the second approach, they suggest that while computer models certainly have their advantages, they should not completely replace the real tangible models in mathematics teaching. They give examples where students were encouraged to build their own models, which they could then physically touch and hold in their hands. Such an approach was shown to improve engineering students' understanding of abstract notions and calculus facts. They suggest it may also promote creative thinking and stimulate students' interest in learning and the applications of calculus.

Ho, Quek, Leong and Lee (in Hannah et al., pp. 129-138) describe an innovative instructional strategy they call *guided-exercise*. It is designed to facilitate and enrich student learning, and takes the form of a series of partially completed worked examples with instructional scaffolding. The students use these alongside the lectures, with assistance from the lecturers when necessary. The authors describe the rationale, design and use of this strategy, and report strongly-positive feedback from students with respect to its effects on learning. In another novel approach in the calculus context, Craig (in Thomas & Hannah, pp. 867-878) studied the effects of explanatory writing on students' problem-solving behaviours. Her study used examples from calculus involving the convergence of improper integrals, and she notes that the scheme successfully observed positive changes over the experimental period in students' level of engagement with the mathematical material and with their stance towards knowledge.

Finally, two papers examine aspects of mathematical knowledge and understanding. Klymchuk and Thomas (in Thomas & Hannah, pp. 1011-1020) compare the attention teachers and lecturers pay to mathematical knowledge in their use of mathematical procedures. They examined the responses from 178 secondary teachers and 25 university lecturers to four calculus-based questions, and their results reveal that many teachers and lecturers fail to notice the necessary conditions for problems that imply certain procedures are not always applicable. They conclude that these findings suggest explicit training in the discipline of noticing could be a useful addition to professional development of both school teachers and university lecturers, especially those in the beginning of their career. Yoon, Thomas and Dreyfus (in Thomas & Hannah, pp. 891-902) consider a generally unrecognised aspect of mathematics instruction when they look at the role gestures play in advanced mathematical thinking. They argue that the role of gestures goes beyond merely communicating thought and supporting understanding – in some cases, gestures can help generate new mathematical insights. Their results suggest that gestures are a productive, but potentially under-tapped resource for generating new insights in advanced levels of mathematics, specifically calculus in this instance.

Statistics

The keynote presentation by Jennifer Brown provided a fitting introduction to the focus on critical thinking, and increased use of online- and other technologies in the delivery and assessment of statistics, which was a common theme at the conference. Brown described major changes to their first-year statistics course at the University of Canterbury, with the shift to an emphasis on teaching critical thinking (Brown & David; David & Brown; in Hannah et al., p. 424; pp. 53-59). They have moved from thinking about statistical skills needed for a statistician, to skills needed to participate in today's society, with less emphasis on lectures, and more on computer-based tutorials, Excel, computer skills testing, and written assignments which demand greater descriptive and analytical writing.

Pfannkuch, Regan, Wild, Budgett, Forbes, Harraway and Parsonage (in Thomas & Hannah, 2011, pp. 903-913) describe the rationale and arguments for their shift: from the teaching of inference based on the normal distribution, the Central Limit Theorem and the sampling distributions of estimates; to the use of computers to embrace randomization and bootstrapping methods, pioneered in 1979 by Efron. The current theory-based approach with its mathematical procedures of inference as well as its multiple underpinning theories is seen as an obstacle to understanding. Bootstrapping and the randomization method offer simplicity and direct access to the logic of inferential reasoning, with opportunities for better conceptual understanding of the thinking underpinning inference.

Cheang (in Hannah et al., pp. 44-52) explores ways in which the simulation and graphing capabilities of the statistical software package R may be used to enhance students' conceptual understanding of the inference methodology. He provides examples of simulations that may be used in the teaching of confidence intervals and hypothesis testing; for example the simulation of t-intervals for μ from a normal population with unknown variance. This may help address common misconceptions such as the belief that the population parameter is a random quantity because it is unknown, while the interval is fixed because it is calculated.

Dunn, Richardson, McDonald, Oprescu and Fairweather (in Hannah et al., pp. 69-77) promote the use of mobile phones as a Classroom Response System (CRS). CRS systems in general have been shown to foster student engagement through the provision of immediate feedback. Mobile phones provide a much cheaper and more accessible approach to CRS, than the earlier systems using clickers. Their choice (from many options) is VotApedia (http://urvoting.com), which is free to use and implement. The usual way to use a CRS is to pose a question, generate student responses, and then reveal and discuss the correct answer. Uses in their study included asking questions at the start of a lecture to review important concepts from the previous lecture; and posing questions to promote discussion during the mid-lecture break and refocus the class afterwards.

Edwards, Fitch, Heath, Jones, Jones, Leader and Stirling (in Hannah et al., pp. 86-92) describe their use of an interactive e-book instead of a traditional textbook in a large service Statistics paper. They developed their own e-book based on the e-learning software framework CAST, since it could be customised with exactly the topics that were required for their course syllabus. While there has been little published evidence to support claims of improved student performance through using e-books instead of paper-based textbooks, they suggest several reasons as a rationale for their decision, such as accessibility to students, interactive features, and the use of tools (e.g. Java) that allow quick, immediate, and visual feedback. While student feedback to the project was mixed, the authors nevertheless believe that continued development and feedback is warranted.

Stewart and Stewart (in Hannah et al., pp. 355-364) provide an alternative spin on the teaching of statistical inference, when they describe an innovative approach using ventriloquist dolls to grab students' attention and embed important ideas in revealing the differences between the Bayesian and classical paradigms. While Bayesian methods lie outside the domain of most secondary statistics syllabi, this study nevertheless provides a refreshing approach to teaching difficult new concepts, and suggests that the novel approach resonated with the controversial ideas presented in a Bayesian statistics course.

Gunn (in Hannah et al., pp. 120-128) describes her development of a rich statistical *sampling* activity in first-year statistics. She presents her reflections on this activity as a meaningful statistical learning task, using a technique to describe the task using *points-of-noticing,* and their relevance to the design of statistical tasks. Her discussion is informed by the notion of embodied knowing; ideas from complexity science; established criteria for good learning activities; understandings of mathematical task design such as those proposed by Mason and Johnston-Wilder (2004); and research into creativity in higher education.

Bilgin, Jersky and Petocz (in Hannah et al., p. 422) describe the development of their introductory course in statistical consulting. The main aim of this unit is to prepare graduates for the workforce by providing them opportunities to carry out authentic statistical consulting. Their unit covered topics such as: the human side of statistical consulting; asking the right questions of clients so that we can translate their problem into a statistical problem; learning to work in a group - after all, no statistician works in isolation except perhaps theoretical statisticians; and writing a statistical report that makes sense to a client. While they have managed to prepare an appropriate series of lectures for the course, they note that important questions remain regarding what to assess, how to assess, and how not to overdo assessment, all while paying attention to correct technical skills.

Fletcher and Reyneke (in Hannah et al., p. 431) describe various intervention strategies adopted by the Department of Statistics at the University of Pretoria to address the entrenched problem of poor pass-rates in compulsory first year Statistics courses. They give a brief explanation of the set of problems unique to South Africa, and then discuss the effects of a succession of mediations over the past couple of years, starting with compulsory homework assignments introduced in 2006, found to be flawed as an aid to learning. It was thus replaced by a system of class tests in 2007. Finally, a modified matriculation curriculum was introduced in 2008, and they provide an analysis of students' performance before and after, to illustrate the impact of this change. A rationale is provided for a more interactive intervention where students will be required to complete assignments online, with immediate feedback, to be implemented in 2012.

Finally in this section of presentations focused on statistical issues, Penny (in Hannah et al., p. 457) raises questions about sampling theory, survey methodology and survey theory. Sampling theory is a major part of what is taught at university, but the practice of surveying requires knowledge of survey methodology which is often seen as just another bundle of techniques. Many of the methods require good knowledge of statistical theory and he suggests they should be taught along with sampling theory as a coherent and statistically based theory of surveying.

References

Hannah, J., Thomas, M. O. J., & Sheryn, L. (Eds.) (2011). *Te Ara Mokoroa: The long abiding path of knowledge. Proceedings of Volcanic Delta 2011, The Eighth Southern Hemisphere Conference on Teaching and Learning Undergraduate Mathematics and Statistics*, Rotorua, New Zealand, 27 November - 2 December 2011: Auckland, New Zealand: The University of Auckland and The University of Canterbury. Available online from: www.math.canterbury.ac.nz/~j.hannah/Delta11/VolcanicDelta2011Proceedings.pdf

Thomas, M. O. J., & Hannah, J. (Eds.) (2011). Special Issue: Volcanic Delta 2011: The Eighth Southern Hemisphere Conference on Teaching and Learning Undergraduate

Mathematics and Statistics, Rotorua, New Zealand, 27 November - 2 December 2011, *International Journal of Mathematical Education in Science & Technology, 42*(7), 831-1021.

Author

Greg Oates, The University of Auckland, NZ, Email: g.oates@auckland.ac.nz

On Knowing in Mathematics

John Mason
The Open University, UK

Introduction

It is a well known and all too common phenomenon that students often do not check the conditions of a theorem or a technique before applying it. For students of pure mathematics this is bad practice; for engineers and other users of mathematics it can be disastrous. A second, less well recognised phenomenon is that many students have no idea how to study mathematics, and in particular, how to study for a mathematics exam.

Both of these phenomena are for me instances of the difference between *knowing-about* (a spectrum which includes knowing-that, knowing-how, knowing-when and even knowing-why) and *knowing-to* act in the moment. The latter requires that an appropriate action comes-to-mind (to check the conditions, to apply a particular theorem, to use a particular technique, to make use of a range of study techniques) in the moment when appropriate.

Being satisfied with knowing-about is typical of students who have adopted or migrated to an epistemological stance which Ference Marton and Roger Saljö (1976a, 1976b) called *surface* learning, in contrast with *deep* learning. People who treat mathematics as a tool are more likely to migrate to surface learning more quickly than students of mathematics, but as a course progresses and as the concepts and techniques mount up, many students, despite their original intentions, migrate to a form of surface learning. For example, Open University students tend to begin courses with the intention of deep learning, working on the study material and then preparing their responses to the assigned assessment tasks, but as the pressure mounts up and they fall behind, they begin to look at the assessment tasks before working on the study material, and eventually their pragmatic study technique is to look through the materials to find something matching the assessment.

My aim here is to highlight what is for me the primary distinction, between *knowing-about* and *knowing-to* and to suggest ways in which students can also be supported in knowing-to act as well as developing their knowing-about mathematical content. I use the gerund *knowing* rather than the noun *knowledge* to indicate that the various types of knowing are not static but rather are dynamic, always in flux and capable of extending as well as contracting, and not always available to be called upon in any case. The ideas are developments from the notion of *inert knowledge* used by Alfred Whitehead (1932, pp. 8-10) and distinctions between types of knowing considered by Gilbert Ryle (1949).

Knowing About

There is a broad spectrum of 'knowings' (not necessarily confined to one dimension) that are consequences of English prepositions and hence are enculturated distinctions.

Knowing-That

Memorising formulae and procedures is an age-old approach to passing examinations. In a Confucian-influenced education culture, memorisation is an initial step to provide the fodder for deeper analysis and contemplation. In Western culture a similar strategy underpinned Jesuit instruction, with vestiges remaining in the USA in the form of *recitation* as the name for tutorials: in the 19th century reciting of lecture content by students was common practice. The question is not whether memorisation is good or bad, but rather what students are encouraged or prompted to do with what is memorised, that really matters.

For example, internalising the expansion formulae for $\sin(A + B)$ and $\cos(A + B)$ seems useful for anyone needing to deal with trigonometry; but memorising dozens of trigonometric identities seems fruitless. On the other hand, having to re-derive it for oneself every time is a waste of time energy, but not having access to how to derive it when necessary is a form of dependency.

Knowing a definition (being able to recite it, a form of knowing-that) is one thing, being able to re-construct it is quite another, and having it come-to-mind when needed is different again. David Tall and Shlomo Vinner (1981) captured this nicely in their distinction between *concept definition* (a formal definition) and *concept image* (the collection of associations, images, links, words and phrases, techniques and other actions, contexts where met etc.). Most mathematicians will re-construct a definition from their concept-image. But if students' concept images are impoverished, they may not have enough connection, or in the metaphor proposed by Yuichi Handa (2011), a deep enough relationship with the concept to re-construct it. They are then equally unlikely to have it come-to-mind when needed.

Many of the facts that we have at our finger tips (that readily come-to-mind) have been internalised through a process which Caleb Gattegno (1970) called 'internalisation through subordination'. He was referring to the way an expert requires very little attention to carry out an established action (such as algebraic manipulation), and he proposed that this is achieved most efficiently not through mindless rehearsal of standard tasks, but through having attention directed away from the 'doing'. One way to achieve this is to set tasks in which students are prompted to construct and calculate with particular examples for themselves as part of their natural process of specialising in order to locate underlying structure which they then articulate as their own (re)generalisation. During the process attention is directed away from the practice, which is itself mathematically purposeful.

Having their attention withdrawn from the action and reflected onto the action, here, of specialising in order to recognise structural relationships, can enrich their experience of this entirely natural but often overlooked strategy of sense-making. At the same time, the rehearsal of some calculation or technique supports its internalisation, providing both a local 'purpose' and an experience of 'utility', in the language of Ainley and Pratt (2002).

Knowing-How

Successful completion of routine tasks is taken as evidence that students know-how to do something. But it is entirely possible and frequently resorted to, to train behaviour to react to various cues in order to initiate an action. For example, the words 'extreme values', 'maximum' and 'minimum' are cues to embark on a calculus routine. A great deal of energy has gone into trying to train students to solve word problems by getting them to look for arithmetic-operation cues; relatively little has gone into getting students to 'enter the situation' using their mental imagery and their past experience and to seek out and express structural relationships. There is a long history of teachers prompting students to train their behaviour by rehearsing procedures. However the issue is whether students will recognise the appropriateness of one of their trained behaviours in some fresh and slightly novel situation. This is the role of 'awareness', which in the language of Gattegno (1987) means 'that which enables action'. It is becoming aware of the features which suggest the appropriateness of a particular procedure that really matters, and this means working on *knowing-to* act.

For over 3000 years students of mathematics have been given worked examples and then sets of exercises to do. The teacher-tutor works through an exercise, and then abjures the students to "do thou likewise" or "in this way is it done" (Gillings, 1972; Cardano, 1545/1968). The teacher-tutor is instantiating a general method or technique in a particular instance; the student is immersed in the particulars. The student has to discern the difference between structural constants and parameter values. Indeed, seeing parameter values as values of parameters already opens the way to recognising the particular task as an example of a class of similar tasks. However it is not always clear to students what exactly is being exemplified (Watson & Mason, 2005).

Extensive research on worked examples by Alexander Renkl and colleagues (see for example Atkinson, Derry, Renkl & Wortham, 2000, or Renkl, Stark, Gruber & Mandl, 1998) has isolated the features of a worked example that actually enhance student appreciation of what is being exemplified. In particular, students want to know not just ‘what to do next’ but ‘how you know what to do next’. One approach is to allocate a short period of time to having a tutor work an exercise publicly, while trying to bring to articulation the thoughts and incantations that come-to-mind. Using the worked example as a template is one thing, but deepening your appreciation of how other people know-to act in certain ways at each step is what is required for successful performance.

Ference Marton (Marton & Booth, 1997; Marton & Pang, 2006) articulated *Variation Theory* as a way of determining from a session what is available to students to learn, because he considers (with Aristotle) that to know a concept is to know what it is that can be changed (‘dimensions of possible variation’) and over what range (‘range of permissible change’).

For example, knowing the fact that “the sum of the internal angles of a planar Euclidean triangle is 180°” is one step, but not the same as being aware of how triangles can be varied (very large even astronomical, very small, even microscopic; nearly equilateral and extremely ‘pointed’). Many students seem to think that if one vertex of a three sided figure falls off the screen or is otherwise extreme, the figure becomes a ‘stick’ and is not strongly associated with ‘triangle’ (Sfard, 2008). They have not developed a richly inhabited example-space for triangles.

I have observed a lesson in which all of the students, when they heard “vertically opposite” could complete the utterance “angles are equal”. However few if any recognised a pair of vertically opposite angles when they were displayed, vertically; none recognised other pairs to which the same label applies even though the quality of verticality is absent. They need to be exposed to a variety of diagrams and situations in which a pair of lines cross and in which the angle at the crossing is important.

Students trained to Lagrange multipliers to find extremal values of a function of two or more variables are likely to get into trouble when the situation is not quite standard, and few have any images or other appreciation to fall back on in order to work out how to apply the technique in unfamiliar situations.

According to variation theory, recognising aspects that can be changed requires experiencing them as changing within a local neighbourhood of space and time. A concomitant is to have attention drawn out of the action and reflexively directed to the actions themselves.

It is traditional to assess students in other disciplines (including education) by getting them to write essays. The aim is to encourage them to coordinate and connect different sets of distinctions, and even to crystallise their network of associations and connections so as to inform their practice in the future. However, writing an essay about how an issue in teaching and learning is at best an initial step on the road to informing future practice.

Writing an essay about 'how to do a particular type of question' leaves untouched the experience of having-come-to-mind when required. However, reflecting on this after work on a problem, and placing oneself mentally in the future having the same idea come-to-mind is perhaps the only method people have of preparing for knowing-to.

Knowing-Why

We would all like students to know-why they are invoking a technique or a theorem, and to know-why a theorem is true or why a technique works. The classic distinction between *instrumental* and *relational* understanding made by Richard Skemp (1976) which admits plenty of overlap on a continuum between extremes, has been transmuted into a gap between *procedural* versus *conceptual* which dogs the teaching of mathematics at all levels (Hiebert, 1986). Service course students want to be 'given the tool kit' that they need, while mathematicians want students to appreciate the uses and the limitations of specific techniques, and to do this, indeed to use tools appropriately, it is valuable if not essential to know-why they work. Again, all teachers and lecturers run into the surface-deep distinction mentioned earlier: despite intentions otherwise, deep approaches to learning are frequently displaced by surface approaches.

Handa (2011) describes his own development from a surface-dominated approach to a deep personal relation with mathematics, both with respect to its actions and its content. He roots *understanding* etymologically as 'standing in the midst of' rather than being separate from. If students do not develop a relationship with mathematics, through, for example, developing richly structured example spaces (Watson & Mason, 2002) with corresponding construction techniques, developing concept images and a disposition to have-a-go, make a conjecture, try to see from some examples what might be going on structurally in general, then their relational understanding will remain at arms length, fragmentary and disconnected. For example, students whose concept-diagrams remained static, fragmentary or even quite different over time were the ones who dropped out of mathematics courses, while those whose concept diagrams grew in richness and complexity over time were much more succeed (McGowen, 1998). This is not simply an indicator of the individual, but suggests ways that students who may not have tuned into how mathematics calls upon them to use their natural powers in specific ways, can be supported, encouraged, and can make significant progress.

One of the problems with distinctions is that they can raise inappropriate questions, which appear to be important. Here there is a perennial question of whether it is better to teach procedures first and to expect conceptual understanding to grow, or to aim for conceptual understanding so as to inform the use and carrying out of procedures. Not only is there no definitive 'best way', the very separation exaggerates and exacerbates rather than clarifies the problem. The issue is how the two are entwined together, and how the disposition and intentions of students may need to be worked on in order to jog them out of a dependency rut. It often appears that mastering the how involves least effort, whereas in fact it can be a lot more efficient to develop both in concert: knowing what to do and how to do it at the same time, and appreciating why it is appropriate and what can sometimes go wrong all contribute to successful performance.

Knowing-When and Knowing-Where

You might think that *knowing-when* or its variant, *knowing-where* to use a technique or theorem is what we are all aiming for. But it comes as a bit of shock to find that people can write essays or discourse knowledgably about some topic without actually being able to do what they talk so well about. The root phenomenon is that 'knowing-when' to use a formula all too readily masquerades for *knowing-that*; a summary of experience in the form of a label becomes the thing in itself to be learned. Thus the school students mentioned earlier can tell you that 'when you have vertically opposite angles, then they are equal' but not actually recognise vertically opposite angles when they encounter them. In studying the appreciation of integration by students of engineering, education and pure mathematics, Shafia Abdul-Rahman (2005) found that although everyone claimed that they thought of integration as finding the area under a curve, very few interpreted a given definite integral as an area when asked to work with the integral or to construct other ones like it Their claims to *know-when* did not align with their observed behaviour.

In studying changes in undergraduate apprehension of mathematics, Scataglini-Belghitar and Mason (2011) found that their subjects, despite seeing a proof of the theorem that a continuous function on a closed and bounded interval of the reals attains its extrema, did not have it come-to-mind as a possible action in a situation in which they had to construct the interval for themselves, even after being shown how to do this in a 'similar' question in a tutorial. The central issue is in what is appreciated as 'similar'.

As soon as something that needs to be experienced (a mathematical theme such as doing-and-undoing or invariance-in-the-midst-of-change; a mathematical heuristic such as 'try working backwards'; a form of reasoning such as mathematical induction) is turned into something to be learned, perhaps joining a list of things to be learned, then opportunities to integrate these into one's functioning as *knowing-to* can all too easily be deposited in a surface level of knowing-when or knowing-what. Learning a list of 'things to do when you are stuck' is of little use, as is a printed list of mathematical heuristics: if you have to run down a list to look for something helpful, you are unlikely to recognise something appropriate when you get to it.

Knowing-What

Sometimes students have a sense of what they want or are required to do, but have not a clear sense of how to go about it. For example, undergraduates confronted with the need to formulate a proof of a statement in early analysis that seems to them to be obvious may know-what is wanted in some vague general sense, namely a 'proof', but not have much idea of how to set about constructing it. This state serves as a halfway house between knowing-that a proof is required and knowing-to act in ways that will develop into a proof.

Similarly, when students arrive at university, they are sometimes quite unclear as to whether or not their own response to a task is what is actually being sought. They know-that a proof is required, but are not yet clear what constitutes a proof. They do not have actions coming-to-mind that will enable them to check that something is a proof within the requirements of the course. Often lecturers will assert that an early course in analysis is really about inducting students into mathematical proof, without recognising that student attention is very often immersed in details about what is being proved, and not on the criteria for what makes a line of reasoning part of an acceptable proof, nor even on how one knows what to write 'next'.

Of course if knowing-what includes knowing-what-to-do next, then it is very close to the full power and effect of knowing-to act in the moment.

Knowing-To

It is one thing to act when cued; it is quite another to have an action come-to-mind when appropriate. Lev Vygotsky (1978) coined the term *Zone of Proximal Development* to describe actions which students can enact when cued, and which they are on the edge of initiating for themselves (van der Veer & Valsiner, 1991) see also Mason, Drury & Bills, 2007). The *ZPD* is often misconstrued as what students can achieve when guided or cued by a (relative) expert, but the important part of Vygotsky's construct is that the students are sufficiently fluent in the action that they are on the verge of being able to initiate it for themselves.

The question of interest is how students can be encouraged to develop the independence, the spontaneity of acting from and by themselves rather than depending on cues and clues, such as finding a worked example of a task similar to their assessment task and so falling into the rut of using worked examples as templates to follow. It is clear that if students come to rely on a teacher-tutor to initiate acts or on previously worked examples as templates, they are likely to be come dependent on being cued rather than internalising the action for themselves.

The term *scaffolding* was introduced by David Wood and Jerome Bruner (Wood, Bruner & Ross, 1976) to describe what a teacher can do to support a student, acting as 'consciousness for two' (Bruner, 1986) by maintaining attention on global goals when activity becomes focused on a local goal, and bringing to mind actions that are in the student's repertoire but not yet independently active. Because of the dependency issue, Seeley Brown, Collins and Duguid (1989) augmented scaffolding to *scaffolding and fading*: only when the teacher cues have faded away does it make sense to say that the student has been effectively scaffolded, has become independent rather than remaining independent. A different and independent articulation of the same awareness is captured by the framework *directed–prompted–spontaneous* (Floyd, James, Burton & Mason, 1981; see also Love & Mason, 1981; Mason & Johnston-Wilder, 2004/2006). This can act as a reminder that by increasingly indirect prompts to action, students can be encouraged, even enculturated into spontaneously initiating actions for themselves. Indirect prompts include things like "read that out loud for me" through to "what did you do last time?" or "what question am I going to ask you?". These meta-comments serve to draw the student out of immediate action and to direct attention to the actions themselves. The idea is to prompt the strengthening of an inner witness (Mason, Burton & Stacey, 1982/2010) or monitor (Schoenfeld, 1985) and this applies not only to specific techniques but also to instances of mathematical heuristics (Pólya, 1945), powers and themes (Mason, 2008) and study techniques.

Knowing-to act in the moment is so often so automatic for the expert that they do not even recognise there might be an issue for students. That is why it can be so helpful to work on mathematics at your own level so as to keep fresh about what it might be like for students. Bringing to the surface the awarenesses that enable expert action, and trapping the features and aspects of a situation that trigger these awarenesses, is no small matter. It is the mark of an excellent teacher to be in touch with these sufficiently so as to construct tasks and to interact with students in such a way that students become aware of them, at whatever level of consciousness is necessary.

Knowing-to act in the moment is enhanced and enriched when students are able to imagine themselves in some future situation undertaking a relevant action. Being told (and so *knowing-that* or otherwise *knowing-about*) is rarely sufficient. An experiential approach seeks out tasks in which students spontaneously enact useful strategies, powers, themes or heuristics, and then have their attention drawn to them. For example, the technique of colouring a grid of cells as in a chess board when tackling domino-covering problems is a powerful combinatorial technique. Few students are likely to come up with it themselves, but being told it without having struggled for oneself reduces the impact, the awareness of effectiveness. Applying it in other circumstances, (and preferably, as soon as possible on variations of one's own devising) is more likely to enrich the network of connections the students have than simply being given a list of 'now try these' problems. Subsequence experience of novel uses of the same technique are more likely to have an impact if the 'trick' has become internalised and is richly connected to other techniques in students' minds.

Students' impressions of what mathematics is about as a human endeavour comes from what they experience, most particularly through the questions they are asked and the tasks they are set. If they never encounter a relative expert asking themselves what can be varied about an example of a concept and still it remains an example, or asking themselves why it is that certain conditions are required for a theorem to apply or a technique to work they are unlikely to pick up this important aspect of mathematical practice. If students never encounter a relative expert posing themselves questions through generalising or otherwise extending a solved problem, they are unlikely to appreciate the scope and nature of mathematical thinking. All of this can be summed up in asking whether students are in the presence of relative experts *being mathematical with as well as in front of* them, and most particularly, being invited to withdraw from activity and consider what it is that has been effective or ineffective about various actions undertaken, and what they would like to have come-to-mind in the future in similar circumstances.

References

Abdul-Rahman, S. (2005). Learning with examples and students' understanding of integration. In A. Rogerson (Ed.), *Proceedings of the Eighth International Conference of Mathematics Education into the 21st Century Project on "Reform, Revolution and Paradigm Shifts in Mathematics Education"*. Johor Bahru, Malaysia: UTM.

Ainley, J., & Pratt, D. (2002). Purpose and utility in pedagogic task design. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 17-24) Norwich, UK: PME.

Atkinson, R. K., Derry, S. J., Renkl, A., & Wortham, D. (2000). Learning from examples: Instructional principles from the worked examples research. *Review of Educational Research, 70*(2), 181-214.

Bruner, J. (1986). *Actual minds, possible worlds*. Cambridge: Harvard University Press.

Cardano, G. (1545/1968). T. Witmer, (Trans.). *Ars magna* or *The rules of algebra*. New York: Dover.

Floyd, A., Burton, L., James, N., & Mason, J. (1981). *EM235: Developing mathematical thinking*. Milton Keynes, UK: Open University.

Gattegno, C. (1970). *What we owe children: The subordination of teaching to learning*. London: Routledge & Kegan Paul.

Gattegno, C. (1987). *The science of education part I: Theoretical considerations*. New

York: Educational Solutions.
Gillings, R. (1972). *Mathematics in the time of the pharaohs*. Reprinted 1982. New York, USA: Dover.
Handa, Y. (2011). What does understanding mathematics mean for teachers?: Relationship as a metaphor for knowing. *Studies in Curriculum Theory Series*. London: Routledge.
Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Erlbaum.
Love, E., & Mason, J. (1992). *Teaching mathematics: Action and awareness*. Milton Keynes: Open University.
Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ, USA: Lawrence Erlbaum.
Marton, F., & Pang, M. (2006). On some necessary conditions of learning, *Journal of the Learning Sciences. 15*(2), 193-220.
Marton, F., & Säljö, R. (1976a). On qualitative differences in learning – 1: Outcome and process. *British Journal of Educational Psychology, 46*, 4-11.
Marton, F., & Säljö, R. (1976b). On qualitative differences in learning – 2: Outcome as a function of the learner's conception of the task. *British Journal of Educational Psychology, 46,* 115-27.
Mason, J. (2008). From concept images to pedagogic structure for a mathematical topic. In C. Rasmussen & M. Carlson (Eds.) *Making the connection: research to practice in undergraduate mathematics education* (pp, 253-272). MAA Notes. Washington: Mathematical Association of America.
Mason, J., Burton L., & Stacey K. (1982/2010). *Thinking mathematically*, London, UK: Addison Wesley.
Mason, J., Drury, H., & Bills, E. (2007). Explorations in the Zone of Proximal Awareness. In J. Watson & K. Beswick (Eds.) *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 42-58). Adelaide, SA, Australia: MERGA.
Mason, J., & Johnston-Wilder, S. (2004/2006). *Designing and using mathematical tasks*. Milton Keynes, UK: Open University; (2006 reprint) St. Albans, UK: QED.
McGowen, M. (1998). *Cognitive units, concept images, and cognitive collages*. Unpublished PhD thesis. Coventry: Warwick University.
Pólya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, USA: Princeton University Press.
Renkl, A., Stark, R., Gruber, H., & Mandl, H. (1998). Learning from worked-out examples: The effects of example variability and elicited self-explanations. *Contemporary Educational Psychology, 23*, 90-108.
Ryle, G. (1949). *The concept of mind*. London: Hutchinson.
Scataglini-Belghitar, G. & Mason, J. (2011). Establishing appropriate conditions: Students learning to apply a theorem. *International Journal of Science and Mathematics education*. DOI 10.1007/s10763-011-9305-0
Schoenfeld, A. (1985). *Mathematical problem solving*. New York: Academic Press.
Seeley Brown, J., Collins A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-42.
Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing (Learning in doing: Social, cognitive and computational perspectives).* Cambridge: Cambridge University Press.
Skemp , R. R. (1976). Relational and instrumental understanding. *Mathematics teaching, 77*, 20-26.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*(2), 151-169.

Vygotsky, L. (1978) *Mind in society: The development of the higher psychological processes*. London: Harvard University Press.

van der Veer, R., & Valsiner, J. (1991). *Understanding Vygotsky*. London, UK: Blackwell.

Watson, A., & Mason, J. (2002). Extending example spaces as a learning/teaching strategy in mathematics. In A. Cockburn & E. Nardi (Eds.). *Proceedings of PME 26* (Vol 4 pp. 377-385), University of East Anglia, Norwich

Watson, A., & Mason, J. (2005). *Mathematics as a constructive activity: Students generating examples.* Mahwah: Erlbaum.

Whitehead, A. (1932). *The aims of education and other essays*. London: Williams and Norgate.

Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry, 17*(2), 89-100.

Author

John Mason, The Open University, UK. Email: j.h.mason@open.ac.uk

A Course on Experimental Design for Biotechnology Students

Victor Martinez-Luaces[1], Blanca Velázquez[2]
[1] University of the Republic of Uruguay, [2] ORT University, Uruguay

We analyse an Experimental Design course, for Biotechnology students. Students were assessed at the end of the course, by carrying out individual projects on real-life problems, which was innovative for universities in this country. A concrete example of one of these assessment projects is described. Students produced high quality work, some of which formed the basis of future papers. In a feedback loop, the student projects were used in later courses as examples to illustrate the use of different techniques. A detailed analysis of students' opinions and suggestions is presented.

Introduction and Theoretical Framework

This paper analyses an Experimental Design course taught for Biotechnology students at the Faculty of Sciences of the Uruguayan public University (UdelaR).

To analyse this course, we have chosen to adopt the treatment suggested by Roiter and Petocz (1996). As these authors explicitly acknowledge, the genesis of their ideas was an earlier article by Blum and Niss (1991). Their article addresses the topic of general mathematics instruction, discussing five theoretical approaches that consider course design in terms of what its teaching objectives are, how it is organised, and the previous educational background of the students. Roiter and Petocz (1996) adapted the ideas of Blum and Niss (1991) to the specific area of introductory statistics courses at the tertiary level.

As our course falls within the area of statistics education, we will base our course description on the theoretical work of Roiter and Petocz (1996) who distinguish four different approaches to statistics courses:

- Statistics as a branch of mathematics
- Statistics as data analysis / a laboratory subject
- Statistics as a tool for planning research / experimental design
- Statistics as a problem-based subject.

In the following table, the main characteristics of the different approaches are summarized.

We will now analyse the course, in terms of the framework provided by Roiter and Petocz (1991).

The Characteristics of the Experimental Design Course

Our course has been modified several times, but right from the start, the final assessment of students attending the course has been carried out in what is for Uruguay a non-traditional way. In this and several other Latin American countries, university courses are normally assessed by a written test consisting of multiple-choice questions, marked by computer, followed by individual oral exams. In contrast, we have always insisted that the students carry out a monograph or short project on a real-world problem they are personally interested in. As other authors have observed, this is very important for getting students to recognise that mathematical knowledge is relevant to their specialist subject (McAllevey & Sullivan, 2001). In addition, if real problems and using computers are an integral part of the course, it is sensible to make them part of student assessment as well (Martínez-Luaces, 2005). In fact, efforts to change the contents or teaching methods in a course, without adapting student assessment methods accordingly, are incomplete (Smith & Wood, 2000).

This particular type of assessment has motivated students to produce high quality work, some of which has been published in international journals. For instance, two pieces of research work on biotechnology were published in the Journal of Data Science (Martínez-Luaces, Guineo, Velázquez, Chabalgoity & Massaldi, 2006) and the Journal of Applied Quantitative Methods (Velázquez, Martínez-Luaces, Vázquez, Dee & Massaldi, 2007) among other publications.

Over the years, real-life examples have been used not only for student assessments, but have also been gradually introduced into the various Experimental Design courses themselves, in a continuous feedback process similar to the principles governing Action Research, in which knowledge arising from a research intervention serves to inform continuous critical reflection on, and improvement of, the research design (O'Brien, 2001).

Many of these examples have proved to have excellent didactic potential when explaining and exemplifying techniques customarily used in Experimental Design. In some cases it was possible to use illustrations from the assessment projects directly, whereas in others a didactical transposition (Chevallard, 1985), was necessary.

This form of assessment fully corresponds to the Roiter and Petocz's third approach to statistics teaching, (C): Statistics as a tool for planning research, or experimental design (Table 1), where the typical forms of assessment are lab assignments, non-mathematical exams and reports on research papers. This is exactly the kind of assessment that we have used in our Experimental Design course from the outset.

In contrast to the above, planning of the course has changed significantly over time. Initially we used the planning typical of the first approach, (A), moving on to planning modes of the second and third approaches (B and C), as described by Roiter and Petocz (1996) and summarised in Table 1.

Table 1. Summary of Characteristics of the Four Approaches to Statistics Courses, According to Roiter and Petocz (1996)

	Approach	Planning	Content	Activities	Assessment
A	Statistics as a branch of mathematics	Weekly lectures, some tutorials	Combinatory, theory of probability, random variables	Proofs and derivations, advanced mathematical skills	Mid-semester and end-of-semester exams
B	Statistics as analysis of data / as a laboratory subject	Weekly labs with tutorials, class discussion, and group interaction	EDA, methods of data collection, hypothesis testing, regression and correlation	Collecting, investigating and analysing data; confirming hypotheses	Regular class tests, lab reports, assignments
C	Statistics as research design / experimental design	Discussion groups, lab work, group discussion and interaction	Analysis of the effect of variables on a response, critical analysis of published papers, understanding regression and ANOVA, interpreting p-values	Designing an experiment and collecting data, interpreting results; theory is discovered rather than presented	Lab assignments, non-mathematical exams, reports on research papers
D	Statistics as a problem-based subject	Group discussion, project, or consulting work	EDA, design of experiments, ANOVA, consultation, and report writing	Students solve problems from their field, working as consultants. Theory is introduced and developed as needed	Progress reports, essays, final reports, and presentations

ANOVA: Analysis of variance EDA: Exploratory data analysis

A, B, C and D refer to Roiter and Pecotz's (1991) classification of types of Statistics courses, see Table 1.

The contents of the course have also changed, from a more mathematical content at the beginning and more research design, data collecting and analysis of results at the end of the course. Table 2 is a summary of the above in diagrammatic form.

Table 2. Characteristics of the Experimental Design Course

Place/Department	Planning	Typical content	Typical activities	Typical assessment
Faculty of Science / Biotechnology	A-B-C	B-C	B-C	C

As seen in Table 2, over the years our course has been taught, changes have been made in its presentation, the use of technology, the examples used to illustrate concepts and techniques, the course load in terms of total hours of class time, and its contents and teaching methods. Basically the only thing that has remained constant throughout is the method of course assessment.

Final course assessment was in all cases based on the solution of real-life problems, including design, data collection and analysis, drawing conclusions, and writing a report or monograph on the experiment undertaken. An example of one assessment task will be described in the next section.

A Concrete Example of an Assessment Problem

For the final assessment after taking a one-semester (42 hours) Experimental Design course as part of the coursework, one student carried out the following work project, a piece of experimental design work applied to a real life problem posed by a practical laboratory project.

In the monographic manuscript presented by the student, the main purpose was to optimize the production of a biotechnological product, recombinant streptolysin-O (rSLO) (Velázquez, Battistoni, Massaldi & Chabalgoity, 2005), in a fermenter in order to obtain a production protocol. Particularly, the influence of feeding glucose and the oxygen transfer to the system were examined at controlled temperature and constant *pH*.

For this purpose, the volumetric rate of dissolved oxygen transfer ($k_L a C_L^*$) and glucose feeding are the variables studied. Both variables are presented in two different variants during the fermentation process: in the growth phase and induction phase. Finally, the system self-induction with isopropyl-β-D-1-thiogalactopyranoside (IPTG) is another variable and its effect on the production of rSLO is analysed.

In fact, this is a problem of nested variables (Montgomery, 2005), but in this first approach – just a monograph for the course approval – the analysis was done in groups of three different variables each time. For this purpose, 2^3 full factorial designs (Montgomery, 2005), were utilized. Based on the results of these experiments, the author was able to determine the best conditions for obtaining the rSLO. The recommended conditions are the following:

The fermentation process must be carried out with a $k_L a C_L^*$ 80 mmol/L/h in the growth phase and 6 mmol/L/h in the induction phase. These results were suggested by a positive principal effect in growth phase and a negative one in the induction phase.

The feeding with glucose is important through its interactions with $k_L a C_L{}^*$ and induction. In fact, when studying the experimental results – obtained with and without glucose feeding in the induction phase – the positive interaction of glucose feeding with $k_L a C_L{}^*$ was proved.

Finally, the induction factor was showed to be important in the rSLO production.

These conditions were finally recommended for the production of this product under the conditions of temperature, *pH* and volume of the fermenter descript above and were presented in a Latin American Statistics Conference (Martínez-Luaces & Velázquez, 2010).

We believe this process serves the purposes of assessment by demonstrating the extent of students' ability to apply experimental design knowledge, but also reinforces their knowledge and their ability to use it. Experience of applying experimental design knowledge to real problems shows them its relevance as a research tool, and increases their confidence to apply it in future, which are the desired outcomes of the Experimental Design course.

Results of Students' Opinion Surveys for the Purpose of Course Evaluation

Students were asked to evaluate the courses, in order to continually improve them. At the end of each course a written questionnaire was issued to students to be filled out anonymously. These questionnaires covered a range of specific items on how the course had been conducted, and perceptions of teacher effectiveness, using structured questions susceptible to quantitative analysis of responses. These parts of the questionnaires offered four categories of answer: 'Bad', 'Neither Good nor Bad', 'Good' and 'Very Good'.

One question, about whether the contents of the course had satisfied the students' expectations, had three possible answer categories: 'Yes', 'No' and 'Partly'. There were also a number of semi-open questions, where students could freely express their opinions and contribute suggestions, criticisms, and so on.

The course evaluations presented here are for two courses taught for biotechnology students at the Faculty of Science (Tables 3 and 4).

In their answers to open-ended questions that were part of the evaluation, 100% of student respondents said that the course had met their expectations. Their suggestions for improving the course included increasing the course length by 10 hours, and more dynamic presentation of the material. They indicated that the course objectives had been met, and that it was a very important university extension topic.

Table 3.Students' Evaluation of Different Aspects of the Course

	Bad (%)	Neither Good nor Bad (%)	Good (%)	Very Good (%)
Selection of topics covered	-	-	83.3	16.7
Quality of the methodology	-	-	66.6	33.4
Time assigned to each topic	-	33.2	66.8	-
Clarity of the initial objectives	-	-	50	50
Course dynamics (group work)	-	16.6	66.8	16.6
Course material supplied	-	-	83.4	16.6
General appreciation of the course	-	-	33.3	66.7

Table 4. Students' Evaluation of Teacher Effectiveness on the Course

	Bad (%)	Neither Good nor Bad (%)	Good (%)	Very Good (%)
Competence in theoretical knowledge	-	-	22.2	77.8
Skill and clarity of explanations	-	12.5	25	62.5
Ability to motivate and interest students	-	-	50	50
Interest in clarifying students' problems	-	-	50	50
Ability to communicate theoretical information	-	12.5	62.5	25
Management of the class	-	12.5	37.5	50

The results of the student evaluation showed an acceptable level of general satisfaction with the course, its contents, teacher effectiveness, and most other items asked about in the questionnaire. However, the need for a greater connexion between Experimental Design and specific topics was stressed. This was reflected in two parts of the questionnaire: the open-ended part, where more examples applied to students' fields were explicitly requested, and the part asking about time allocated to different activities, where several students mentioned the need for more hours of practical work and computer laboratory access to deal with these examples.

It should be noted that the example worked through in Section 3 was a work project carried out after the first course for biology students, and in later courses it served as a source of example problems that aroused the interest of this group of students. In fact, in evaluations of more recent courses, taught in 2006 and 2007, students' perceptions improved with respect to applicability to their main subject, time allocated to practical work, use of appropriate software, and other points.

An important result that can be observed in the student evaluations of the Experimental Design courses is that while their expressions of satisfaction or dissatisfaction have varied for several items, this has not been the case in their evaluation of the course assessment. Indeed, assessment by means of practical projects investigating real-life problems in the students' areas of expertise has consistently elicited their approval and enthusiasm (personal communications).

Every course and every target group of students pose specific contexts and different challenges, and designing a course must take these specific characteristics into consideration, so that it is tailor-made for the specific context and circumstances.

The other elements of the course (planning, contents and activities) have been adapted to the course audience and the means available. For example, courses taught to biotechnology students contained substantially less pure mathematics, and the students worked on real life problems, concentrating on how to design experiments, collect data, analyse them, and so on.

Conclusions

The Experimental Design course assessment is based on real-life problems, and almost every class also involves discussing possible designs, how best to collect the data, how to analyse them, and so on.

The course has been followed with increasing interest and approval by the students who have attended it. The use of relevant applied examples has played a major role throughout the life of the course.

We found the following to be good practices for increasing both students' and teachers' learning and satisfaction: a dynamic feedback cycle of course improvement and methodological changes, using materials based on students' own work projects; the linking of student assessment to solving real-world problems; careful attention to students' opinions through regular evaluations; and ensuring closer application of examples to students' specific specialties.

These have been part of the process of continuous improvement of the Experimental Design course which began several years ago and is still continuing to evolve.

References

Blum, W., & Niss, M. (1991). Modelling, applications, and links to other subjects - State, trends and issues in mathematics instruction. *Journal of Educational Studies in Mathematics*, *22,* 37-68.

Chevallard, Y. (1985). *La transposition didactique. Du savoir savant au savoir enseigné.* Grenoble: La Pensée Sauvage.

Martínez-Luaces, V. (2005). Engaging secondary school and university teachers in modelling: Some experiences in South American countries. *International Journal of Mathematical Education in Science and Technology, 36*(2-3), 193-205.

Martínez-Luaces, V., Guineo, G., Velázquez, B., Chabalgoity J. A., & Massaldi, H. (2006). Bifactorial design applied to recombinant protein expression. *Journal of Data Science*, *4*(2), 247-255.

Martínez-Luaces, V., & Velázquez, B. (2010). Un problema de optimización en Biotecnología utilizando técnicas de diseño experimental. *Resúmenes del Noveno Congreso Latinoamericano de Sociedades de Estadística*, Santiago de Chile, p. 72.

McAllevey, L. G., & Sullivan, J. C. (2001). Making statistics more effective for business. *International Journal of Mathematical Education in Science and Technology*, *32*, 425-438.

Montgomery, D. C. (2005). *Design and analysis of experiments* (6th ed.). New Jersey: J. Wiley & Sons.

O'Brien, R. (2001). Um exame da abordagem metodológica da pesquisa ação [An overview of the methodological approach of action research]. In Roberto Richardson (Ed.), *Teoria e prática da pesquisa ação [Theory and practice of action research]*. João Pessoa, Brazil: Universidade Federal da Paraíba. (English version available: www.web.ca/~robrien/papers/arfinal.html).

Roiter, K., & Petocz, P. (1996). Introductory statistics courses - A new way of thinking. *Journal of Statistics Education, 4.2*. Retrieved from: www.amstat.org/publications/jse

Smith, G., & Wood, L. (2000). Assessment of learning in university mathematics. *International Journal of Mathematical Education in Science and Technology*, *31*, 125-132.

Velázquez, B., Battistoni, J., Massaldi, H., & Chabalgoity, J. A. (2005). Construction and expression of recombinant Streptolysin-O; Pre-evaluation of its use in immunoassays. *Clinical Laboratory Diagnostic Immunology, 12*, 683-684.

Velázquez, B., Martínez-Luaces, V., Vázquez, A., Dee, V., & Massaldi, H. (2007). Experimental design techniques applied to study of oxygen consumption in a fermenter. *Journal of Applied Quantitative Methods*, *2*(1), 135-141.

Author

Victor Martinez-Luaces, Faculty of Engineering, University of the Republic of Uruguay
Email: victorml@fing.edu.uy
Blanca Velázquez, Faculty of Engineering, ORT University, Uruguay

THE UNIVERSITY OF AUCKLAND
NEW ZEALAND
Te Whare Wānanga o Tāmaki Makaurau