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Community for Undergraduate Learning in the Mathematical Sciences

The CULMS Newsletter

CULMS is the Community for Undergraduate Learning in the Mathematical Sciences.

This newsletter is for mathematical science providers at universities with a focus on teaching and learning.

Each issue will share local and international knowledge and research as well as provide information about resources and conferences.

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Guest Editorial

This issue's Guest Editorial is by Carl Winsløw, Professor of Didactics of Mathematics in the Faculty of Science, University of Copenhagen.

Didactics – The Missing Link Between Mathematics and Education

As mathematicians, we sometimes seem to have a schizophrenic relationship with teaching. On the one hand, most of us have a much more than "scientific" attitude to our field – without being religious about it, most of us do not hesitate to use words like "beauty" and even "love" when we talk about mathematical results and reasoning. And I have yet to meet a mathematician who does not know the gratification from *sharing* this joy of mathematics with others, even if we don't all find that easy to attain. On the other hand, we also seem to find the act of teaching something "old" to be somehow inferior to the creative business of pushing the boundaries of what is known (not just to us or our students but to any mathematician). Systematic gatherings and analyses of such points of view can even be found in the literature (eg. Burton, 2004; Madsen & Winsløw, 2007).

Paul Halmos (1985) is one who was very explicit about the feeling of "guilt" he attached to his evident inclination for teaching, along with other activities which he summarizes in the expression, "sharpening pencils" (as opposed to "work", meaning to do research):

> Despite my great emotional involvement in work, I just hate to start doing it; it's a battle and a wrench every time. Isn't there something I can (must?) do first? Shouldn't I sharpen my pencils, perhaps? (…) Yes, yes. I may not have proved any new theorems today, but at least I explained the law of sines pretty well, and I have earned my keep. (Halmos, 1985, 321f)

For a mathematician, the business of explaining, constructing problems, and adapting it all to particular students, may, indeed, seem less worthy of glory than that of thinking about an open problem, while producing a small lemma on the way.

But it is not easy to draw the lines between the noble, "truly" creative activity called research, and the act of reorganizing "old hat" contents in view of its teaching. One easy way to demonstrate this is to look at the long history of "famous" mathematical treaties, gathering and presenting contents which at least in some sense were known already, but which became accessible and coherent to the new generations of mathematicians through these exquisite presentations. The list is long, from the *Elements* (300 BC) collecting the results and methods of antique mathematics, to the long series of textbooks on modern analysis written by figures like Halmos and Rudin. Any of us can think of other examples in more specialized and recent fields. And most of us will also have no difficulty to think of teachers whose "way" to share mathematics was determinant for our engagement in the field. Such "ways" would not, by the way, be confined to clear and coherent presentation. Our best teachers led us to experience the creative practice of identifying, attacking and solving mathematical challenges.

This note exists to explain how the term "didactics" brings a new perspective on this quandary (cf. Chevallard, 1999; Lijnse, 2000). My own interest in it began with two related experiences. The first consisted in vain attempts to apply principles of "university pedagogy" (encountered during a course of this name, mandatory for junior faculty members in Copenhagen in the early 90's) to the concrete problems of teaching undergraduate mathematics. The second was my somewhat coincidental discovery of the works of a Frenchman, Guy Brousseau (1997), which by the way deal mostly with the problems of teaching mathematics to primary school kids. To make a long story short, Brousseau devoted his professional career to lay the grounds of what he wanted to become a scientific approach to the conditions and mechanisms of mathematics teaching – the *theory of didactic situations in mathematics.* It is radically different from what I found – and still mostly find – in mainstream Anglophone literature on "mathematics education", with its applicationism appearing in those magic sticks (from constructivist learning theory to the latest fads in socio-cultural theory), which are claimed to serve as "scientific bases".

Brousseau, in one of his foundational papers (original 1986; translated in Brousseau, 1997, Chap. 1) explains the relation – but also the difference – between "the work of the mathematician", and "the work of the teacher" (which, understood widely, constitutes the object of study for didactics). Roughly put, the mathematician begins with a mathematical situation in which he identifies a problem to be solved, and then produces (some sort of) knowledge or "solution". The teacher will be faced with a kind of inverse task. He must teach a corpus of – typically old and heavily transformed – knowledge, and to do so, he must establish a mathematical situation that will allow the student to adapt his "old" knowledge to the "new" knowledge. The conditions under which this becomes possible for the student – and hence, under which the situation "works" – are not, *a priori*, solely linked to the target knowledge and the "old" knowledge of the student. Nevertheless, it is a fundamental hypothesis in Brousseau's theory that with very precise descriptions of these two, one may indeed construct a situation in which the only way for the student to succeed is to adapt his knowledge in such a way that the target is reached. Brousseau's work provides a number of demonstrations both of this point, and of other properties a situation may turn out to have, in precise experimental settings.

We now come back to our schizophrenic mathematician, culpabilized with the hours he spent to teach. Indeed, creating new mathematics is not exactly the same as finding a good situation (explanations, problems and so on) for students to learn the law of sines. However, it is also not something entirely different: "80 percent of mathematics research consists in reorganizing, reformulating, and "problematizing" mathematics that has already been "done", by the researcher himself or by others" (Brousseau, 1999). Mathematicians need to come to terms with "the didactical" which is, indeed, everywhere dense in mathematical activity. My aim is not to suggest that the collected works of Brousseau or Chevallard will be necessary or sufficient to do so. But we must give up two illusory shortcuts. The first one is to claim that didactic phenomena are either just transparent acts of "repetition" or conversely opaque "wisdom" of the gifted, about which nothing firm can be known. The second one is the, less prevalent but not uncommon, expectation that mathematics could wait for some general and progressive pedagogy to marry, in view of solving its manifest didactic challenges. Brousseau (1999) notes, already in the context of primary school mathematics, "the core of didactics of mathematics is necessarily the work of mathematicians". To me this holds clearly, and *a forteriori,* in the context of university mathematics (cf. eg., Artigue, 1994).

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Power and Passion: Why I Love Mathematics

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Mathematics has beauty and power. This paper looks at a few recent advances in mathematics and describes some ways to incorporate them into the classroom at first year level. I suggest a way to structure learning tasks to assist students to develop confidence in their ability to contribute to mathematics as a discipline.

Introduction

Mathematics is joy and exasperation. It is beauty and power. As mathematicians we love elegant solutions and the way that the same mathematics can be applied to different contexts. In this article I will take an eccentric look at areas of mathematics that I find fascinating, fun and frustrating. I will give ideas on teaching mathematics using these diverse topics in mathematics.

Designing learning tasks

For designing examinations, the MATH Taxonomy (Smith et al., 1996) has been shown to identify students who use deep and surface approaches to learning (Wood, 2002). To assist with the layout and wording of examinations, Hoadley (2008) has done an analysis of 50 papers in a variety of content areas and has written a guide to help with setting examinations. In examinations the conditions restrict the types of questions that can be asked, so this article will concentrate more on extended learning tasks and investigations.

For extended tasks I use the TALC approach. TALC stands for:

- Techniques – basic knowledge and skills
- Applications – structured approach to problems; modelling
- Life/career – linking what students have learnt to external
- Creativity – developing something new, innovative or original

TALC is useful because of the students' affective response to mathematical tasks.

Table 1. Talc Categories and Student Response

Categories	**Response**
Techniques	Skill
Applications	Structure
Life/career	Communication
Creativity	Confidence

Using categories to design tasks assists lecturers to cater to the different learning needs of students. Most mathematics courses emphasise *techniques* and students gain skills in being able to perform routine tasks easily. However this is not the only role of mathematics and the technical aspects are those that are more easily done by computer so students need to move onto higher level learning tasks.

Applications of techniques include modelling and problem solving and help students develop a structure around which to hang their learning. Good applications help make links between different areas of mathematics and so aid students to see the structure of their discipline. A structured learning approach, such as the “analytical, numerical and graphical” approach to problem solving of Hughes-Hallett et al. (1994, 1996) works to give students a ordered way of thinking and tackling mathematical applications. This structured, analytic way of thinking is what many graduates state is the main outcome from their mathematical education.

Surveys of mathematics students (e.g., Reid et al., 2005) show that student learning is strongly affected by ideas of their future careers. Students who could articulate their careers more clearly connected to their studies more deeply. It is therefore important that the learning tasks chosen orient students to their *life and career* after study (Wood & Solomonides, 2008). This is challenging for mathematics because the career possibilities are so wide and unspecified. The language and communication aspects of learning tasks reflect the need from employers for graduates who are able to articulate their knowledge and communicate in the workplace (Wood, 2005).

Finally, opportunities should be presented to encourage students to think *creatively* about their discipline and to apply their knowledge to new areas. When we surveyed 1200 undergraduate mathematics students (Reid et al., 2005) only 2 stated that they could contribute in an original way to their discipline. While most of us will not become Fields medallists, from our interviews with students (Reid et al., 2005) and our surveys of undergraduate students, we find that the vast majority have no idea that mathematics has open problems – that mathematics is a living, changing, vibrant field. Many did not realise that mathematicians think creatively and look for opportunities to expand the discipline. Students should be given the opportunity, however small, to demonstrate that they are able to make a contribution to the discipline – not just regurgitate the knowledge of others.

To illustrate these I will consider several areas of mathematics, Sudoku, image processing and wavelets.

Sudoku and Latin Squares

I have chosen to examine Latin squares because of my interest in the game Sudoku. Paraphrasing Weisstein (2009), a Latin square consists of *n* sets of the numbers 1 to *n* arranged in such a way that no orthogonal (row or column) contains the same number twice. For example, the two Latin squares of order two are given by:

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (1)$$

The 12 Latin squares of order three are given by:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix},$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix},$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}. \quad (2)$$

As you can see, the numbers of Latin squares grow quickly with increased size. There are some cool results, generalisations and unsolved problems that are fascinating, and described in Weisstein. The statistical analysis of Latin square designs is explained in Montgomery (2005), and a short history is given in Petocz and Sowey (2006).

Applications of Latin squares

Latin squares are used extensively in experimental design and coding. Latin square designs are used when the factors of interest have more than two levels and you already know that there is little or no interaction between factors. For example, if you wanted to test the effect of 4 different solar panels on the production of electricity and had 4 test sites and 4 ways to install the panels, you could test every combination (a 4×4×4 factorial design) but this is very expensive. Using Latin square designs you are able to do a much smaller number of experiments with excellent statistical power. The main factor you are examining is the effect of the different solar panels on the production of electricity. You are not interested in the interaction between the panels and the sites or the panels and the ways they are installed but you want to reduce the bias caused by the different sites and installation.

Label the solar panels with the letters A, B, C, and D, and you get a 4×4 Latin square which allows you to plan which panel gets which treatment (see Figure 1).

You can see that there would be other possible designs for this experiment (how many?) but you can also see that every combination is covered. Therefore you are able to answer your experimental question with 16 instead of 64 experiments. It is clear that for larger experiments that huge savings in efficiency can be made.

	Test site			
Installation	A	B	C	D
	C	D	A	B
	B	A	D	C
	D	C	B	A

Figure 1. Latin square design for 4 factors.

Sudoku

Sudoku (su = number, doku = single) is an application of Latin squares that uses a 9×9 grid with the digits 1 to 9 in each row and column. Additionally, the board is broken further into 3×3 squares that also must contain the digits 1 to 9 (see Figure 2). Sudoku is extremely popular with puzzles in daily newspapers around the world, a plethora of books, websites and computer games. There are many derivative games, such as KenKen (www.kenken.com), which the author invented to encourage students to enjoy mathematics and mathematical ideas.

6	7							
	1						4	2
		3		1		6		
					3	7		6
4	2			8			1	3
3		5	2					
		9		2		8		
7	8						5	
							6	9

Figure 2. Example of a Sudoku game (diabolical).

Learning tasks

One mathematical context, Latin squares, can be used in many situations as a learning task. Even preschool children can play with grids and coloured squares, letters or shapes.

A	B	C
C	A	B
B	C	A

Figure 3. Latin square with letters.

At a senior school level, students can work with a simple experimental design to give them a taste of a research tool that is used in many contexts. They can have fun with Sudoku and KenKen and explore theoretical combinatorial concepts. A project on the history of Latin squares will reveal important characters in the history of mathematics.

At university level, there are many opportunities for the use of Latin squares in biology, health sciences, engineering and mathematics. The theoretical concepts of matrices, orthogonality and combinatorics are evident.

Techniques

- Find the number of 4×4 Latin squares. List several examples.
- Solve a Sudoku puzzle.

Applications

- Find an appropriate Latin square design to test the growth of genetically modified beans using 4 test sites and 4 planting techniques.
- What is a Graeco-Latin square? And how is it used?
- Write a computer program to solve Sudoku puzzles.

Life/career

- Investigate how Latin squares are using in your discipline area (eg biology, engineering).
- Investigate the history of the puzzle Sudoku. The puzzle was marketed in an innovative way. Investigate how this was done and why it was so successful.

Creativity

- Design an experiment that will answer a question you are interested in.
- Design a new puzzle.

Gaussian Operators

The normal (Gaussian) distribution is familiar to students of statistics. Often this is studied at school and then used in applications at tertiary level and in industry. An excellent summary of the distribution is available at mathworld.wolfram.com/NormalDistribution.html

What is less familiar is that Gaussian operators (sometimes called filters or kernels) are used extensively in image processing. There are other operators but I will concentrate on this operator for reasons that will become obvious later. A Gaussian operator can be formed from a convolution of two operators. Convolutions save on computing time, which is significant when the operators are large. There is not limit on the size of the operators—except computing power.

A 3×3 Gaussian operator can be given by:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \tag{3}$$

A 5×5 Gaussian operator can be given by:

$$\begin{bmatrix} 1 \\ 4 \\ 6 \\ 4 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix} \tag{4}$$

Image processing

An image is made up of a huge number of pixels in an array—the latest cameras are around 18 megapixels. These pixels are activated by light (or other radiation) falling on a sensor and therefore exciting an electron, which triggers a response. A digital camera has a sensor that is overlaid with a filter consisting of red, blue and green to allow you to register colour in your photos. The camera interpolates between these pixels to create the image. Not all the pixels are excited by real information—there is some background noise when electrons can be excited randomly. This is where the Gaussian operators come into use. It is, of course, more complex than this and there are many excellent books for further explanations such as Berry and Burnell (2005).

The exciting aspect of digital data is the ability to work on these pixels mathematically. The ideas are well within the scope of students with some knowledge of randomness and the normal distribution.

The computer package *Photoshop* uses operators to work on images, though these are called filters. The following is adapted from Berry and Burnell (2005, p. 373). Think about the array of pixels that make up the image and imagine the Gaussian operator superimposed on the image array, one pixel at a time with the zero-zero element of the operator over the current pixel. Each element of the operator is multiplied by the pixel value directly underneath it. The products are then summed and divided by the sum of the elements in the operator. This is done for each pixel in the image.

For a Gaussian operator, this has the effect of smoothing out the noise in the image. It is the most useful in processing astronomical images because it tapers evenly at the edges, which enhances high spatial frequencies (Berry & Burnell, 2005, p. 390). Figure 4 shows two images; Figure 4a shows an image of the Moon before processing and Figure 4b shows the Moon after processing. Note that it looks clearer but there is exactly the same amount of data – the details have been embellished. The images were created from 6 images @1/160 sec taken through a 12" reflector.

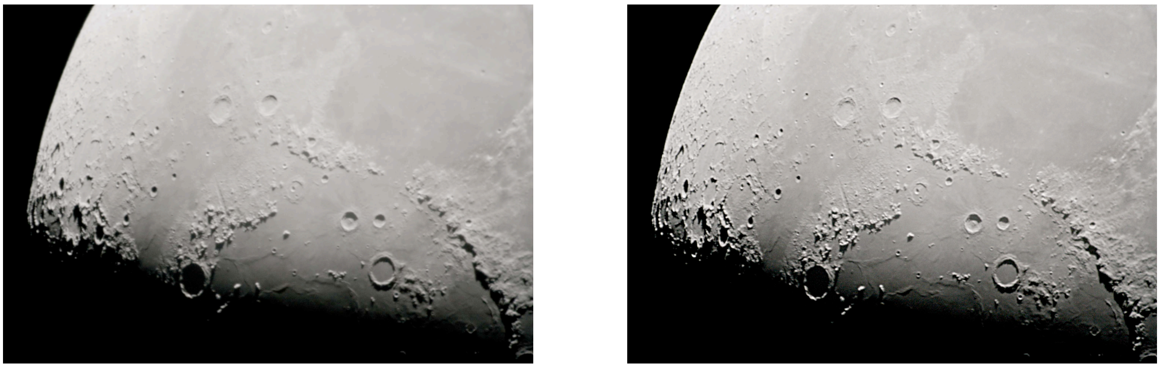

Figure 4a. Moon original image. *Figure 4b.* Moon after processing using unsharp masking.

Learning tasks

Techniques

- Describe the difference between a discrete and continuous distribution.
- What is the connection between the normal distribution and a Gaussian operator?
- Convolution examples from various areas.
- Use the 3×3 Gaussian operator in the method described above to enhance the set of pixels:

$$\begin{bmatrix} 1 & 4 & 4 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Applications

- Apply other operators to photographs and relate the different results to the types of operator used. Explain why the results are different.
- Use Photoshop to enhance photographs.
- Applications using convolution in different areas.

Life/career

- One of the skills that graduates require in the workplace is the need to instruct others in terms of procedures (Wood & Smith, 2005). Imagine you are at work and need to run a workshop to instruct your team about operators in image processing. Design a 4-page handout to achieve this.
- Investigate the history of the normal distribution.
- Find two careers that lead from your degree and investigate the use of the normal distribution of Gaussian operators in those areas.
- Convolution is used in different ways in different contexts. Find at least two way that convolution is used in different disciplines.

Creativity

- Find a new application for these operators.

Wavelets

Wavelets are used in a variety of applications including, again, image processing. These are new mathematical tools that are often dependent on the quality of the data to be analysed – which has improved significantly. Traditional Fourier techniques, applied to real data, give very spiky spectra, in which the separation of real maxima and high harmonics can be difficult. For data of sufficient quality, this new mathematical tool allows us to detect structures of different scales in data sets.

Here is an image of the planet Saturn taken with a webcam (Philips ToiuCam). Three hundred frames at 5 frames per second were taken with a 4" refractor, then stacked and processed (Figure 5).

Figure 5a. Saturn original image. *Figure 5b.* Saturn after processing using wavelets.

Many computer packages have options for wavelets such as: www.wolfram.com/products/applications/wavelet. In a previous article (Wood & Smith, 2007, p. 722), I gave an example of an assignment using wavelets with the following student quote:

> One of the things that I find interesting about wavelets is the fact that they are such a recent notion. Although Alfred Haar discovered what is now recognized as the first wavelets in 1909 (and many other rediscoveries have been made since), wavelets have only really been used and developed since the 1980's. As a math student, I am used to hearing about functions and formulas that came into existence hundreds of years ago, so the idea that wavelets came into use within my lifetime was a nice change.

Conclusion

Mathematics is a wonderful discipline, ever changing and evolving. It has the advantage of beauty, elegance and simplicity—and also the distinction of being messy, frustrating and impossible. How often do we reveal these traits to our students? How often do we tell them about new and exciting discoveries and current open problems in mathematics?

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Teaching and Learning of Undergraduate Mathematics and Statistics (Kenyan Scene)

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Many university lecturers of Mathematics look back to the period before 1990, as a golden era of university teaching, when the students were well prepared for a rigorous degree programme, and would work hard, and enjoy their programme. These students had passed the Kenya Advanced Certificate of Education (KACE). However the admission of students holding the Kenya Certificate of Secondary Education (KCSE) from 1990 has presented problems for university lecturers, especially in mathematics, where entrants are younger, less mature, and not so well prepared for their degree programme. This paper seeks to identify the major problems, mentions steps taken to solve these problems, and suggests further steps that should be taken to reduce these problems.

The Golden Era of University Teaching

In the early days of university education in Kenya, in the 1950's and 1960's, the disciplines of mathematics and statistics found their applications almost solely in the context of degree programmes in physical sciences and engineering. However, the importance of these disciplines in other areas, mostly in agriculture, business, medicine, biological sciences and social sciences, grew considerably in the 1980's and 1990's.

Until 1990, transition from school mathematics to university mathematics was very smooth. The Kenyan Advanced Certificate of Education (KACE) programme, studied intensively with three or four principal subjects over two years following Form 4 in school, provided a very good method of sorting out which school leavers could proceed to a university degree, which ones could proceed to a diploma, and which ones could proceed to a certificate. It was a rigorous syllabus, in which most subjects would be taught for ten periods a week, with at least ten private study periods a week, and a lot of intensive private study during preparation time and a student's spare time. Students were thus well prepared for an intensive degree or diploma programme and found the transition from school to be very easy. On leaving school they had a good grasp of the basic essentials of university mathematics, in the case of those who achieved a Principal Pass in Mathematics.

Many university lecturers, who taught these students before 1990, look back to these years as a golden era, especially in the teaching of mathematics and statistics. While those majoring in these disciplines would be extra keen to sharpen all the skills they learnt, even those in other disciplines would work hard and enjoy their units of mathematics and statistics. One professor of Computer Science, who has also had his share of teaching university mathematics, had shared his experience of teaching mathematics units to students of Electrical Engineering in the University of Nairobi during the 1980s. If he started with first year students, he would teach the same class all the way to third year, in each of the units of mathematics that they were taking. He could even stretch their minds by including extra topics that were outside the syllabus. They soaked up everything he gave them, and they really enjoyed the work. This professor's experience was typical of the real joy that lecturers in university mathematics and statistics experienced, when they taught KACE holders.

The Challenges From The Nineties

The early 1990s brought an abrupt halt to this idealistic state of affairs that had been a delightful reality to the lecturers for 40 years. An overhaul of the Kenya education system meant that, from 1990 onwards, students admitted to university came as holders of the new Kenya Certificate of Secondary Education (KCSE) directly from Form 4. These students had studied between 8 and 10 subjects in Form 4. There were no private study periods and no time for intensive concentration on only 3 subjects. They were now admitted to university degree programmes at a young age, and were thus less mature than their predecessors. A young Form 4 leaver had to be very self-disciplined in order to succeed at university. Unlike school, there were no strict rules to tell you in which place you should be at a particular time. It was very easy to misuse newfound freedom for which the young university student was not ready, resulting in the student drifting into all kinds of unhealthy activities and hence failing to concentrate on studies. Sadly, some end up as cases of discontinuation or have to repeat the year of study, even with good counselling mechanisms put in place by the university.

In the early nineties each university in Kenya had to meet the challenge of re-designing the syllabi for the degree programmes. The previous 3-year programme would be replaced by a 4-year programme (in the case of most faculties) or by a 5-year programme (Engineering) or a 6-year programme (Architecture or Medicine). The content of the KCSE syllabus, especially in Mathematics, had very little of the very substantial former KACE syllabus. Bridging this gap proved to be a big challenge for the universities to handle. Most of the time in the early nineties was spent in drafting and re-drafting syllabi for degree programmes, in judging how much content could be put in a unit (which comprised 45 lecture hours), and in ensuring that all the gaps could be covered. Unlike in a school set-up, where a teacher would meet a class of 30 to 40 students, university classes in mathematics and statistics over the first two years of any programme would normally be between 100 and 200 students. Of course, the philosophy in university from time immemorial has been that the lecturer is only there to guide not spoon-feed the student, as happens in some schools. In particular students have come to university to learn to think for themselves, not to expect ready answers to every question. Needless to say, in a large class there is little chance for individual attention, and lecturers, while not being deliberately unhelpful, tell students that they have to study and work on their own. If we can use the illustration of a large family – a university student is like a four year old who has to learn to stand on his/her own two feet in that large family. One lecturer in statistics in Jomo Kenyatta University of Agriculture and Technology (JKUAT) uses that illustration regularly when first meeting a class in first year and many in the class follow this advice.

Jaworski (2009) has recently addressed the problems of teaching large classes, and also handling less able students, when teaching mathematics in a UK university. Fox and Hackerman (2003), and Holton (2001) have also addressed these problems, when teaching mathematics in a USA university. In JKUAT, in the Physical Sciences programmes, most students take up a good attitude and appreciate that they have to do most of the work on their own. Mathematics and statistics are essential ingredients in the degree, and so they take all units in these disciplines seriously.

In Biological Sciences, there is one faculty unit in mathematics, which caused a problem initially. Possibly the speed of presentation of the topics caused problems, the fact that it was NOT a specialist unit for the students, and also that some of the students held relatively low grades (D+, D or D-) in KCSE Mathematics, resulted in these students having difficulties, and so there was a high failure rate among these students the first year they took the unit. However, lecturers in the Biological Sciences appreciated the students' difficulties and since then have given these students a lot of counselling, that the unit provides essential mathematics for their studies in any of the Biological Sciences and the unit must be taken seriously. This counselling has been of great help, and the mass failure of this unit has never occurred again.

In Engineering, students are required to take many units of mathematics and one unit of statistics. Overall, these students perform well in the unit of statistics. However, often there has been a trend for some of the students to perform poorly in the units of mathematics, especially the units of Calculus and Differential Equations. Yet students joining Engineering normally have better grades in Mathematics than those joining Science Programmes. Often students in Engineering do not put effort into their units in mathematics, physics and chemistry, as they think that these are not their main subjects. As a result they perform poorly in units of mathematics and sometimes in units of physics and chemistry. This trend became even more noticeable when private students were admitted, who had lower grades in mathematics, physics and chemistry, and who learnt along side the Government of Kenya sponsored students.

The admission of private students has certainly helped universities to establish a good financial base at a time when Government funding alone is unable to sustain the cost of degree programmes. However, private students have generally presented their own problems on enrolling for a degree programme. Some come to be able to get a degree by any means and will resort to all kinds of dishonesty in order to achieve this goal. Some have been spoon-fed at school, or have been used to attending coaching sessions after school in order to boost their KCSE grades. These students have never really learnt to work on their own, and they find the university environment quite hostile. Some come to misuse their newfound freedom, and arrange to enjoy themselves outside the campus for most of the semester. They tend to miss lectures, and then do not copy notes from their friends, and know hardly anything about the unit they are supposed to study, when the coursework assessment tests (CATs) are given. As a result, they fail the CATs. Then, just before examination time, they run to a friend, beg for the friend's complete set of notes, rush to get all the notes photo-copied, and try to cram all the notes for a few days before the examination. Most of these students end up, at best with having to sit for supplementary (re-sit) examinations, or at worst, find themselves discontinued from their programme. Some students, in a unit system, adopt the Form 4 bonfire mentality to some, if not all of the units they study. This is where, in Kenya, some students, on completing Form 4 or their KCSE course, have a big bonfire to burn all the notes they have made at school, thinking (mistakenly) that these notes will not be useful for them in the future. In the same way, some students will study simply to pass a unit, and then make a point of forgetting all the work in that unit, even though the unit is a pre-requisite for a unit to be taken the following semester. For example, a student, who deliberately forgets everything studied in Calculus I, will have great difficulty in following the work in Calculus II and Calculus III. One of the major weaknesses noticed in some students of Engineering and Science, is a failure to grasp the Differential and Integral Calculus of the first year of study. This work is pivotal in all kinds of applications in Mathematics, Statistics, Physics, Chemistry, Engineering, Business, Agriculture and Biology.

Solutions To The Challenge

What solutions have been put in place? What solutions could be put in place? One question raised has been training of lecturers, who may not be trained teachers. In the early nineties, when JKUAT recruited staff, most of those recruited into the Faculty of Science already held a BEd Degree from the mother, Kenyatta University. However, in JKUAT the Faculty of Science soon recruited their own BSc holders as Assistant Lecturers or Teaching Assistants. The lack of teacher training has not been a serious hardship, as the young recruits could be mentored and in many cases a unit they were allocated could be a unit they had studied at undergraduate level, and they already had a good set of notes for the unit, which could form the basis of the material they could deliver to their students. In recent years, JKUAT has held some very good in-service training for their lecturers by faculties, where an enterprising lecturer from the University of Nairobi who is also a trained teacher, presented a very stimulating workshop, which generated a lot of good ideas for enhancing effective teaching at university level.

However, an observant employer will always look beyond the paper qualifications of an applicant for employment, since great importance will be attached to the personality and the character of the applicant. Two examples of employees who lacked such paper qualifications are given in the next two paragraphs.

In one high school, the headmaster was given a teacher of Chemistry with a BEd qualification, who did not appear to be confident in handling KACE Chemistry, and who eventually transferred to another school. A few years later, another teacher of Chemistry joined the school with a good Honours Degree in Chemistry and Biochemistry but NO qualification in Education. However, he was launched at once into teaching KACE Chemistry, which he did very well over the five years he spent at the high school.

A young man was recruited as a laboratory attendant in a university computer laboratory, holding only his KCSE certificate. However, once he had completed his other duties in the laboratory, he observed keenly the computer practicals, and the way the technicians helped students in these practicals, especially any exercises in word processing and spreadsheets. Later on, he was also able to assist the students when they needed help in practicals. He could even prepare difficult research papers, which included using the Microsoft Equation, very accurately for senior university staff.

There has been much discussion recently on the use of E-learning. However, currently this approach would be used most effectively for postgraduate programmes. JKUAT has pioneered this form of learning for one of its postgraduate programmes and there are plans to extend it to other postgraduate programmes. However, this form of learning may not be suitable for undergraduate students as yet. One factor is the poverty level of some students who may not be able to afford their own computer. The other factor is the presence of ‘malingerers’ in every class, who may not take their work seriously and present all kinds of excuses for not completing assignments and tests on the computer. In any case, when students have queries that even an e-learning program may not be programmed to answer; they need the face-to-face contact with the teacher, who can explain things personally.

For large classes, tutorials are already in place. These should provide the opportunity for students to ask the teacher about topics they do not understand. From experience, most of the questions are raised by the top performers, who have usually researched some difficult questions, which they have attempted but failed to solve completely. There is no doubt that these students benefit from discussing the hard questions in the tutorial. However, the weaker students, for whom the tutorial is a golden opportunity to grasp concepts they do not understand, either sit passively through the tutorial or absent themselves from the tutorial. Regrettably, you can take the cow to the water, but you cannot force her to drink!

Every semester, students are given evaluation forms for assessing the way each unit has been taught. However, it is questionable whether any meaningful assessment can be given, using the format of the current forms used in JKUAT. For each unit, students are asked to assess approximately 40 features of the teaching of the unit on a scale of 1 (poor) to 5 (excellent). With every student taking between 6 and 8 units in a semester, this assessment must be a real burden, and there will be the temptation just to choose a number from 1 to 5 at random, so as to enable the student to fill in the form as quickly as possible. Other institutions use a much simpler form of evaluation, with ten pertinent questions requiring an assessment of 1 to 5 on each, and space for general comments. If this were to be adopted at JKUAT, it would make the whole exercise more meaningful. In any case, 'malingerers' in a class are likely to under-rate the lecturer in a difficult unit, which the 'malingerer' does not understand, despite the lecturer doing his or her best to make the content as clear as possible. However, from experience, many lecturers have found that serious hard working students will often come to thank them for their presentations, as a mark of appreciation after the teaching of the unit has been completed.

The Way Forward

What further steps can be taken to motivate students to work hard and be serious with units of Mathematics and Statistics in a degree programme? Five points are given below, to conclude the paper.

Motivation by the lecturers who own the degree programme

The experience of lecturers in the Biological Sciences, who have constantly emphasized to students taking their programmes, that units in

Mathematics have to be taken seriously, should be followed by other departments. Indeed any department that owns a degree programme should ensure that the students appreciate the relevance of ALL units in the programme whether taught by the owning department, or serviced by another department.

Good parental guidance

Parents in a particular profession sometimes want to force all their children to follow them in the same profession. Unless the children have shown interest in that profession, this attitude is wrong. Parents should endeavour to find out what career really interests their children, and encourage the children to follow that career. Students who fail to reach the required standard at KCSE, but who qualify for a diploma or certificate programme in the line of their desired career, should be encouraged to apply for that programme. A good performance in a certificate may gain admission to a diploma programme, and a good performance in a diploma programme may gain admission to a degree programme.

School leavers need to think beyond money

Degree programmes in Commerce, Purchasing and Supply, Actuarial Science and Financial Engineering may attract applicants, who are simply looking for a way of getting rich quickly. Without a real interest in such a programme, because it attracts the applicants, the student may not be motivated to work really hard and may end up with what he/she calls a "boring job". School leavers need to follow a career that really interests them, rather than choose a career or Degree programme, which they think will land them in a very well paid job straight after graduation. Kenya needs doctors, nurses, engineers, teachers and hotel managers, just as much as if not more than, she needs accountants, bankers and financial experts.

Understanding, not merely memorizing

At degree level, all aspects of Mathematics and Statistics require understanding, not cramming. Even a piece of theory to be proved should be understood line by line, so that students can see the main line of attack but they should never attempt to memorize. Understanding of any topic is, of course, improved by a lot of practice!

Restoring KACE ('A' level) in the school syllabus

As mentioned earlier, weakness in first year Differential and Integral Calculus is a serious handicap. Students should be encouraged to do a lot of practice in first year problems in Calculus, especially in mastering all the fundamental techniques of integration. The former KACE Mathematics syllabus really stressed this, which is why KACE holders could follow lectures of a high mathematical content in a degree or diploma programme so well in the period before the nineties. Hence, the Ministry of Higher Education should seriously consider bringing back the former KACE for students who have completed KCSE and scored good grades in key subjects such as English, Mathematics and the Sciences.

Hon. Mwai Kibaki, the current President of Kenya, was quoted by the Daily Nation (one of the popular Kenya dailies) on 26/5/97, when he was the Leader of the then non-governing Democratic Party, as saying that there should be a total overhaul of the 8-4-4 education system and dismissed it as 'useless'. He was further quoted as saying that the syllabus 'tormented' students as their brains could not accommodate all the subjects, and asking Christians (he was attending an Education Sunday service) to pray for those entrusted with education matters to revive the old system of education.

Recently on 8/2/2011 in the same daily newspaper, Hon Joseph Kamotho, a current Kenyan MP and a former Minister for Education, writes in favour of not overhauling the 8-4-4 system but doing a SWOT analysis and recommends ways of eliminating the weaknesses and threats while building on the strengths and opportunities. However, the beauty of the former KACE was that the two years of intensive study trained students to do a lot of private study, and also helped them to mature and be more focussed on their preferred career. He mentions that the proposal to scrap KACE was made in 1976, on the argument that there were at that time very few places available for those who had completed Form 4. However, during his tenure as Minister of Education during the eighties, the number of schools offering places for KACE mushroomed, and there were more chances offered for students to compete for a university programme. At the same time, three new public universities were launched, together with a university college, hence more places available for the school leavers, who had performed well in KACE. Currently seven public and twenty-three private universities should be able to absorb many well-qualified students, whatever the system of education.

Mr David Aduda, an editor at the Nation Media Group, has also contributed to the debate in the Daily Nation of 15/2/2011, posing questions that a task force on education reform recently set-up, to be headed by Professor Douglas Odhiambo, needs to address. These include the questions of the kind of education that best suits Kenya in the twenty-first century, how to integrate technology into the learning process (with clear budget constraints likely to feature in the debate), making education accessible to all as enshrined in the August 2010 Constitution for Kenya, what governance and institutional structures are needed in education, and whether Kenyans have the political will to carry out educational reforms.

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Complex Issues of Technology Use in Undergraduate Mathematics

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The effective integration of technology into the teaching and learning of mathematics remains one of the critical challenges facing contemporary tertiary mathematics. This paper reports on some significant findings of a wider study investigating the use of technology in undergraduate mathematics. It proffers a means of examining the complex issues confronting those wishing to implement and sustain integrated technology in undergraduate mathematics. Key findings include: the importance of mandating technology use in official departmental policy; paying attention to consistency and fairness in assessment; re-evaluating the value of topics in the curriculum; re-establishing the goals of undergraduate courses; and developing the pedagogical technical knowledge of teaching staff.

Introduction

Despite many years of technological innovation, the effects of technology on the mathematics curriculum remain limited. Arnold, for example, notes in 2004 that:

> Even today, there remain widespread reservations regarding not only the best ways to incorporate such tools into teaching and learning, but fundamental questions regarding their appropriateness…The questions which practitioners have been asking (What will be left to teach if students have access to tools which factorise, solve, and do calculus? What about their manipulative skills? What will we ask them to do in examinations?) were the same questions asked a few years ago regarding graphing calculators. In fact, they were precisely the same questions asked twenty years ago regarding student access to traditional calculators (p. 21).

Oates (2009a) describes a longitudinal investigation of technology use in undergraduate mathematics courses at The University of Auckland, which sought to identify ways in which effective technology implementation may be facilitated. This study was accompanied by two international surveys of technology use in undergraduate mathematics, used to identify the essential features of an *Integrated Technology Mathematics Curriculum* (ITMC). The taxonomy developed from these surveys and the literature may then be used as a means of characterising and comparing technology use in and between courses. The first exploratory survey of technology use in undergraduate courses in 2004 elicited responses from 31 mathematics educators, representing 21 different tertiary institutions from five countries. Issues identified in this study were then related to the literature, to inform the development of the second survey. Thomas and Holton (2003) identify a wide range of such factors associated with the use of technology at the tertiary level, for example student instrumentation (Artigue, 2002; Stewart, Thomas and Hannah, 2005); the affordances, constraints and obstacles encountered with different technologies (Brown, Stillman & Herbert, 2004); research mathematicians' beliefs about mathematical knowledge, technology and pedagogy, and the effects of these beliefs on both their own, and their students' use of technology (Keynes & Olson, 2001; Kendal & Stacey, 2001; Kersaint, Horton, Stohl & Garofalo, 2003); changes to the relative epistemic, pedagogical and pragmatic values of curriculum topics when using computer algebra systems (CAS) (Artigue, 2002; Stacey 2003); assessment issues such as curricular congruency and the nature of questions asked in formal assessments (Leigh-Lancaster, 2000; Flynn & McCrae 2001); and the relationship between mathematicians' experience with different technologies in their research domains and their pedagogical technology knowledge (PTK). PTK is characterised as the necessary knowledge of the principles and techniques required to teach mathematics using a given technology (Hong & Thomas, 2006).

This paper provides a brief summary of the development of the taxonomy of integrated technology, and presents a strategy developed to describe and compare technology integration for different courses and institutions using this model (Oates, 2009a). It then examines one particular sub-category of the taxonomy more closely, namely content issues, within the *Mathematical Factors* component of the taxonomy (Oates, 2009b).

A Model for Integrated Technology

A full description of the overall methodology of this study, including the construction and administration of the second survey used to develop the taxonomy of integrated technology, is provided by Oates (2009a). Respondents to this latter survey represented 72 different undergraduate courses, from 31 tertiary institutions in 8 countries (Australia, Canada, France, New Zealand, South Africa, United Kingdom, United States, and Uruguay). Responses to the survey were coded as described in Oates (2009a), with four survey respondents being asked to review the subsequent classification of their responses, as a check on the degree to which the assigned categories agreed with their intentions.

Table 1 below summarises the six main components of the taxonomy, with sample responses from the survey to illustrate the focus for each component. The full taxonomy (Oates, 2009a, pp. 205-206) describes a more extensive, complex range of inter-related factors that should be considered within each of the components in Table 1, along with studies from the literature that reference each. The greater complexity of the taxonomy is illustrated in Table 2, which lists sub-category details for the *Mathematical Factors* component examined later in this discussion.

Table 1. A Taxonomy for Integrated Technology

Taxonomy Component	Characteristic Survey Response for Taxonomy Component
Access	"It has many benefits if all the students can reach almost the same technology; otherwise it creates important differences between them. I would like to see all my students using laptops, as in the private universities." (Uruguay)
Assessment	"Students may use any hand held calculator, but in exams they must show full written working to reach the answer. Calculators are often used to check results". (Australia)
Organisational Factors	"Bureaucracy slow to change. Use often isolated to single course." (South Africa)
Mathematical Factors	"Less emphasis on techniques, more powerful visualisation." (New Zealand)
Staff Factors	"Technology should be integrated only by staff who believe it is useful. Imposition of technology seems to have a negative effect on all involved." (Australia)
Student Factors	"It's difficult (for students) to make sense of the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology." (Canada)

Despite the complexity and inherent inter-dependence of the taxonomy elements, Oates (2009a) describes a strategy using radar diagrams, which provides an effective visual comparison of overall technology integration, useful for example in bench-marking between courses, documenting the progression of technology integration over time, and a means to identify specific areas within the curriculum where attention to technology issues may be needed (Oates 2009a p. 207). An example using this strategy, for technology use in one particular first-year Calculus course in the United States, is shown in Figure 1. The extent of technology integration for each radial on the diagram is measured on a 5-point scale (low to high), estimated by the researcher from the responses to the appropriate survey questions as described in Oates (2009a). This diagram suggests there is a lack of organisational support for technology in this otherwise integrated-technology course. In his survey responses, the teacher of this course described his frustration at the wider department's lack of interest in technology, noting that "since I am almost the only instructor in our department who uses technology consistently, I have to use the same book as is used in other sections of the course" (Oates, 2009a, p. 207). Other radials also fall short of a fully integrated value of five due to such effects, for example Assessment, since the students in this course are required to sit the same examination as other students in the department, without access to their usual technology.

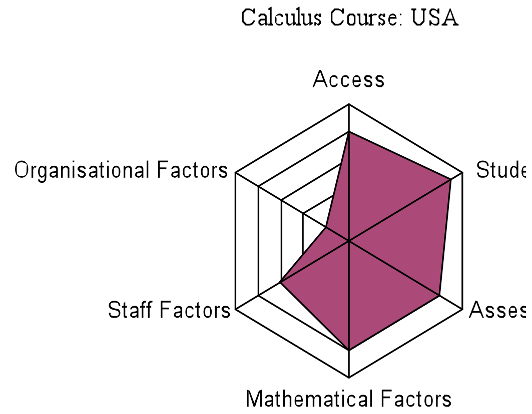

Figure 1. First-year Calculus course from the United States.

Possible causes for the technology position that underlies the under-represented areas may be seen later in the same survey response:

> The department has discussed the possibility of introducing technology in its pre-calculus courses, although the last official discussion was about 10 years ago. The final decision has always been to maintain the status quo; i.e. no technology. The two reasons which seem to carry weight are: (a) our pre-calculus courses are precisely that: preparation for calculus; and students need to improve their (pencil-and-paper) algebraic skills in order to succeed at calculus; and (b) purchase of technology, such as a graphing calculator or student copy of Maple, would be an additional financial burden on many of our students (Oates, 2009a, p. 207].

With respect to a closer examination of course content, Table 2 describes four main issues with respect to technology and mathematical knowledge and learning, listed as sub-categories of the *Mathematical Factors* component of the taxonomy.

However, as suggested earlier, the full taxonomy provides considerably more detail for each sub-category, than it is possible to reproduce in this table, or consider in detail in these discussions. For each sub-category, the taxonomy provides several indicative responses from the survey and a sizeable representative list of associated studies from the literature, to support each component, representing such issues as PTK discussed in the introduction (Hong & Thomas, 2006), and "*IA, MK* and APOS theory" as listed in Table 2 below (Oates 2009a). So while it might seem issues such as didactical factors affecting both staff and students are missing from the categories depicted in Table 2, they are considered elsewhere in the study (see e.g. *Staff Factors* and *Student Factors*, Oates 2009a). One of the more significant overall findings of this study lay in the interdependency observed between the complex elements detailed in the taxonomy. The results suggest that it is essential to recognise the inter-related structure of the taxonomy, and that addressing the factors in a comprehensive fashion leads to higher and more sustainable levels of technology integration. While attendance to some elements in isolation, as in the course depicted in Figure 1, may obviously stimulate change, it is difficult to achieve effective, sustainable technology integration through such a piecemeal approach (Oates, 2009a, p. 252).

Table 2. Sub-Categories of the Mathematical Factors Component

1.	Content: Order and value of topics; Subject Imperatives: e.g. for Algebra, the availability and accepted use of CAS & symbolic manipulation tools; Differing technology demands of Pure, Applied, and Service courses; Staff research domain-specific technology;
2.	Cognition, Reasoning and Skills: Technical versus Conceptual understanding, *IA* (Instrumental Ability) versus *MK* (Mathematical Knowledge;
3.	Representational Versatility; APOS theory; Objects & Procepts; Technology Design Limitations;
4.	Mathematical Knowledge: Nature of Mathematics, Objectives and Goals; Needs of users versus learners.

Content Value

While the order and value of curriculum topics is included in the taxonomy under *Mathematical Factors*, as shown in Table 2, arguments about the worth of particular topics are also closely associated with elements under *Staff Factors,* such as the beliefs of staff about the nature of mathematics, technology and learning; the effects of these beliefs on their teaching and students learning; and the relationship between their specialist research domains and these beliefs (Keynes & Olson, 2001; Kendal & Stacey, 2001; Kersaint, Horton, Stohl & Garofalo, 2003). Harman (2003, p. 93), for example, questions the traditional approach of teaching anti-differentiation first when introducing the fundamental theorem of Calculus. The advanced numerical methods and graphing packages now readily accessible to many students mean that we can address concrete *summative* problems of integral calculus first, following on with the more abstract notions of rates of change and the fundamental theorem later. With respect to the effect of technology on content value, the following issues may be considered:

- (Some) content areas may be trivialised or made redundant by the use of the graphics calculator. This could include for example some routine algebraic skills.
- New content areas or richer conceptual understanding becomes accessible using the graphics calculator.
- The possibility (exists) that some of the new content areas opened up by the graphics calculator may be trivial in nature and of limited educational value.

Artigue (2002) and Stacey (2003) suggest a means of examining such aspects, developing a framework that considers the pragmatic, epistemic and pedagogical value of individual topics. The pragmatic value recognises the usefulness of a topic, the epistemic value measures the importance of a topic's place in the structure and development of mathematical knowledge, and pedagogical value considers whether a topic serves a purpose not related to the content itself, such as providing an opportunity to practice skills (Stacey, 2003, p. 6). The use of CAS may considerably change the relative values of a topic, often reducing the pragmatic value, and sometimes questioning the epistemic value. For example, using calculus to make a linear approximation to a function with the formula $f(x+h) \approx f(x) + h.f'(x)$ once had a pragmatic value in permitting ready approximation to the values of complicated functions, but even a simple scientific calculator removes the pragmatic value of this formula. However, Stacey (2003) believes it still has immense epistemic value, because it is central to the principles underlying calculus:

> The curriculum value of topics is markedly changed by the introduction of CAS. Old justifications for teaching topics, especially pragmatic justifications, will not necessarily apply...The educational community needs to build up sophisticated rationales for curriculum areas that were not debated in the past. Justifications may be on pragmatic, epistemic, or pedagogical grounds. (p. 7)

Assessing the relative values of particular topics is a complex task, even for a researcher of Stacey's considerable experience, as illustrated in several examples of the study at The University of Auckland. Diminished pragmatic value can be perceived in the removal of teaching techniques for solving differential equations from the syllabus of one of the applied mathematics courses:

> Before computers, there used to be a big emphasis on special techniques for special differential equations, …students had to recognise some 15 different types of differential equation, you had no options, you had to solve it

> explicitly, there was no numerical option. You had to know the technique, all that's gone, if you don't recognise a differential equation, you whack it on a computer (Oates, 2009b, p. 423).

The value of technology is seen here not just in its computational capabilities, but also in the opportunity it allows to investigate problems and develop flexible solving strategies, as opposed to learning a catalogue of instrumental techniques. The effect of technology on calculating eigenvalues and eigenvectors is seen as less clear by another respondent. While she concedes that some pragmatic value has definitely been lost with the ability of technology to compute these directly, she still perceives some pragmatic and epistemic value in teaching these procedures. "Matlab does not easily find families of eigenvectors, and often presents the results of complex eigenvectors in an unusual structure that requires considerable understanding from students to become recognisable" (Oates, 2009b, p. 424). Hence, while students are encouraged to use technology for such calculations in tutorials and assignments, manual procedures are still taught and examined.

Gaussian elimination provides another contentious topic for evaluating relative values. Hillel (2001) suggests that there is a clear epistemic value in the process of Gaussian elimination, but he also considers there may be a reduction in the pragmatic value when he observes that:

> Grasping the relation between elementary row operations and equivalent systems is the key notion, not the actual procedure for row-reducing matrices. Once understood, I see no reason why students should not be given free rein to use CAS and go directly to row reduced echelon form of a matrix without actually performing the row operations (and later, to go directly from a system of equations to a CAS-generated solution). (p. 374)

All interview respondents in the observational study agreed there is a reduction in the pragmatic value, but there was considerable argument about the continued value of Gaussian elimination from an epistemic or pedagogical perspective. While one respondent believes that the "black-box that provides a solution to a set of linear equations is probably all most people need", he also sees little value in technology for the more important role of developing critical thinking and effective problem-solving strategies:

> Depends what one wants, I can't see how a student can understand the process by pushing a button, it may be OK for an engineer who just needs a seriously good program to provide the numbers at the end, they don't need to know anything about Gaussian elimination, …but most students haven't got a clue what their answer means, they know nothing more about their solutions than that they are a result of what they do (Oates, 2009b, p. 425).

This respondent sees little pragmatic value for Gaussian elimination in such an environment, regardless of whether technology is used. Students who have learnt to row-reduce flawlessly without technology are frequently no better off than those who perform the operations using a calculator, so technology is seen as having no significant learning advantage other than to save time, check on working or avoid arithmetic errors. Another respondent observed with respect to his marking of examination questions on Gaussian elimination that:

> The main problem doesn't seem to be that they can't do the operations (for which the calculator can help them); it's that they don't know what operations to do. They'll do three pages of working and still won't have any zeros in their matrix…the students don't understand what the goal is. I'm not sure how technology can help with this (Oates, 2009b, p. 425).

Here, the respondent clearly sees an epistemic value in Gaussian elimination, but he questions whether technology can help students achieve such a goal. The examples presented here demonstrate that reaching consensus on exactly which elements of a particular topic may retain their pragmatic or epistemic value and which may be assigned to technology, is a complex and challenging task. It appears that decisions about the relative value of a given topic often depend on who is making them.

Summary

These discussions have highlighted the complexity of issues confronting those wishing to integrate technology into the undergraduate mathematics curriculum. In particular, notwithstanding the holistic appreciation of the taxonomy that is advocated, several issues were identified as having a significant impact on the technology implementation at The University of Auckland. Content issues were singled out for particular attention here, as they were seen as one area requiring attention for the effective and sustainable integration of technology. The complexity of the task of assessing the relative values of topics as demonstrated supports the conclusion that a re-examination of the changing pragmatic and epistemic values of specific topics, and the goals of mathematics education, within a rapidly evolving technological environment, remains a pressing challenge for undergraduate mathematics educators.

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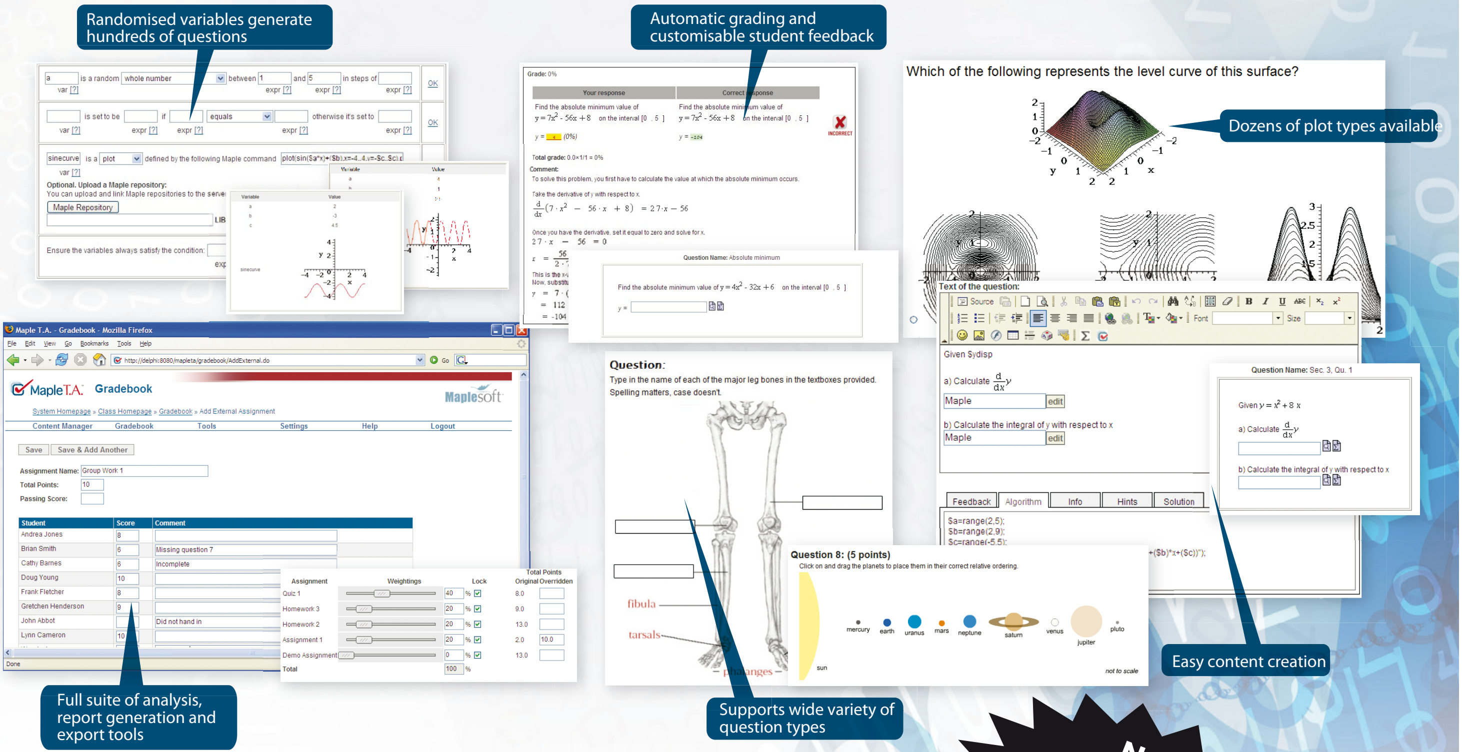

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Programme

Sunday 27th November:

Registration
Opening Ceremony & Opening Address
Welcome Reception

Monday 28th November:

Keynote Address
Parallel Sessions
Happy Hour
Local Restaurant Night

Tuesday 29th November:

Teachers' Day
Keynote Address by John Mason
Parallel Sessions
New Delegates Reception
Optional Trip to Rainbow Springs & Gondola/Luge

Wednesday 30th November

Conference Excursion. Wai-O-Tapu thermal area, with options for Waimangu Valley, Hell's Gate & Mt Tarawera (Details to be posted soon on to the website)

Thursday 1st December

Keynote Address
Parallel Sessions
Conference Dinner at the Historic Blue Baths

Friday 2nd December

Keynote Address
Parallel Sessions
Closing Ceremony and Lunch

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